STRESSES

IN WIRE-WRAPPED GUNS AND IN GUN CARRIAGES

BY

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PREFACE TO SECOND EDITION.

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PREFACE.

In this text the author has endeavored to explain and illustrate a number of the important engineering principles underlying the design of wire-wrapped guns and of gun carriages, and in its preparation he has made free use of the methods of officers of the Ordnance Department, U. S. Army, who have been engaged on such work and of the various publications on the subject written by them or translated by them from foreign sources.

The deductions of the formulas relating to wire-wrapped guns and their application to a 12-inch gun on which the wire is wrapped under constant tension have been taken from *Notes on the Construction of Ordnance Nos.* 38 *and* 87 written by General William Crozier, Chief of Ordnance, U. S. Army, when a junior officer of the Ordnance Department, the formulas giving the tensions in the wire envelope and the pressures produced by it being, as stated by General Crozier, mainly those of Longridge, an English engineer. Some shortening of the mathematical work involved in the deductions of the formulas has been effected by the author of this text by starting with the assumption that the modulus of elasticity is the same for the steel wire as for the steel tube of the gun, and the formulas have been slightly extended to include the radial stresses in the wire envelope.

In the preparation of Chapter II much assistance was derived from the ordnance pamphlet entitled *A Discussion of the Methods Proposed to Increase the Rapidity of Fire of Field Guns* by Captain (now Lieutenant Colonel) Charles B. Wheeler and Captain (now Major) William H. Tschappat, Ordnance Department; and from the original calculations made in the Office of the Chief of Ordnance in connection with the design of the 3-inch field carriage, model of 1902, and the 5-inch barbette carriage, model of 1903.

Much assistance was likewise derived in the preparation of

Chapter III from the original calculations made in the Office of the Chief of Ordnance in connection with the design of the 6-inch disappearing carriage, model of 1905 MI. The computations relating to the throttling grooves of this carriage given in the text (which are the same in principle as those made for the throttling grooves of a number of earlier models of disappearing carriages) are practically identical with those made for this carriage in the Office of the Chief of Ordnance by Captain James B. Dillard, Ordnance Department, acting under the direction of Major John H. Rice, Ordnance Department, the present chief of the gun-carriage division of that office.

The sources of the formulas used in Chapter IV are given in the text.

In the preparation of Chapter V the author has freely consulted various standard works on applied mechanics and mechanical engineering, the greatest assistance having been derived from the works of the International Library of Technology.

The methods outlined in Chapter VI are largely based upon the practice of the Ordnance Department.

The thanks of the author are due to Major Tracy C. Dickson, Ordnance Department, for advice and information in connection with the preparation of the text; and to Captain Otho V. Kean, Ordnance Department, 1st Lieutenant Ned B. Rehkopf, 2nd Field Artillery, and 1st Lieutenant George R. Allin, 6th Field Artillery, instructors in the Department of Ordnance and Science of Gunnery, U. S. Military Academy, for suggestions tending to add to the clearness of the text, for checking and correcting where necessary the results of the computations, and for reading the proofs.

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CHAPTER I.

ELASTIC STRENGTH OF WIRE-WRAPPED GUNS.

1. General Construction. — Wire-wrapped guns consist of (a) an inner steel tube which forms the support on which the wire is wrapped and in which the rifling grooves are cut; (b) the layers of wire wrapped upon the tube to increase its resistance by the application of an exterior pressure as well as to add to the strength of the structure by their own resistance to extension under fire; and (c) one or more layers consisting of a steel jacket and hoops placed over the wire with or without shrinkage. The jacket is generally relied upon to furnish the longitudinal strength of the gun since this feature is lacking in the wire envelope. The breech-block is therefore ordinarily screwed into the jacket, or into a breech bushing screwed into the jacket, in order that the latter may resist the longitudinal stress due to the pressure at the bottom of the bore, the consequent rearward acceleration of the gun in recoil, and the resistance to recoil of the recoil brake applied to the gun through the trunnions or through a lug forming part of the jacket.

2. An Important Principle. — A very important principle in wire gun construction is that enunciated in paragraph 121, page 216, Lissak's Ordnance and Gunnery, to the effect that the stresses produced by any pressure applied to a compound cylinder are exactly the same as would be produced by the same pressure applied to a single cylinder of the same dimensions. This refers to the stresses and strains produced by the pressure under consideration only, and if before the application of this pressure there existed in the compound cylinder stresses and strains produced by shrinkage or otherwise, the resulting stresses are the algebraic sum of those previously existing and those induced by the application of the pressure. It follows from this that if we know the tangential and radial stresses and strains existing at any radius r in a compound cylinder before the application of an interior or exterior pressure, the resultant tangential and radial stresses and strains therein at any radius r when an interior or exterior pressure or both are acting can be readily obtained by adding algebraically to those previously existing the stresses and strains computed from the appropriate one of equations (9) and (10), paragraph 104, page 195, Lissak's Ordnance and Gunnery, which are as follows:

$$El_t = S_t = \frac{2}{3}\,\frac{P_0R_0^2 - P_1R_1^2}{R_1^2 - R_0^2} + \frac{4}{3}\,\frac{R_0R_1^2\,(P_0 - P_1)}{R_1^2 - R_0^2}\,1/r^2 \qquad *(1)$$

$$El_p = S_p = \frac{2}{3}\,\frac{P_0R_0^2 - P_1R_1^2}{R_1^2 - R_0^2} - \frac{4}{3}\,\frac{R_0^2R_1^2\,(P_0 - P_1)}{R_1^2 - R_0^2}\,1/r^2 \qquad (2)$$

in which R_0 and R_1 are the inner and outer radii, respectively, of the compound cylinder and r is the radius of any point within the wall of the cylinder. If the compound cylinder is composed of any number of layers n and its radii are designated as R_0, R_1, R_2, . . . R_n, R_0 in formulas (1) and (2) will be the R_0 of the compound cylinder but the R_1 of these formulas will be the R_n of the compound cylinder. r in the formulas may have any value between R_0 and R_n of the compound cylinder.

Similarly if we know the resultant tangential and radial stresses and strains at any radius r in a compound cylinder when acted upon by an interior or exterior pressure, or by both, the tangential and radial stresses and strains that will remain at any radius r when these pressures are removed can be readily obtained by subtracting algebraically from the resultant stresses and strains those computed from the appropriate one of equations (1) and (2) as due to the pressures that have been removed.

3. Difference Between Tangential Tension (or Tension Simply) and Tangential Stress and Strain; and Between Radial Pressure and Radial Stress and Strain. — Equations (7) and (8), page 194, Lissak's Ordnance and Gunnery, which are as follows:

* The equations in this discussion will be renumbered consecutively as used regardless of their number in Lissak's Ordnance and Gunnery.

$$t = \frac{P_0R_0^2 - P_1R_1^2}{R_1^2 - R_0^2} + \frac{R_0^2R_1^2(P_0 - P_1)}{R_1^2 - R_0^2} 1/r^2 \qquad (3)$$

$$p = -\frac{P_0R_0^2 - P_1R_1^2}{R_1^2 - R_0^2} + \frac{R_0^2R_1^2(P_0 - P_1)}{R_1^2 - R_0^2} 1/r^2 \qquad (4)$$

differ from equations (1) and (2) in that they give the tangential tension t and the radial pressure p at any radius r produced by the application of interior and exterior pressures to a cylinder whose inner and outer radii are R_0 and R_1, respectively. In the deduction of these equations tensile forces and tensile strains have been considered positive and compressive forces and compressive strains negative. The radial pressure in the walls of a gun is always a compressive force. The difference between the tension and the tangential stress and strain, and between the radial pressure and the radial stress and strain should be carefully noted. The tangential strain l_t is due both to the tangential tension (or compression) and the radial pressure and is equal to

$$1/E\,[t - (-p/3)] \quad \text{or} \quad 1/E\,(t + p/3)$$

obtained from the first of equations (4), page 192, Lissak's Ordnance and Gunnery, by making $q = 0$; and the tangential stress El_t is equal to this strain multiplied by the modulus of elasticity E, or to $(t + p/3)$. If the tension and the pressure have different signs, that is, if one is a tensile and the other a compressive force, the tangential stress will be greater numerically than the tension by one-third of the radial pressure; but if they have like signs, that is, if both are tensile or compressive forces the tangential stress will be equal numerically to the difference between the tangential tension and one-third of the radial pressure. Similarly the radial strain l_p is due both to the tangential tension and the radial pressure and is equal to

$$1/E\,(-p - t/3) = -1/E\,(p + t/3)$$

obtained from the second of equations (4), page 192, Lissak's Ordnance and Gunnery, by making $q = 0$; and the radial stress El_p is equal to this strain multiplied by the modulus of elasticity E, or to $-(p + t/3)$. If the tension and the pressure have different signs the radial stress will be numerically greater than the radial pressure by one-third of the tangential tension; but if they have like signs the radial stress will be equal numerically to the difference between the radial pressure and one-third of the tangential tension.

4. The Elastic Strength of a Gun is Reached when Either El_t or El_p is Equal to the Elastic Limit of the Material. — Tension and Pressure at any Radius r of a Compound Cylinder, System in Action or at Rest. — While as a matter of fact, neglecting the longitudinal force, the only forces acting on a particle in the walls of a gun are the tangential tension and the radial pressure, the strain produced by these forces acting together is, in the tangential or radial direction, the same as would be produced by a force El_t or El_p, respectively, acting alone. It is generally accepted that if the material is subjected to a compressive or tensile strain equal to that occurring at the elastic limit when a single force is applied in the direction of the strain, it will yield no matter how the strain may be produced; and therefore the stresses El_t and El_p corresponding to the strains l_t and l_p instead of the tangential tension and the radial pressure are considered as determining whether or not the metal of the gun is being worked within the elastic limit.

As in the case of the stresses and strains, the tensions and pressures produced by any interior or exterior pressure or both applied to a compound cylinder are exactly the same as would be produced by the same pressure or pressures applied to a single cylinder of the same dimensions, and if we know the tension and pressure at any radius r of the compound cylinder before the application of the pressures, the resultant tension and pressure at that radius when the pressures are acting may be determined by adding algebraically to those previously existing, those due only to these pressures, computed from equations (3) and (4). Also if we know the resultant tension and pressure which exist at any radius r of a compound cylinder when interior or exterior pressures are acting, those which will remain at the radius r when the interior or exterior pressures are removed can be obtained by subtracting algebraically from the resultant tension and pressure those computed by equations (3) and (4) as due only to the pressures which have been removed.

5. The Elastic Strength of a Compound Cylinder, Properly Assembled to Secure the Maximum Resistance to an Interior Pressure, Depends only on the Sum of the Elastic Limits for

Compression and Tension of the Material of the Tube and the Thickness of the Wall in Calibers. — Compression of the Tube, System at Rest, Beyond its Elastic Limit. — It follows from the above discussion that if a compound cylinder has been so assembled as to compress the inner surface of the inner cylinder to the elastic limit ρ for that cylinder, the elastic strength of the compound cylinder can be determined from equation (1) by finding that interior pressure which acting alone on the compound cylinder will cause a stress $\rho + \theta$ on its interior surface, which stress will overcome the initial compression ρ and cause a tangential extension equal to θ, the elastic limit for tension of the material.

Making $P_1 = P_n = 0$, $R_1 = R_n$, $r = R_0$, and $S_t = \rho + \theta$ in equation (1) and solving for P_0, we have

$$P_0 = \frac{3(R_n^2 - R_0^2)(\rho + \theta)}{4R_n^2 + 2R_0^2} \qquad (5)$$

which gives the maximum interior pressure which the compound cylinder can withstand without exceeding the elastic limit for tension of the inner cylinder. In this discussion it is assumed that the compound cylinder whether wire-wrapped or of the built-up construction has been properly assembled, in which case the stresses in the layers outside the inner cylinder will not exceed the elastic limit whether in the state of rest or action. As shown in paragraph 121, page 217, Lissak's Ordnance and Gunnery, the greatest value of P_0, corresponding to $R_n = \infty$, is $.75(\rho + \theta)$. Making $R_n = 3R_0$, corresponding to a thickness of wall of one caliber,

$$P_0 = .63(\rho + \theta)$$

and, therefore, comparatively little advantage results from increasing the thickness of wall beyond one caliber. On the other hand, whatever be the mode of assembling the compound cylinder, by wrapping the tube with wire or otherwise, the elastic strength of the cylinder, if properly assembled to secure the maximum resistance to an interior pressure, will depend only on the sum of the elastic limits for compression and tension of the tube and the thickness of the wall in calibers.

Many designers of wire-wrapped guns have compressed the tube in the state of rest beyond the elastic limit for tangential compression, and on the assumption that the elastic limit for tension has not been lowered thereby, have computed the elastic strength of the gun from equation (1), substituting for ρ the value of the actual stress of compression of the bore of the tube. Many experiments, however, indicate that if the metal is compressed beyond the elastic limit, its elastic limit for tension is lowered, and vice versa. Therefore, if, when such a gun is fired, the stress at the bore is raised to the elastic limit for tension θ as determined in the testing machine, it will in reality have passed the actual elastic limit, due to the over-compression at rest. As a result the tube will be worked beyond its elastic limit both in tension and compression. Such treatment is known to be most injurious if the repetitions of stress are sufficiently numerous, but owing to the comparatively limited number of rounds fired in any gun no trouble has so far resulted in wire-wrapped guns from over-compression of the tube.

6. Two Principal Methods of Wrapping Wire on a Gun Tube. Special Formulas Relating to Layers of Wire Wrapped on a Gun Tube. — Let Fig. 1 represent a section of a wire-wrapped gun

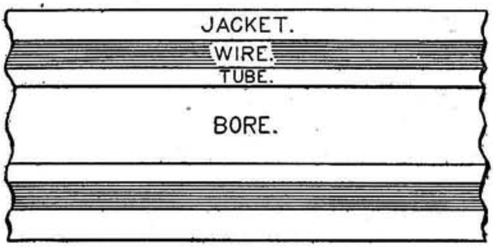

Fig. 1.

consisting of tube, wire envelope, and jacket. Represent the inner and outer radii of the tube by R_0 and R_1, respectively, the radius of the outer surface of the wire envelope by R_2 and that of the outer surface of the jacket by R_3. When wire is wrapped under tension around a tube the effect is to cause a pressure on its exterior surface which compresses the metal tangentially, the greatest stress occurring as has been shown (paragraph 108, page 200, Lissak's Ordnance and Gunnery) at the bore. The pressure on the tube and the resulting compression in a tangential direction increase with the number of layers of wire applied. As each layer of wire is applied its tension is that of wrapping, but this is gradually reduced by the compression of this layer due to the pressure on its surface caused by the application of the succeeding layers of wire. There are two principal methods of wrapping wire on a tube; one is to wrap the wire at constant tension and the other is to wrap it at such varying tension that, when the gun is fired with the prescribed pressure, all the layers of wire shall be subjected to the same tangential stress. The latter method is theoretically the better, but owing to the greater convenience of wrapping the wire at constant tension and to its great elastic strength, which permits the tube to be compressed to its elastic limit by wrapping wire thereon at constant tension without causing the stress in the wire to approach too close to its elastic limit when the gun is fired, the former method is generally used. Formulas have been deduced giving

(*a*) the uniform tension T at which an envelope of wire of thickness equal to $R_2 - R_1$ must be wrapped on a tube of radii R_0 and R_1 to produce a pressure P_1' on its exterior.

(*b*) the pressure at any radius r of the envelope of wire when the gun is in the state of rest due to the wrapping thereon of the outer layers of wire.

(*c*) the resultant tension at any radius r of the envelope of wire due to the tension of wrapping and to the compression caused by the wrapping of the layers from that radius out.

(*d*) the tangential and radial stresses and strains at any radius r of the wire envelope after the wrapping has been completed.

(*e*) the uniform tangential stress $El_t = S_t$ which must occur in all the layers of a wire envelope of thickness equal to $R_2 - R_1$ to produce a pressure P_1 on the exterior of a tube of radii R_0 and R_1 when the gun is fired.

(*f*) the variable tension of winding which must be used in order that when the gun is fired all the layers of wire will be subjected to a uniform tangential stress S_t.

(*g*) the pressure, the tension, and the radial stress and strain which exist at any radius r of the wire envelope when the gun is fired.

The formulas referred to under (*a*) to (*d*), inclusive, pertain only to the case where the wire is wrapped under constant tension and relate exclusively to the gun in the state of rest. Those referred to under (*e*) to (*g*), inclusive, pertain only to the case where the wire is wrapped at such varying tension that when the gun is fired with the prescribed powder pressure all layers of wire shall be subjected to the same tangential stress; and they relate exclusively to the gun in the state of action.

CASE I. WIRE WRAPPED UNDER CONSTANT TENSION.

7. General Method of Design. — Suppose it is desired to construct a wire-wrapped gun as shown in Fig. 1, in such manner as to give to it the maximum elastic strength permitted by the quality of the metal in the tube and the total thickness of wall $R_3 - R_0$, with the condition that the wire shall be wrapped on the tube under constant tension. The first thing to determine is the maximum interior pressure which the gun can support in action on the assumption that in the state of rest the inner surface of the tube is compressed to its elastic limit. This is given by equation (5). The exterior pressure on the tube which will cause its interior surface to be compressed to the elastic limit when in the state of rest is then determined from equation (29), page 206, Lissak's Ordnance and Gunnery, which, after replacing a by its value R_1^2/R_0^2 is as follows:

$$P_{1_\rho} = \frac{R_1^2 - R_0^2}{2\,R_1^2}\,\rho \qquad (6)$$

Part of this pressure P_{1_ρ} will be due to the wire envelope and part to the shrinkage of the jacket on the wire. If the jacket is assembled without shrinkage, as is sometimes the case, all of the pressure P_{1_ρ} will be due to the pressure exerted by the wire envelope. Supposing the jacket assembled with shrinkage, the amount of the shrinkage must be determined and from that the pressure on the exterior of the tube due to such shrinkage. Subtracting this pressure from P_{1_ρ} there results the pressure which must be exerted by the wire envelope. To utilize the full strength of the jacket its inner surface must be extended in the state of action to the elastic limit. Part of this extension will be due to the action of P_0 and the remainder to the shrinkage. The part due to P_0 is obtained from equation (1) by making $P_1 = 0$, $R_1 = R_3$, and $r = R_2$. The difference between this part and the tensile elastic limit θ_3 is due to the shrinkage. Calling this difference S'_{t3}, the pressure P'_2 to produce it is obtained from equation (1) by making $P_1 = 0$, $S_t = S'_{t3}$, $P_0 = P'_2$, $R_1 = R_3$, $R_0 = R_2$, and $r = R_2$, and solving for P'_2, whence

$$P'_2 = \frac{3\,(R_3^2 - R_2^2)}{4\,R_3^2 + 2\,R_2^2} S'_{t3} \qquad (7)$$

The absolute shrinkage to produce this pressure is given by equation (54), page 218, Lissak's Ordnance and Gunnery, which, after replacement of the radius ratios by their values in terms of radii and making $R_1 = R_2$, $R_2 = R_3$, $P_{1s} = P'_2$, and $S_1 = S_2$, becomes

$$S_2 = \frac{4\,R_2^3\,(R_3^2 - R_0^2)\,P'_2}{E\,(R_2^2 - R_0^2)\,(R_3^2 - R_2^2)} \qquad (8)$$

The pressure P'_2 due to the shrinkage of the jacket on the wire envelope causes a pressure p''_1 on the exterior of the tube which is obtained from equation (4) by making $P_0 = 0$, $P_1 = P'_2$, $R_1 = R_2$, and $r = R_1$.

Whence

$$p''_1 = \frac{P'_2 R_2^2}{R_2^2 - R_0^2}\left(1 - \frac{R_0^2}{R_1^2}\right) \qquad (9)$$

The pressure p'_1 to be produced by the wire envelope is, therefore,

$$p'_1 = P_{1_\rho} - p''_1$$

8. Intensity of the Constant Tension of Wrapping Necessary to Produce a Given Pressure on the Exterior of the Tube, System at Rest.—It now remains to deduce the formulas referred to under (*a*) to (*d*), inclusive, article 6. To do this let us suppose the wrapping of the wire on the tube, Fig. 1, to be finished to a radius r producing a pressure on the exterior of the tube p_t and then to be continued to any other radius r', the pressure on the exterior of the tube changing to p'_t; the additional wrapping will also produce a pressure p_r on the wire at r. The pressure p_r on the wire at r then produces a pressure $p'_t - p_t$ on the tube. Applying equation (4), making $p = p'_t - p_t$, $P_0 = 0$, $P_1 = p_r$, $r = R_1$, $R_1 = r$, we obtain

$$p'_t - p_t = \frac{p_r r^2}{r^2 - R_0^2} - \frac{p_r r^2 R_0^2}{(r^2 - R_0^2)\,R_1^2} \qquad (10)$$

which, by reduction becomes,

$$p'_t - p_t = \frac{(R_1^2 - R_0^2)\, r^2 p_r}{R_1^2\,(r^2 - R_0^2)} \tag{11}$$

Let τ be the mean tension of the wires between r and r', then $p_r \times 2\,r = \tau \times 2\,(r' - r)$ whence

$$p_r = \frac{r' - r}{r}\tau$$

Substituting this value for p_r in equation (11) and dividing by $r' - r$, we get

$$\frac{p'_t - p_t}{r' - r} = \frac{(R_1^2 - R_0^2)\, r\tau}{R_1^2\,(r^2 - R_0^2)} \tag{12}$$

Passing to the limit of both members, we have, since τ then becomes the uniform tension of wrapping T,

$$dp_t/dr = \frac{(R_1^2 - R_0^2)\, rT}{R_1^2\,(r^2 - R_0^2)} \tag{13}$$

Multiplying by dr and integrating between the limits r and R_2

$$p'_{tR_2} = \frac{R_1^2 - R_0^2}{2\,R_1^2} T \log_e \frac{R_2^2 - R_0^2}{r^2 - R_0^2} \tag{14}$$

which gives the pressure on the tube produced by wrapping from r to R_2. Making in this $r = R_1$ we have the total pressure of the wire envelope on the tube or

$$p'_1 = \frac{R_1^2 - R_0^2}{2\,R_1^2} T \log_e \frac{R_2^2 - R_0^2}{R_1^2 - R_0^2} \tag{15}$$

and solving this for T we have for the uniform tension of wrapping to produce the required pressure p'_1 on the exterior of the tube

$$T = \frac{2\,R_1^2 p'_1}{(R_1^2 - R_0^2) \log_e \dfrac{R_2^2 - R_0^2}{R_1^2 - R_0^2}} \tag{16}$$

If the jacket is assembled without shrinkage on the wire envelope, or if there is no jacket, p'_1 becomes the total pressure $P_{1\rho}$ required on the exterior of the tube to compress its inner surface to the elastic limit when the gun is in the state of rest.

9. Intensity of Pressure at any Radius r of the Wire Envelope Due to Wrapping, System at Rest. — In equation (11) p_r is the pressure at r due to wrapping from r to any other radius r'. If r' be taken equal to R_2, p_r will be the pressure at r due to wrapping all the wire from r out, or the final pressure at rest at r due to the wrapping, and $p'_t - p_t$ will be the pressure on the exterior of the tube produced by the same wrapping. This pressure is the p'_{tR_2} of equation (14). Substituting for $p'_t - p_t$ in equation (11) the value of p'_{tR_2} from equation (14) we have

$$\frac{R_1^2 - R_0^2}{2\,R_1^2} T \log_e \frac{R_2^2 - R_0^2}{r^2 - R_0^2} = \frac{(R_1^2 - R_0^2)\, r^2 p_r}{R_1^2\,(r^2 - R_0^2)}$$

and solving for p_r

$$p_r = \frac{r^2 - R_0^2}{2\,r^2} T \log_e \frac{R_2^2 - R_0^2}{r^2 - R_0^2} \tag{17}$$

which gives the pressure p_r due to the wrapping at any radius r of the wire envelope when the gun is in the state of rest. If, in equation (17) r becomes R_1, p_r becomes p'_1 the total pressure of the wire envelope on the tube. Making these substitutions in equation (17)

$$p'_1 = \frac{R_1^2 - R_0^2}{2\,R_1^2} T \log_e \frac{R_2^2 - R_0^2}{R_1^2 - R_0^2}$$

the same as given in equation (15), as it should be.

10. Intensity of Tension at any Radius r of the Wire Envelope Due to Wrapping, System at Rest. — Let t_r be the tension at r due to the wrapping after it is completed. Then calling τ the mean tension of the wires between r and r',

$$p'_r r' - p_r r = -\,\tau\,(r' - r) \quad \text{or} \quad \frac{p'_r r' - p_r r}{r' - r} = -\tau$$

or passing to the limit, when τ becomes $-t_r$,

$$\frac{d\,(p_r r)}{dr} = -t_r \tag{18}$$

Substituting in equation (18) the value of p_r from equation (17) we have

$$-t_r = \frac{d\left[\dfrac{r^2 - R_0^2}{2\,r} T \log_e \dfrac{R_2^2 - R_0^2}{r^2 - R_0^2}\right]}{dr}$$

Performing the differentiation indicated

$$-t_r = \frac{T}{dr}\left[\log_\epsilon \frac{R_2^2 - R_0^2}{r^2 - R_0^2}\left\{\frac{4\,r^2dr - (r^2 - R_0^2)\,2\,dr}{4\,r^2}\right\} - \left\{\frac{r^2 - R_0^2}{2\,r}\right\}\left\{\frac{R_2^2 - R_0^2}{(r^2 - R_0^2)^2}2\,rdr\,\frac{r^2 - R_0^2}{R_2^2 - R_0^2}\right\}\right]$$

and reducing

$$t_r = T\left[1 - \frac{r^2 + R_0^2}{2\,r^2}\log_\epsilon \frac{R_2^2 - R_0^2}{r^2 - R_0^2}\right] \qquad (19)$$

Equation (19) gives the tension due to the wrapping at any radius r of the wire envelope when the gun is in the state of rest.

11. Tangential and Radial Stresses and Strains at any Radius r of the Wire Envelope Due to Wrapping, System at Rest. — The tangential stress at any radius r of the wire envelope due to the wrapping, system at rest, is

$$S_{tw} = t_r + p_r/3 \qquad (20)$$

and substituting in this the values of t_r and p_r from equations (19) and (17)

$$S_{tw} = T\left[1 - \frac{r^2 + R_0^2}{2\,r^2}\log_\epsilon \frac{R_2^2 - R_0^2}{r^2 - R_0^2} + \frac{r^2 - R_0^2}{6\,r^2}\log_\epsilon \frac{R_2^2 - R_0^2}{r^2 - R_0^2}\right]$$

reducing

$$S_{tw} = T\left[1 - \frac{r^2 + 2\,R_0^2}{3\,r^2}\log_\epsilon \frac{R_2^2 - R_0^2}{r^2 - R_0^2}\right] \qquad (21)$$

The radial stress at any radius r of the wire envelope due to the wrapping, system at rest, is

$$S_{pw} = -\,p_r - t_r/3 \qquad (22)$$

and substituting in this the values of p_r and t_r from equations (17) and (19)

$$S_{pw} = -\frac{T}{3}\left[1 + \frac{r^2 - 2\,R_0^2}{r^2}\log_\epsilon \frac{R_2^2 - R_0^2}{r^2 - R_0^2}\right] \qquad (23)$$

The tangential and radial strains can be obtained by dividing S_{tw} and S_{pw}, respectively, by the modulus of elasticity E.

12. Stresses in Jacket, Wire Envelope, and Tube, System at Rest and in Action. — Equations (17), (19), (21), and (23) give the radial pressure, the tangential tension, the tangential stress, and the radial stress due to the wrapping, at any radius r of the wire envelope when the system is at rest, in terms of the uniform tension of wrapping T and the radii of the tube and wire envelope. The shrinkage of the jacket also produces radial pressure, tangential tension, and tangential and radial stresses in the wire envelope which must be added algebraically to those produced by the wrapping to obtain the final state of the wire envelope, system at rest. The stresses in the jacket, system at rest, are due only to the shrinkage pressure P'_2 and those in the tube, system at rest, only to the exterior pressure on the tube P_{1_p}. The radial pressure, tangential tension, and tangential and radial stresses in both jacket and tube, system at rest, can be calculated from equations (4), (3), (1), and (2), respectively, after proper substitutions therein.

The tangential tension, radial pressure, and stresses in the tube, wire envelope, and jacket having been calculated for the gun in the state of rest, the tensions, pressures, and stresses in action corresponding to any pressure P_0 may be obtained by adding algebraically to those in the state of rest the tensions, pressures, and stresses computed from equations (3), (4), (1), and (2), respectively, by considering the gun as a single cylinder acted on only by an interior pressure P_0.

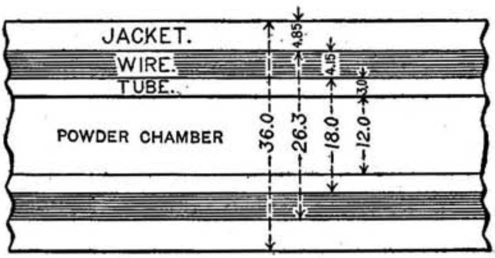

Fig. 2.

EXAMPLE.

13. Figure 2 represents a section through the powder chamber of a 12-inch wire-wrapped gun.

The material and physical qualities of the tube, wire envelope and jacket are shown in the following table:

Part	Material	Elastic limit in tension θ	Elastic limit in compression ρ	Modulus of elasticity E
Tube	Forged steel	45,000	60,000	30,000,000
Wire	Steel wire	100,000	100,000	30,000,000
Jacket	Cast steel	30,000	30,000	30,000,000

It is required to determine,

(*a*) the maximum powder pressure which the gun may withstand without the tangential stresses in any part thereof exceeding its elastic limit;

(*b*) the shrinkage of the jacket;

(*c*) the uniform tension of wrapping of the wire envelope;

(*d*) the stresses at the interior and exterior surfaces of each part in the state of rest; and

(*e*) the stresses at the interior and exterior surfaces of each part in the state of action corresponding to the maximum permissible powder pressure and also to a powder pressure of 42,000 lbs. per sq. in.

Maximum Permissible Powder Pressure — Shrinkage of Jacket — Uniform Tension of Wrapping. — From Fig. 2

$$R_0 = 6;\ R_1 = 9;\ R_2 = 13.15;\ R_3 = 18.$$

From equation (5) the maximum permissible powder pressure is

$$P_0 = \frac{3[(18)^2 - (6)^2]\,[60{,}000 + 45{,}000]}{4\,(18)^2 + 2\,(6)^2} = 66{,}316 \text{ lbs. per sq. in.} \quad (24)$$

The pressure on the exterior of the tube to compress its interior surface in the state of rest to the elastic limit in compression, 60,000 lbs. per sq. in., is, from equation (6),

$$P_{1\rho} = \frac{(9)^2 - (6)^2}{2\,(9)^2}\,60{,}000 = 16{,}667 \text{ lbs. per sq. in.} \quad (25)$$

Part of this pressure is due to the shrinkage of the jacket. To determine this shrinkage it is first necessary to find the intensity of stress at the interior of the jacket in the state of action due to the powder pressure of 66,315 lbs. per sq. in., and to subtract this stress from the elastic limit in tension of the material in the jacket. The difference will be due to the shrinkage pressure and from it the shrinkage pressure can be determined. Having the shrinkage pressure, the pressure which it transmits to the exterior of the tube may be obtained. Substituting in equation (1) $R_3 = 18$ for R_1, $R_2 = 13.15$ for r, and making $P_1 = 0$ and $P_0 = 66{,}316$, the tangential stress at the inner surface of the jacket due to P_0 is

$$S''_{t3} = \frac{2}{3}\frac{66{,}316\,(6)^2}{(18)^2-(6)^2} + \frac{4}{3}\frac{(6)^2\,(18)^2\,66{,}316}{[(18)^2-(6)^2]\,(13.15)^2}$$
$$= 26{,}235 \text{ lbs. per sq. in. tension.} \qquad (26)$$

The elastic limit in tension of the jacket being 30,000 lbs. per sq. in., 30,000 − 26,235 or 3765 lbs. per sq. in. will be the stress due to the shrinkage pressure. Substituting in equation (1), $R_2 = 13.15$ for R_0, $R_3 = 18$ for R_1, making $P_1 = 0$, $P_0 = P'_2$, and $S_t = 3765$, and solving for P'_2, the shrinkage pressure,

$$P'_2 = \frac{3\,[(18)^2-(13.15)^2]}{4\,(18)^2+2\,(13.15)^2}\,3765 = 1039 \text{ lbs. per sq. in.} \qquad (27)$$

The absolute shrinkage to produce this pressure is from equation (8)

$$S_2 = \frac{4\,(13.15)^3\,[(18)^2-(6)^2]\,1039}{30{,}000{,}000\,[(13.15)^2-(6)^2]\,[(18)^2-(13.15)^2]}$$
$$= .00439 \text{ inch.} \qquad (28)$$

The pressure on the exterior of the tube due to the shrinkage pressure of the jacket from equation (9), derived from equation (4), is

$$p''_1 = \frac{1039\,(13.15)^2}{(13.15)^2-(6)^2}\,[1-(6)^2/(9)^2] = 729 \text{ lbs. per sq. in.} \qquad (29)$$

Subtracting this from the total exterior pressure on the tube, equation (25), when the system is at rest, we have $p'_1 = 16{,}667 - 729 = 15{,}938$ lbs. per sq. in. as the part of the pressure on the exterior of the tube, system at rest, which must be due to the wire envelope. Substituting this value of p'_1 in equation (16)

$$T = \frac{2\,(9)^2\,15{,}938}{[(9)^2-(6)^2]\log_e\dfrac{(13.15)^2-(6)^2}{(9)^2-(6)^2}} = 51{,}566 \text{ lbs. per sq. in.} \qquad *(30)$$

* The Naperian logarithm of a number is equal to the common logarithm of the number divided by .4343.

which is the uniform tension under which the wire must be wrapped on the tube.

STRESSES AT REST.

(See figures 3 and 4.)

14. Inner Surface of Tube. — The tangential stress, see equation (25), is one of compression equal to 60,000 lbs. per sq. in. The radial stress obtained from equation (2) by making $P_0 = 0$, $P_1 = 16{,}667$, $r = R_0 = 6$, and $R_1 = 9$, is

$$S_p = \frac{2}{3}\frac{-16{,}667\ (9)^2}{(9)^2-(6)^2} - \frac{4}{3}\frac{-(9)^2\,16{,}667}{(9)^2-(6)^2}$$
$$= +20{,}000 \text{ lbs. per sq .in.} \tag{31}$$

and since the result is positive the stress is one of tension.

Outer Surface of Tube. — The tangential stress, obtained from equation (1) by making $P_0 = 0$, $P_1 = 16{,}667$, $R_0 = 6$, and $r = R_1 = 9$, is

$$S_t = \frac{2}{3}\frac{-16{,}667\ (9)^2}{(9)^2-(6)^2} + \frac{4}{3}\frac{-(6)^2\,16{,}667}{(9)^2-(6)^2} = -37{,}777 \text{ lbs. per sq. in.} \tag{32}$$

and since the result is negative the stress is one of compression. The radial stress, obtained by making the same substitutions in equation (2), is

$$S_p = \frac{2}{3}\frac{-16{,}667\ (9)^2}{(9)^2-(6)^2} - \frac{4}{3}\frac{-(6)^2\,16{,}667}{(9)^2-(6)^2} = -2223 \text{ lbs. per sq. in.} \tag{33}$$

and the stress is one of compression.

Inner Surface of Wire Envelope. — The tangential stress due to the layers of wire only, obtained from equation (21) by making $R_0 = 6$, $r = R_1 = 9$, $R_2 = 13.15$, and $T = 51{,}566$, see equation (30), is

$$S'_{tw} = 51{,}566\left\{1 - \frac{(9)^2 + 2\,(6)^2}{3\,(9)^2}\log_e\frac{(13.15)^2-(6)^2}{(9)^2-(6)^2}\right\}$$
$$= +15{,}439 \text{ lbs. per sq. in.} \tag{34}$$

and the stress is one of tension.

This stress is decreased by that due to the shrinkage pressure of the jacket which is obtained from equation (1) by making

$R_0 = 6$, $R_1 = R_2 = 13.15$, $r = R_1 = 9$, $P_1 = P'_2 = 1039$, and $P_0 = 0$.

Whence

$$S''_{tw} = \frac{2}{3}\,\frac{-(1039)\,(13.15)^2}{(13.15)^2 - (6)^2} + \frac{4}{3}\,\frac{-(6)^2\,(13.15)^2\,(1039)}{(13.15)^2 - (6)^2} \times \frac{1}{(9)^2}$$

$$= -1653 \text{ lbs. per sq. in. compression.} \qquad (35)$$

The final tangential stress at rest is, therefore,

$$15{,}439 - 1653 = 13{,}786 \text{ lbs. per sq. in. tension.}$$

The radial stress due to the layers of wire only, obtained from equation (23) by making the same substitutions as in equation (21) for the tangential stress, is

$$S'_{pw} = -\frac{51{,}566}{3}\left\{1 + \frac{(9)^2 - 2\,(6)^2}{(9)^2}\log_e \frac{(13.15)^2 - (6)^2}{(9)^2 - (6)^2}\right\}$$

$$= -19{,}314 \text{ lbs. per sq. in. compression.} \qquad (36)$$

To this must be added algebraically the radial stress due to the shrinkage of the jacket which is obtained from equation (2) by making the appropriate substitutions therein.

Whence

$$S''_{pw} = \frac{2}{3}\,\frac{-(1039)\,(13.15)^2}{(13.15)^2 - (6)^2} - \frac{4}{3}\,\frac{-(6)^2\,(13.15)^2\,(1039)}{(13.15)^2 - (6)^2} \times \frac{1}{(9)^2}$$

$$= -97 \text{ lbs. per sq. in. compression.} \qquad (37)$$

The final radial stress at rest is, therefore,

$$-19{,}314 - 97 = -19{,}411 \text{ lbs. per sq. in. compression.}$$

Outer Surface of Wire Envelope. — Before the assembling of the jacket the tangential stress in 51,566 lbs. per sq. in., the tension of wrapping. This is decreased by the compression due to the shrinkage of the jacket which is obtained from equation (1) by making the proper substitutions therein.

Whence

$$S''_{tw} = \frac{2}{3}\,\frac{-(1039)\,(13.15)^2}{(13.15)^2 - (6)^2} + \frac{4}{3}\,\frac{-(6)^2\,(13.15)^2\,(1039)}{(13.15)^2 - (6)^2}$$

$$\times \frac{1}{(13.15)^2} = -1240 \text{ lbs. per sq. in. compression.} \qquad (38)$$

The final tangential stress at rest is, therefore,

$$51{,}566 - 1240 = 50{,}326 \text{ lbs. per sq. in. tension.}$$

Before assembling the jacket the radial stress is minus one-third the tension of wrapping = −51,566/3 = −17,188 lbs.* per sq. in. compression. The radial stress due to the shrinkage of the jacket, obtained from equation (2) by making the proper substitutions therein, is

$$S''_{pw} = \frac{2}{3}\,\frac{-(1039)\,(13.15)^2}{(13.15)^2-(6)^2} - \frac{4}{3}\,\frac{-(6)^2\,(13.15)^2\,(1039)}{(13.15)^2-(6)^2} \times \frac{1}{(13.15)^2}$$

$$= -510 \text{ lbs. per sq. in. compression.} \qquad (39)$$

The final radial stress at rest is, therefore,

$$-17{,}188 - 510 = -17{,}698 \text{ lbs. per sq. in. compression.}$$

Inner Surface of Jacket. — All stresses in the jacket, system at rest, are due to the shrinkage. The tangential stress at its inner surface, see equation (27), is 3765 lbs. per sq. in. tension. The radial stress, obtained from equation (2) by making the proper substitution therein, is

$$S_{pj} = \frac{2}{3}\,\frac{(1039)\,(13.15)^2}{(18)^2-(13.15)^2} - \frac{4}{3}\,\frac{(13.15)^2\,(18)^2\,(1039)}{(18)^2-(13.15)^2} \times \frac{1}{(13.15)^2}$$

$$= -2178 \text{ lbs. per sq. in. compression.} \qquad (40)$$

Outer Surface of Jacket. — The tangential stress, obtained from equation (1) by making the proper substitutions therein, is

$$S_{tj} = \frac{2}{3}\,\frac{(1039)\,(13.15)^2}{(18)^2-(13.15)^2} + \frac{4}{3}\,\frac{(13.15)^2\,(18)^2\,(1039)}{(18)^2-(13.15)^2} \times \frac{1}{(18)^2}$$

$$= +2378 \text{ lbs. per sq. in. tension.} \qquad (41)$$

The radial stress, obtained from equation (2) by making the proper substitutions therein, is

$$S_{pj} = \frac{2}{3}\,\frac{(1039)\,(13.15)^2}{(18)^2-(13.15)^2} - \frac{4}{3}\,\frac{(13.15)^2\,(18)^2\,(1039)}{(18)^2-(13.15)^2} \times \frac{1}{(18)^2}$$

$$= -793 \text{ lbs. per sq. in. compression.} \qquad (42)$$

* This result can also be obtained from equation (23) by proper substitutions therein.

STRESSES IN ACTION.

(See figures 3 and 4.)

15. Maximum Permissible Powder Pressure P_0 = 66,315.— Making in equations (1) and (2) $P_0 = 66{,}315$, $P_1 = 0$, $R_0 = 6$, and $R_1 = R_3 = 18$, they become, respectively,

$$S_t = \frac{66{,}315\,(6)^2}{(18)^2 - (6)^2}\left\{\frac{2}{3} + \frac{4}{3}\,\frac{(18)^2}{r^2}\right\} = 5526 + \frac{[6.55400]^*}{r^2} \qquad (43)$$

and

$$S_p = \frac{66{,}315\,(6)^2}{(18)^2 - (6)^2}\left\{\frac{2}{3} - \frac{4}{3}\,\frac{(18)^2}{r^2}\right\} = 5526 - \frac{[6.55400]^*}{r^2} \qquad (44)$$

By substituting for r in equations (43) and (44) its values corresponding to the surfaces of the various parts of the gun, the stresses due to the action of P_0 on the gun considered as a single cylinder may be determined, and the algebraic sums of these stresses and those previously determined for the state of rest are the stresses in action.

Inner Surface of Tube. $r = R_0 = 6$.— The tangential stress system in action, see equation (24), is 45,000 lbs. per sq. in. tension. The radial stress is

$$+20{,}000 + 5526 - \frac{[6.55400]}{(6)^2} = +20{,}000 - 93{,}946 = -73{,}946 \text{ lbs.}^\dagger$$

per sq. in. compression. (45)

Outer Surface of Tube. $r = R_1 = 9$.— The tangential stress is

$$-37{,}777 + 5526 + \frac{[6.55400]}{(9)^2} = -37{,}777 + 49{,}735 = +11{,}958 \text{ lbs.}$$

per sq. in. tension. (46)

The radial stress is

$$-2223 + 5526 - \frac{[6.55400]}{(9)^2} = -2223 - 38{,}683 = -40{,}906 \text{ lbs.}$$

per sq. in. compression. (47)

---

* The figures in brackets are logarithms of the numbers.

† This exceeds the elastic limit for compression.

Inner Surface of Wire Envelope. $r = R_1 = 9$. — The tangential stress is

$$+13{,}786 + 5526 + \frac{[6.55400]}{(9)^2} = +13{,}786 + 49{,}735 = +63{,}521 \text{ lbs. per sq. in. tension.} \quad (48)$$

The radial stress is

$$-19{,}411 + 5526 - \frac{[6.55400]}{(9)^2} = -19{,}411 - 38{,}683 = -58{,}096 \text{ lbs. per sq. in. compression.} \quad (49)$$

Outer Surface of Wire Envelope. $r = R_2 = 13.15$. — The tangential stress is

$$+50{,}326 + 5526 + \frac{[6.55400]}{(13.15)^2} = +50{,}326 + 26{,}234 = +76{,}560 \text{ lbs. per sq. in. tension.} \quad (50)$$

The radial stress is

$$-17{,}698 + 5526 - \frac{[6.55400]}{(13.15)^2} = -17{,}698 - 15{,}182 = -32{,}880 \text{ lbs. per sq. in. compression.} \quad (51)$$

Inner Surface of Jacket. $r = R_2 = 13.15$. — The tangential stress is, see equations (26) and (27),

$$+3765 + 26{,}234 = +29{,}999 \text{ lbs. per sq. in. tension.} \quad (51\tfrac{1}{2})$$

The radial stress is

$$-2178 + 5526 - \frac{[6.55400]}{(13.15)^2} = -2178 - 15{,}182 = -17{,}360 \text{ lbs. per sq. in. compression.} \quad (52)$$

Outer Surface of Jacket. $r = R_3 = 18$. — The tangential stress is

$$+2378 + 5526 + \frac{[6.55400]}{(18)^2} = +2378 + 16{,}578 = +18{,}956 \text{ lbs. per sq. in. tension.} \quad (53)$$

The radial stress is

$$-793 + 5526 - \frac{[6.55400]}{(18)^2} = -793 - 5526 = -6319 \text{ lbs. per sq. in. compression.} \quad (54)$$

The tangential stresses, system at rest, and in action when $P_0 = 66{,}315$ lbs. per sq. in. are shown graphically in Fig. 3, and the radial stresses in Fig. 4.

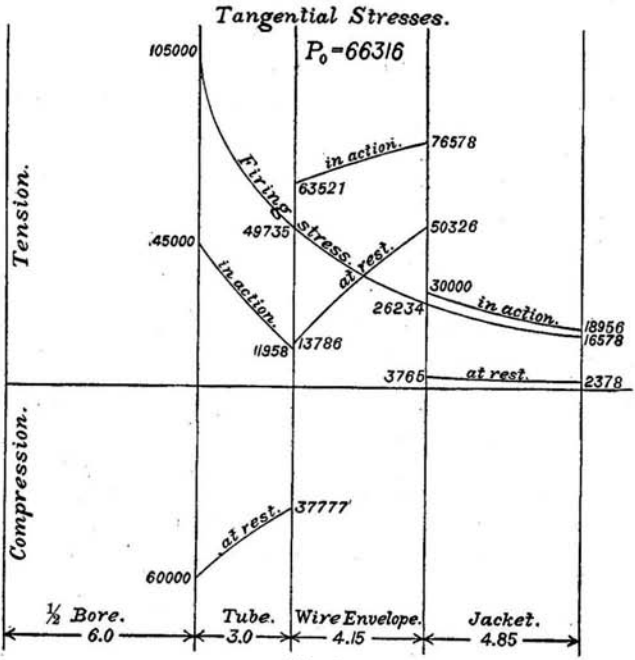
Tangential Stresses.
P_0=66316
Tension.
Compression.
105000
45000
60000
Firing stress.
in action.
at rest.
76578
63521
49735
50326
30000
26234
11958
13786
18956
16578
3765
2378
37777
½ Bore. 6.0
Tube. 3.0
Wire Envelope. 4.15
Jacket. 4.85

Fig. 3.

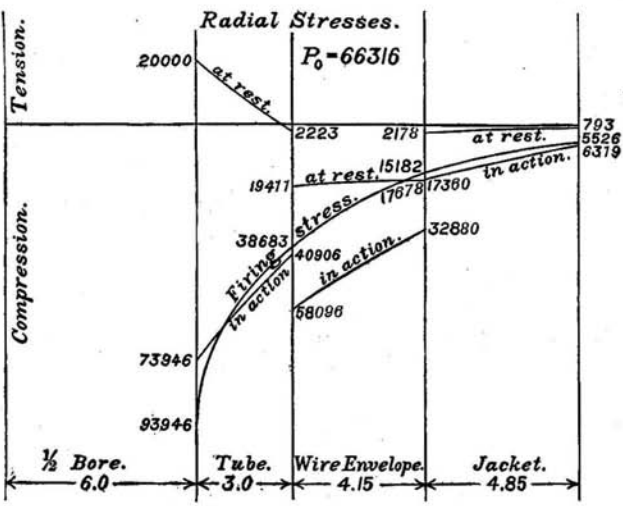
Radial Stresses.
P_0=66316
Tension.
Compression.
20000
at rest.
2223
2178
793
5526
6319
in action.
19411
15182
17678
17360
Firing stress.
38683
40906
32880
58096
73946
93946
½ Bore. 6.0
Tube. 3.0
Wire Envelope. 4.15
Jacket. 4.85

Fig. 4.

STRESSES IN ACTION

(See figures 5 and 6.)

16. P_0 = 42000 lbs. per sq. in. — By reference to equations (1) and (2), or to equations (43) and (44), it will be seen that the stresses due to the action of an interior pressure on a gun considered as a single cylinder are directly proportional to the intensity of the interior pressure. The stresses at the inner and outer surfaces of the various parts of this gun, system in action, under an interior powder pressure of 42,000 lbs. per sq. in. will, therefore, be the stresses at rest plus 42,000/66,316 of the increases in stress in the various parts due to the action on the gun, considered as a single cylinder, of the maximum permissible powder pressure of 66,316 lbs. per sq. in. These increases in stress have already been calculated and are given in equations (45) to (54), inclusive, excepting the increase in tangential stress at the bore of the tube which had been previously determined to be 105,000 lbs. per sq. in.

Inner Surface of Tube. — The tangential stress is

$$-60{,}000 + \frac{42{,}000}{66{,}316} \times 105{,}000 = -60{,}000 + 66{,}501 = +6501 \text{ lbs. per sq. in. tension} \qquad (55)$$

The radial stress is

$$+20{,}000 + \frac{42{,}000}{66{,}316} \times (-93{,}946) = +20{,}000 - 59{,}500 = -39{,}500 \text{ lbs. per sq. in. compression.} \qquad (56)$$

Outer Surface of Tube. — The tangential stress is

$$-37{,}777 + \frac{42{,}000}{66{,}316} \times 49{,}735 = -37{,}777 + 31{,}499 = -6278 \text{ lbs. per sq. in. compression.} \qquad (57)$$

The radial stress is

$$-2223 + \frac{42{,}000}{66{,}316} \times (-38{,}683) = -2223 - 24{,}500 = -26{,}723 \text{ lbs. per sq. in. compression.} \qquad (58)$$

Inner Surface of Wire Envelope. — The tangential stress is

$$+13{,}786 + \frac{42{,}000}{66{,}316} \times 49{,}735 = +13{,}786 + 31{,}499 = +45{,}285 \text{ lbs. per sq. in. tension.} \qquad (59)$$

The radial stress is

$$-19{,}411 + \frac{42{,}000}{66{,}316} \times (-38{,}683) = -19{,}411 - 24{,}500 = -43{,}911 \text{ lbs. per sq. in. compression.} \quad (60)$$

Outer Surface of Wire Envelope. — The tangential stress is

$$+50{,}326 + \frac{42{,}000}{66{,}316} \times 26{,}234 = +50{,}326 + 16{,}615 = +66{,}941 \text{ lbs. per sq. in. tension.} \quad (61)$$

The radial stress is

$$-17{,}698 + \frac{42{,}000}{66{,}316} \times (-15{,}182) = -17{,}698 - 9616 = -27{,}314 \text{ lbs. per sq. in. compression.} \quad (62)$$

Inner Surface of Jacket. — The tangential stress is

$$+3765 + \frac{42{,}000}{66{,}316} \times 26{,}234 = +3765 + 16{,}615 = +20{,}380 \text{ lbs. per sq. in. tension.} \quad (63)$$

The radial stress is

$$-2178 + \frac{42{,}000}{66{,}316} \times (-15{,}182) = -2178 - 9616 = -11{,}794 \text{ lbs. per sq. in. compression.} \quad (64)$$

Outer Surface of Jacket. — The tangential stress is

$$+2378 + \frac{42{,}000}{66{,}316} \times 16{,}578 = +2378 + 10{,}502 = +12{,}880 \text{ lbs. per sq. in. tension.} \quad (65)$$

The radial stress is

$$-793 + \frac{42{,}000}{66{,}316} \times (-5526) = -793 - 3500 = -4293 \text{ lbs. per sq. in. compression.} \quad (66)$$

Figs. 5 and 6 show graphically the tangential and radial stresses, respectively, system in action, when $P_0 = 42{,}000$ lbs. per sq. in.

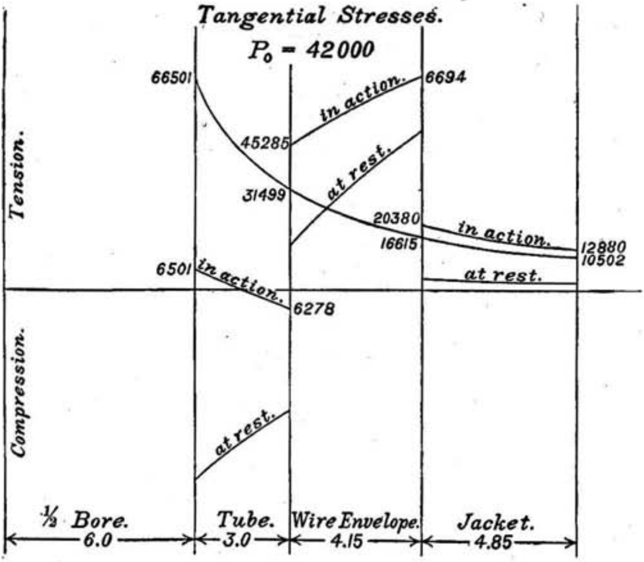
Tangential Stresses.
$P_0 = 42000$
Tension.
Compression.
66501
45285
31499
6501
in action.
6278
at rest.
6694
in action.
at rest.
20380
16615
in action.
12880
10502
at rest.
½ Bore.
6.0
Tube.
3.0
Wire Envelope.
4.15
Jacket.
4.85

Fig. 5.

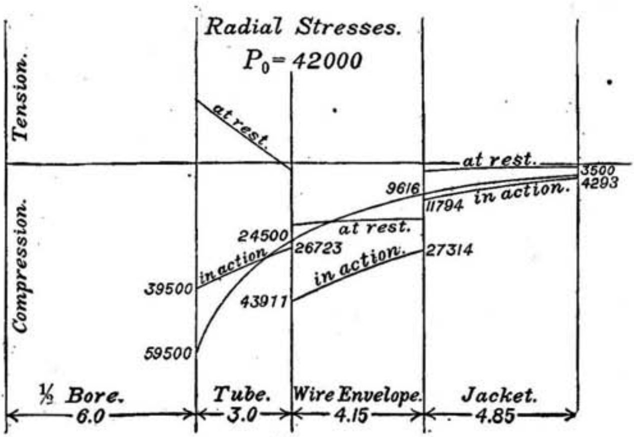
Radial Stresses.
$P_0 = 42000$
Tension.
Compression.
at rest.
24500
in action.
39500
59500
26723
at rest.
9616
in action.
43917
at rest.
11794
in action.
27314
3500
4293
½ Bore.
6.0
Tube.
3.0
Wire Envelope.
4.15
Jacket.
4.85

Fig. 6.

17. Problem. — Fig. 7 shows a longitudinal section through a part of the 6-inch wire-wrapped gun model 1908 just in front of the forcing cone.

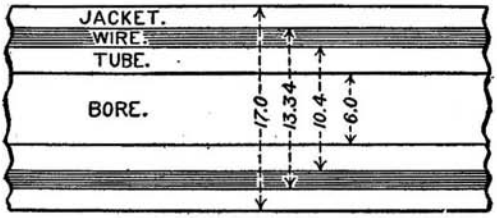

Fig. 7.

Part	Prescribed elastic limit $\rho = \theta$
Tube	50,000
Wire envelope	140,000
Jacket	50,000

The jacket is assembled with an absolute shrinkage of .007 inch. The wire is wrapped under a constant tension of 50,000 lbs. per sq. in.

Find:

(*a*) the maximum powder pressure which the gun can withstand without the tangential stress on any part exceeding the elastic limit in tension or compression;

(*b*) the tangential and radial stresses at the inner and outer surfaces of each part, system at rest;

(*c*) the tangential and radial stresses at the inner and outer surfaces of each part, system in action, when subjected to the powder pressure determined under (*a*).

CASE II. — WIRE WRAPPED UNDER SUCH VARYING TENSION THAT WHEN THE GUN IS FIRED WITH THE PRESCRIBED MAXIMUM POWDER PRESSURE ALL LAYERS OF WIRE WILL BE SUBJECTED TO THE SAME TANGENTIAL STRESS.

18. General Method of Design. — Let it be required to construct a wire-wrapped gun as shown in Fig. 1 or 2 in such manner as to give it the maximum elastic strength permitted by the quality of the metal in the tube and the total thickness of wall $R_3 - R_0$, and with the condition that when the gun is fired with the prescribed maximum powder pressure all layers of wire shall be subjected to the same tangential stress.

First determine from equation (5) the maximum interior pressure which the gun can support in action on the assumption that in the state of rest the inner surface of the tube is compressed to its elastic limit. From equation (6) determine the pressure $P_{1\rho}$ on the exterior of the tube which will compress its inner surface in the state of rest to its elastic limit. With the values of P_0 and $P_{1\rho}$ from equations (5) and (6) determine from equation (48), page 215, Lissak's Ordnance and Gunnery, the corresponding value of the pressure on the exterior of the tube, system in action. The last equation, after replacing the radius ratios by their values in terms of the radii and substituting R_3 for R_2 since there are three parts to the gun instead of two, becomes

$$P_1 = P_{1\rho} + \frac{R_0^2\,(R_3^2 - R_1^2)\,P_0}{R_1^2\,(R_3^2 - R_0^2)} \qquad (67)$$

The next steps require the use of formulas (*e*) to (*g*), inclusive, referred to in article 6 which will now be deduced. Let S_{twa} be the uniform tangential stress in the wire, system in action; S_{pwra} the radial stress at any radius r of the wire envelope, system in action; t_{ra} the tension at any radius r of the wire envelope, system in action; p_{ra} the pressure at any radius r of the wire envelope, system in action; and T_r the tension of wrapping.

19. Radial Pressure at any Radius r of the Wire Envelope, System in Action. Constant Tangential Stress in the Wire Envelope, System in Action. — Substituting for t_{ra} in the expression

$$p_{ra}dr + rdp_{ra} = -t_{ra}dr$$

its value, $t_{ra} = S_{twa} - p_{ra}/3$ obtained from the first of equations (4), page 192, Lissak's Ordnance and Gunnery, by considering $q = 0$, we have

$$p_{ra}dr + rdp_{ra} = -(S_{twa} - p_{ra}/3)\,dr \qquad (68)$$

Whence

$$\frac{dp_{ra}}{S_{twa} + 2\,p_{ra}/3} = -dr/r \qquad (69)$$

Integrating

$$\tfrac{3}{2} \log_e (S_{twa} + \tfrac{2}{3} p_{ra}) = \log_e C / r \qquad (70)$$

Since the wire envelope is covered by a jacket there will, when the system is in action, be a pressure P_2 on the exterior of the wire envelope, which pressure, if the full elastic strength of the jacket is to be utilized, must be such as to extend its inner surface, system in action, to its elastic limit. Therefore, when $r = R_2$, $p_{ra} = P_2$; and the constant of integration C is

$$(S_{twa} + \tfrac{2}{3} P_2)^{3/2} R_2$$

Substituting this value of C in equation (70)

$$\tfrac{3}{2} \log_e [S_{twa} + \tfrac{2}{3} p_{ra}] = \log_e [(S_{twa} + \tfrac{2}{3} P_2)^{3/2} R_2 / r]$$

or

$$(S_{twa} + \tfrac{2}{3} p_{ra})^{3/2} = (S_{twa} + \tfrac{2}{3} P_2)^{3/2} R_2 / r$$

and solving for p_{ra}

$$p_{ra} = \tfrac{3}{2} [(S_{twa} + \tfrac{2}{3} P_2) (R_2 / r)^{2/3} - S_{twa}] \qquad (71)$$

which gives the radial pressure at any radius r of the wire envelope, system in action. Making in this $r = R_1$, the pressure on the exterior of the tube, system in action, is

$$P_1 = \tfrac{3}{2} [(S_{twa} + \tfrac{2}{3} P_2) (R_2 / R_1)^{2/3} - S_{twa}] \qquad (72)$$

and solving for S_{twa} we obtain the constant tangential stress in the wire envelope, system in action, required to produce a given pressure P_1 on the exterior of the tube, system in action.

Whence

$$S_{twa} = \frac{2}{3} \frac{P_1 - P_2 (R_2 / R_1)^{2/3}}{(R_2 / R_1)^{2/3} - 1} \qquad (73)$$

20. Tangential Tension and Radial Stress at any Radius r of the Wire Envelope, System in Action. — The tangential tension t_{ra} at any radius r of the wire envelope, system in action, is $t_{ra} = S_{twa} - p_{ra}/3$ and substituting in this the value of p_{ra} from equation (71)

$$t_{ra} = \tfrac{3}{2} S_{twa} - \tfrac{1}{2} (S_{twa} + \tfrac{2}{3} P_2) (R_2 / r)^{2/3} \qquad (74)$$

The tangential stress S_{twa} throughout the wire envelope, system in action, is constant by hypothesis. The radial stress at any radius r of the wire envelope, system in action, is, from the second of equations (4), page 192, Lissak's Ordnance and Gunnery, when q is taken as zero,

$$S_{pwra} = -(p_{ra} + t_{ra}/3)$$

and substituting for p_{ra} and t_{ra} their values from equations (71) and (74), respectively,

$$S_{pwra} = -\tfrac{4}{3}(S_{twa} + \tfrac{2}{3}P_2)(R_2/r)^{2/3} + S_{twa} \tag{75}$$

The tangential and radial strains can be obtained by dividing the corresponding stresses by 30,000,000, the modulus of elasticity E for steel.

21. Variable Tension of Wrapping at any Radius r of Wire Envelope. — Equation (74) gives the tangential tension t_{ra} at any radius r of the wire envelope, system in action. The increase in tension at the radius r, system in action, over the tension of wrapping T_r is due to the interior pressure P_0 and the pressure p_{ra}, system in action, neither of which was acting when the layer at the radius r was applied. Considering the part of the gun between the interior radius R_0 and the radius r of the wire envelope as a single cylinder, the increase in tension at the radius r over the tension of wrapping due to the pressures P_0 and p_{ra} is obtained from equation (3) by making $P_1 = p_{ra}$ and $R_1 = r$.

Whence

$$t = \frac{2P_0R_0^2 - p_{ra}(r^2 + R_0^2)}{r^2 - R_0^2}$$

Subtracting this increase in tension from the tension at the radius r of the wire envelope, system in action, there results the tension of wrapping at any radius r

$$T_r = t_{ra} - t = t_{ra} - \frac{2P_0R_0^2 - p_{ra}(r^2 + R_0^2)}{r^2 - R_0^2}$$

Substituting in this the values of t_{ra} and p_{ra} from equations (74) and (71), respectively,

$$T_r = \tfrac{3}{2}S_{twa} - \tfrac{1}{2}(S_{twa} + \tfrac{2}{3}P_2)(R_2/r)^{2/3} - \frac{2P_0R_0^2 - \tfrac{3}{2}[(S_{twa} + \tfrac{2}{3}P_2)(R_2/r)^{2/3} - S_{twa}](r^2 + R_0^2)}{r^2 - R_0^2}$$

and reducing

$$T_r = \frac{(S_{twa} + \tfrac{2}{3}P_2)(R_2/r)^{2/3}(r^2 + 2R_0^2) - R_0^2(3S_{twa} + 2P_0)}{r^2 - R_0^2} \tag{76}$$

The tension of wrapping at any radius r may be obtained from equation (76) by substituting therein the particular value of r.

22. Stresses in the Jacket, the Wire Envelope, and the Tube, System in Action and at Rest. — The radial pressure, constant tangential stress, tangential tension, and radial stress in the wire envelope, system in action, may be obtained by proper substitutions in equations (71), (73), (74), and (75), respectively. The stresses in the jacket, system in action, are due only to the pressure P_2 on its inner surface; and those in the tube, system in action, are due to the pressure P_0 on its inner surface and the pressure P_1 on its exterior. The tangential tension, radial pressure, and the tangential and radial stresses in both jacket and tube, system in action, may be calculated from equations (3), (4), (1), and (2), respectively, after proper substitutions therein. The tangential tensions, radial pressures, and stresses actually existing in the various parts of the gun, system in action, corresponding to an interior powder pressure P_0 having been calculated, those at rest may be obtained by subtracting algebraically from the tensions, pressures, and stresses in action those computed from equations (3), (4), (1), and (2), respectively, by considering the gun as a single cylinder acted on only by an interior pressure P_0.

EXAMPLE.

23. Let it be required to determine for the wire-wrapped gun shown in Fig. 2

(*a*) the maximum powder pressure which it can withstand without the tangential stress in any part exceeding its elastic limit;

(*b*) the shrinkage of the jacket;

(*c*) the varying tension of wrapping of the wire envelope so that each layer shall be subjected to the same tangential stress, system in action, under the maximum powder pressure determined under (*a*);

(*d*) the stresses at the interior and exterior surfaces of each part, system in action, under the maximum powder pressure;

(*e*) the stresses at the interior and exterior surfaces of each part, system at rest; and

(*f*) the stresses at the interior and exterior surfaces of each part, system in action, under a powder pressure of 42000, lbs. per sq. in.

Maximum Permissible Powder Pressure. — Pressure on the Exterior of the Tube and on the Interior of the Jacket, System in Action. — Shrinkage of the Jacket. — The maximum permissible interior pressure in this case is the same as when the wire was wrapped under constant tension and is

$$P_0 = 66{,}316 \text{ lbs. per sq. in., from equation (24).}$$

The value of P_{1_ρ} from equation (25) is 16,667 lbs. per sq. in. Substituting this value of P_{1_ρ} and the value of $P_0 = 66{,}316$ in equation (67), the value of the exterior pressure on the tube, system in action, is

$$P_1 = 16{,}667 + \frac{(6)^2[(18)^2-(9)^2]\,66{,}316}{(9)^2\,[(18)^2-(6)^2]} = 41{,}535 \text{ lbs. per sq. in.} \quad (77)$$

The elastic limit of the jacket being 30,000 lbs. per sq. in. the pressure P_2 in action between the jacket and the wire envelope which will extend the inner surface of the jacket to its elastic limit is from equation (1), after making the proper substitutions therein and solving for P_2,

$$P_2 = \frac{3\,[(18)^2 - (13.15)^2]\,30{,}000}{4\,(18)^2 + 2\,(13.15)^2} = 8281 \text{ lbs. per sq. in.} \quad (78)$$

From equation (26) the tangential stress at the inner surface of the jacket due to the action of the powder pressure of 66,316 lbs. per sq. in. was found to be 26,235 lbs. per sq. in., leaving a stress of 3765 lbs. per sq. in. to be produced by the shrinkage of the jacket on the wire envelope. The shrinkage pressure to produce this stress was, from equation (27), found to be 1039 lbs. per sq. in., and the corresponding absolute shrinkage from equation (28) was .00439 inch.

24. Constant Tangential Stress in Wire Envelope, System in Action. — Variable Tension of Wrapping. — The constant tangential stress in the layers of the wire envelope, system in action, may now be obtained from equation (73) by substituting therein the values of P_1 and P_2 from equations (77) and (78), respectively; and then the variable tension of wrapping from equation (76) by substituting therein the values obtained for S_{twa}, P_2, P_0, and the value of r for each layer of the wire envelope.

Solving equation (73) for S_{twa} after substituting for P_1 and P_2 the values 41,535 and 8281 from equations (77) and (78), respectively,

$$S_{twa} = \frac{\frac{2}{3}\left\{41{,}535 - 8281\left(\frac{13.15}{9}\right)^{2/3}\right\}}{\left(\frac{13.15}{9}\right)^{2/3} - 1} = +71{,}578 \text{ lbs. per sq. in. tension.} \qquad (79)$$

which is the constant tangential stress in all the layers of the wire envelope, system in action.

Substituting in equation (76) the values of S_{twa}, P_0, P_2, R_0, and R_2, and reducing

$$T_r = \frac{77{,}099\left(\frac{13.15}{r}\right)^{2/3}(r^2 + 72) - 12{,}541{,}004}{r^2 - 36} \qquad (80)$$

By substituting in equation (80) the value of r for any layer of wire, the tension of wrapping for that layer may be obtained. The wire used on this gun was of square cross section .1 inch on a side. For the first layer of wire the value of r, taken at the center of the thickness of the wire, is 9.05 and the corresponding value of T_r = 58,425 lbs. per sq. in.

For the middle layer of wire, the value of r taken as 11.1 gives T_r = 49,420 lbs. per sq. in.

For the outside layer, the value of r taken as 13.10 gives T_r = 46,375 lbs. per sq. in.

25. Stresses in the Jacket and the Tube, System in Action P_0 = 66,316 lbs. per sq. in., and at Rest. – (See **Figs. 8 and 9.**) — Since the shrinkage and shrinkage pressure between the jacket and wire envelope, and the pressure on the exterior of the tube, system at rest, are the same as when the wire envelope was wrapped under a constant tension of 51,566 lbs. per sq. in., the stresses in the tube and jacket both in action and at rest will be the same as determined for that case.

STRESSES IN THE WIRE ENVELOPE, SYSTEM IN ACTION

P_0 = 66,316 lbs. per sq. in.

(See figures 8 and 9.)

26. The tangential stress in the wire envelope, system in action, has a constant value of 71,578 lbs. per sq. in. The radial stress S_{pwra} in the wire envelope may be obtained by substitution in equation (75) of the proper values of S_{twa}, P_2, R_2, and r.

Outer Surface of the Wire Envelope. $r = R_2 = 13.15$, $S_{twa} = 71,578$, $P_2 = 8281$.

The radial stress is

$$-\frac{4}{3}\left(71,578 + \frac{2}{3}8281\right) + 71,578 = -31,220 \text{ lbs. per sq. in. compression.} \quad (81)$$

Inner Surface of the Wire Envelope. $r = R_1 = 9$. — The radial stress is

$$-\frac{4}{3}\left(71,578 + \frac{2}{3}8281\right)\left(\frac{13.15}{9}\right)^{2/3} + 71,578 = -60,792 \text{ lb. per sq. in. compression.} \quad (82)$$

STRESS IN THE WIRE ENVELOPE, SYSTEM AT REST.

(See figures 8 and 9.)

27. The tangential and radial stresses at any radius r of the wire envelope due to the action of P_0 on the gun considered as a single cylinder may be obtained from equations (43) and (44), respectively, by the substitution therein of the proper values of r. Subtracting the results thus obtained from the corresponding stresses in action, we have

Outer Surface of the Wire Envelope. $r = R_2 = 13.15$. — The tangential stress is

$$+71,578 - \left\{5526 + \frac{[6.55400]}{(13.15)^2}\right\} = +45,344 \text{ lbs. per sq. in. tension.} \quad (83)$$

The radial stress is

$$-31,220 - \left\{5526 - \frac{[6.55400]}{(13.15)^2}\right\} = -16,038 \text{ lbs. per sq. in. compression.} \quad (84)$$

Inner Surface of the Wire Envelope. $r = R_1 = 9.$ — The tangential stress is

$$+71{,}578 - \left\{5526 + \frac{[6.55400]}{(9)^2}\right\} = +21{,}843 \text{ lbs. per sq. in. tension.} \quad (85)$$

The radial stress is

$$-60{,}792 - \left\{5526 - \frac{[6.55400]}{(9)^2}\right\} = -22{,}109 \text{ lbs. per sq. in. compression.} \quad (86)$$

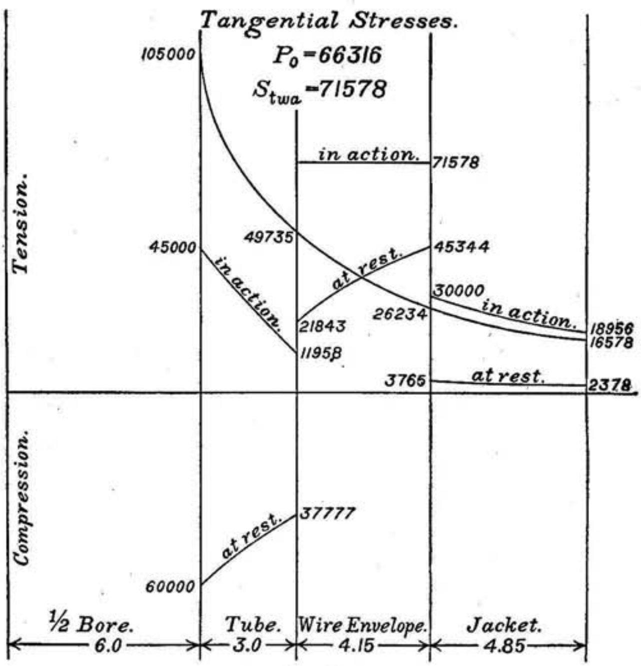

Fig. 8.

The tangential stresses, system in action when $P_0 = 66{,}316$ lbs. per sq. in., and system at rest, are shown graphically in Fig. 8; and the radial stresses in Fig. 9.

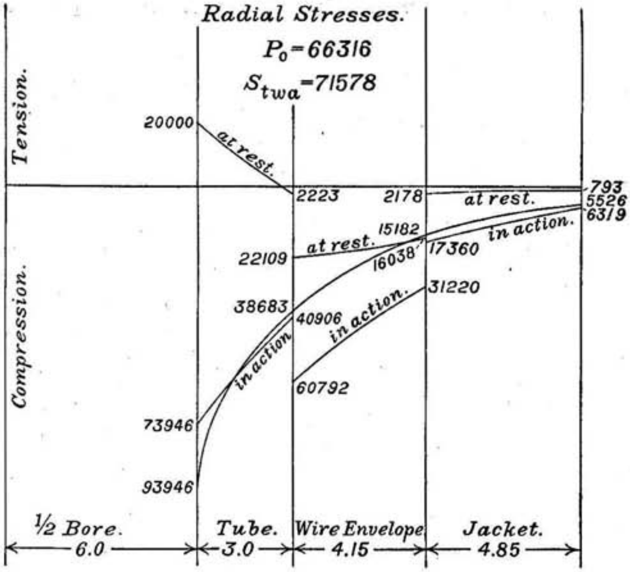

Fig. 9.

STRESSES IN ACTION, $P_0 = 42{,}000$ LBS. PER SQ. IN.

(See figures 10 and 11.)

28. The stresses in action, $P_0 = 42{,}000$ lbs. per sq. in., in the tube and jacket are the same as were obtained when the wire was wrapped under constant tension and are given by equations (55) to (58), inclusive, and (63) to (66), inclusive. The stresses in the wire envelope are obtained by adding to the stresses, system at rest, those due to the interior pressure $P_0 = 42{,}000$ lbs. per sq. in. acting on the gun considered as a single cylinder. The latter stresses have already been determined and used in equations (59) to (62), inclusive, relating to the stresses in action in the wire envelope, wrapped at constant tension, due to $P_0 = 42{,}000$ lbs. per sq. in.

Inner Surface of the Wire Envelope. — The tangential stress is

$$+21{,}843 + 31{,}499 = +53{,}342 \text{ lbs. per sq. in. tension.} \quad (87)$$

The radial stress is

$$-22{,}109 - 24{,}500 = -46{,}609 \text{ lbs. per sq. in. compression.} \quad (88)$$

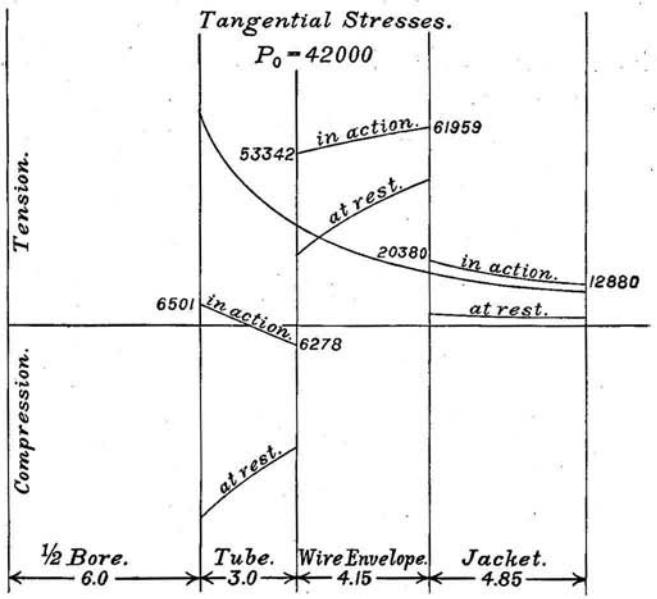
Tangential Stresses.
$P_0 = 42000$
Tension.
Compression.
in action.
61959
53342
at rest.
20380
in action.
12880
6501
in action.
at rest.
6278
at rest.
½ Bore.
6.0
Tube.
3.0
Wire Envelope.
4.15
Jacket.
4.85

Fig. 10.

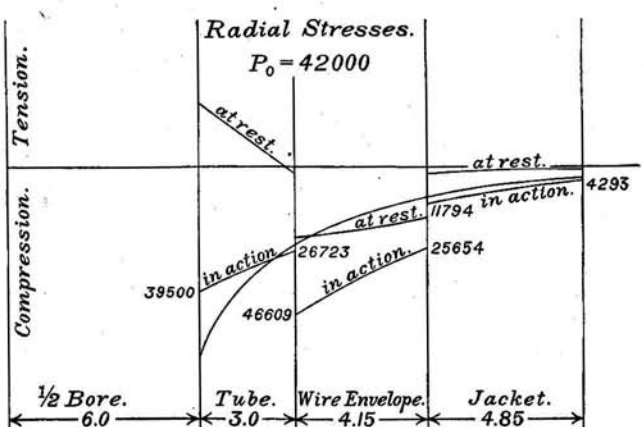
Radial Stresses.
$P_0 = 42000$
Tension.
Compression.
at rest.
at rest.
4293
at rest.
11794
in action.
26723
25654
in action
39500
in action.
46609
½ Bore.
6.0
Tube.
3.0
Wire Envelope.
4.15
Jacket.
4.85

Fig. 11.

Outer Surface of the Wire Envelope. — The tangential stress is

$$+45{,}344 + 16{,}615 = +61{,}959 \text{ lbs. per sq. in. tension.} \quad (89)$$

The radial stress is

$$-16{,}038 - 9616 = -25{,}654 \text{ lbs. per sq. in. compression.} \quad (90)$$

Figs. 10 and 11 show graphically the tangential and radial stresses, respectively, system in action, when $P_0 = 42{,}000$ lbs. per sq. in.

29. Problem. — Find for a 6-inch wire-wrapped gun whose parts have the same dimensions and elastic limits as shown in Fig. 7 and whose jacket is assembled with the same shrinkage, .007 inch, but on the tube of which the wire is wrapped with varying tension so as to obtain a constant tangential stress in the wire envelope, system in action under the required powder pressure:

(*a*) the constant tangential stress in the wire envelope, system in action, that will give to the gun the same tangential elastic strength as when the wire was wrapped under a constant tension of 50,000 lbs. per sq. in.;

(*b*) the tension of wrapping at the inner and outer surfaces of the wire envelope;

(*c*) the tangential and radial stresses at the inner and outer surfaces of each part, system in action under the maximum powder pressure which the gun can withstand without the tangential stress on any part exceeding the elastic limit in tension or compression; and

(*d*) the tangential and radial stresses at the inner and outer surfaces of each part, system at rest.

CHAPTER II.

DETERMINATION OF THE FORCES BROUGHT UPON THE PRINCIPAL PARTS OF THE 3-INCH FIELD CARRIAGE BY THE DISCHARGE OF THE GUN.

30. Stability. — Total Resistance Opposed to Recoil of Gun. — This carriage is so designed that when the gun is fired at 0 degrees elevation the carriage will not move to the rear and the wheels will not rise from the ground. Under these conditions the carriage is said to be *stable*. To prevent the carriage from moving to the rear when the gun is fired, there is provided at the end of the trail a spade of such an area that the horizontal resistance which the ground can exert against it is greater than the sum of the horizontal components of the forces brought upon the carriage by the discharge of the gun; and to prevent the wheels from rising from the ground the moment of the weight of the system, gun and carriage, around the point of support of the trail on the ground is made greater than the sum of the moments about that point of the forces brought upon the carriage by the discharge of the gun. The forces brought upon the carriage by the discharge of the gun will depend on the resistance opposed to the recoil of the gun. If no resistance is thus opposed, no force will be brought upon the carriage by the discharge of the gun except the slight friction between the gun and the carriage due to the weight of the gun and its movement in recoil. The weight of the gun acts on the carriage at all times. To limit the recoil of the gun a resistance must, however, be imposed. This resistance is imposed through the hydraulic brake at a d stance of 7.156 inches below the axis of the gun. The action line of the resistance being below the axis of the gun, it will tend to rotate the gun around its center of mass, assumed to be in that axis. This rotation will be prevented by the clips on the gun engaging with those on the cradle (see Fig. 15), the clip on the cradle exerting a downward force on the forward end of the front clip of the gun and an upward force on the clip of the gun at the rear end of the cradle. These forces will produce friction as the gun recoils, the action line of the friction being parallel to the axis of the gun and along the contact surfaces of the clips. The friction also opposes the recoil of the gun. The direction of the motion of the gun due to the action of the powder gases is in prolongation of its axis, and since this direction is not changed by the forces exerted on the gun by the carriage, it follows that *the resultant of all the latter forces must be a force whose action line coincides with the axis of the gun.* This resultant force is the total resistance which opposes the recoil of the gun.

31. Resultant of Forces Exerted by the Gun on the Carriage. — Since the forces exerted by the carriage on the gun are likewise exerted by the gun on the carriage, but in opposite directions, it follows that the resultant of all the forces exerted by the gun on the carriage when the gun is fired is *a force in the prolongation of the axis of the gun equal and opposite in direction to the resistance which opposes the recoil of the gun.* As the carriage remains in equilibrium when the gun is fired the forces exerted upon it by the gun develop others between the carriage and the ground which oppose and neutralize the effect of the former so far as motion of the carriage is concerned. These forces develop others between the various parts of the carriage which, while they do not cause motion of the parts and, therefore, do not disturb their equilibrium, cause stresses in the materials of which the parts are made, which stresses must not exceed safe limits in order that the parts of the carriage may not be distorted or broken. The resultant of the forces exerted by the gun on the carriage, which is equal and opposite in direction to the resistance opposed to the recoil of the gun, is represented in Fig. 12 by R.

32. Limiting Value of the Resistance R Compatible with Stability of the Carriage. — General Expression for this Resistance. (See Fig. 12.) — Let it be required to determine the greatest value R' which the force R may have without causing the wheels of the carriage to rise from the ground when the gun is fired at 0° elevation. When R has this value the wheels are just about to rise and consequently the pressure between them and the ground is zero. The carriage is prevented from moving to the rear under the action of the force R' by the parallel force S exerted by

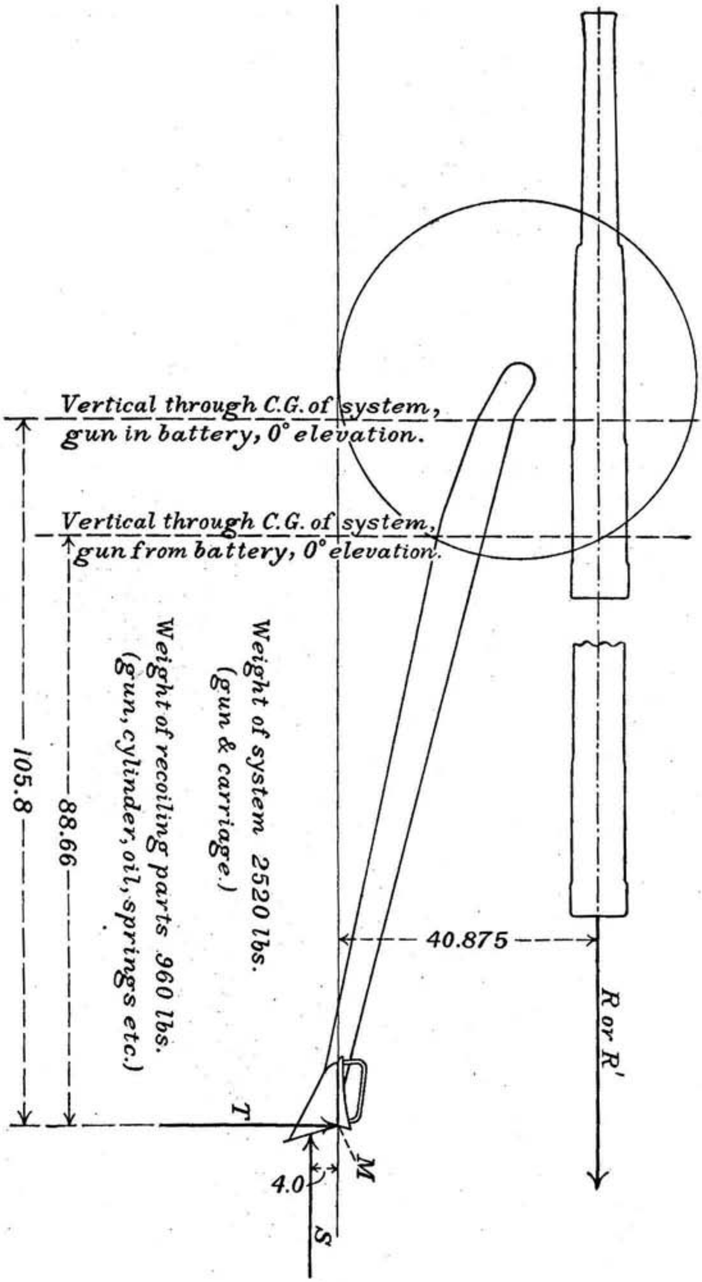
Vertical through C.G. of system, gun in battery, 0° elevation.
Vertical through C.G. of system, gun from battery, 0° elevation.
Weight of system 2520 lbs. (gun & carriage.)
Weight of recoiling parts 960 lbs. (gun, cylinder, oil, springs etc.)
105.8
88.66
40.875
R or R'
T
M
S
4.0

Fig. 12.

the ground on the spade, the center of pressure of the ground, and therefore the point of application of the force S, being at a distance of 4 inches below the surface.

Since the pressure between the wheels and the ground is zero the only force exerted upward on the carriage by the ground to counteract the force of gravity on the system will be the vertical force T assumed to act at the rear point of contact of the float with the ground, the point about which moments will be taken to determine the conditions of stability. The forces acting on the carriage are, therefore, R' acting in prolongation of the axis of the gun; the weight, 2520 lbs., of the system, gun and carriage, acting vertically through its center of mass; and the forces S and T at the spade; as shown in Fig. 12. As the carriage is assumed to be in equilibrium under these forces, which can, because of their symmetrical distribution, be considered as contained in the central plane through the axis of the gun, the sum of the components of the forces in the vertical and horizontal directions must be zero and the sum of the moments of the forces about any point in their plane must be zero.

Whence

$$2520 - T = 0 \text{ or } T = 2520 \text{ lbs.} \tag{1}$$

$$S - R' = 0 \quad \text{or} \quad S = R' \tag{2}$$

and taking moments about the point M and denoting by l the lever arm of the weight of the system with respect to this point,

$$R' \times 40.875 + S \times 4 - 2520 \times l = 0$$

or since $S = R'$ from equation (2)

$$R' = \frac{2520 \times l}{44.875} \tag{3}$$

which is the general expression for the greatest value of the resistance R that can be opposed to the recoil of the gun when it is fired at 0° elevation without causing the wheels to rise from the ground.

Limiting Value of the Resistance Corresponding to $l = 105.8$ ins. — Since the value of l will diminish as the gun recoils, R' will have a maximum value when the gun is in battery and a minimum value when the gun is at its extreme limit of recoil; and it will diminish uniformly with the distance traveled by the gun in recoil from the maximum to the minimum value. The value of l when the gun is in battery is, from Fig. 12, 105.8 ins. and the corresponding value of R', which we will call R'_1, is

$$R'_1 = \frac{2520 \times 105.8}{44.875} = 5941 \text{ lbs.} \tag{4}$$

Limiting Value of the Resistance Corrresponding to l = 88.66 ins. — Limiting Value of the Resistance Corresponding to a Length of Recoil of b ft. — It will be assumed for the present that the length of recoil of the gun on the carriage is not known. Since the value of R' diminishes uniformly with the distance traveled by the gun in recoil its value R'_2 corresponding to any distance b in feet recoiled by the gun can be determined as follows:

Assume any distance traveled by the gun in recoil, as 3.75 ft., and compute the corresponding position of the center of gravity of the system and the value of l. This has already been done and the value of l from Fig. 12 is 88.66 ins. The value of R'_2 when the gun has recoiled 3.75 ft. is, therefore,

$$R'_2 = \frac{2520 \times 88.66}{44.875} = 4979 \text{ lbs.} \tag{5}$$

The difference between R'_1 and R'_2 corresponding to a distance of 3.75 ft. recoiled by the gun is 962 lbs., and, therefore, the difference for a distance of b ft. recoiled by the gun is

$$\frac{962}{3.75} b = 256.54\, b \text{ lbs.}$$

The value of R'_2, then, corresponding to a distance of b ft. recoiled by the gun, is

$$R'_2 = R'_1 - 256.54\, b = (5941 - 256.54\, b) \text{ lbs.} \tag{6}$$

If we set off on a perpendicular the value of R'_1 to any scale from equation (4), and, on a second perpendicular at a horizontal distance from the first representing to any scale a distance of 3.75 ft., the value of R'_2 from equation (5), and join the ends of the perpendiculars by a right line, the ordinates of this line will represent the limiting values of R corresponding to any distance passed over by the gun in recoil which values must not be exceeded if the wheels are not to rise from the ground when the gun is fired at an elevation of zero degrees. This line is shown in Fig. 13 by the full line.

If the resistance opposed to recoil is to be constant it may have any value less than R'_2, but in order to meet all conditions that may arise it is the practice to make it at all points of recoil substantially less than R', its limiting value.

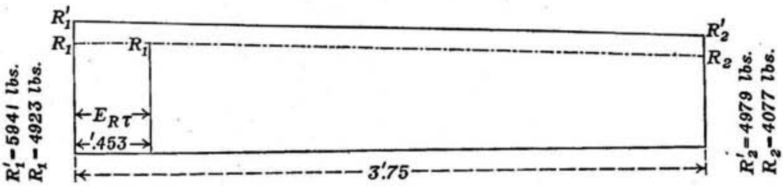

Fig. 13.

Method of Varying the Resistance Opposed to Recoil Followed in the Design of this Carriage. — In the design of the carriage by the Ordnance Department the resistance is made constant during the time the powder gases are acting on the gun, and after they cease to act it is made to decrease with the further distance passed over in recoil as shown by the dot and dash line in Fig. 13, the rate of decrease being the same as that of R', its limiting value. This arrangement not only increases the margin of stability of the carriage during the early stages of recoil, as compared with a constant margin of stability, but facilitates the calculation of the value of the resistance corresponding to a given length of recoil of the gun or of the length of recoil corresponding to any assumed value of the resistance.

33. Relation Between the Varying Values of the Actual Resistance and the Length of Recoil of the Gun on the Carriage. — Determination of the Values of R_1, R_2, and b. — Let V_f be the maximum velocity of free recoil of the gun and recoiling parts determined as described in article 160, page 275, Lissak's Ordnance and Gunnery, τ the time corresponding to the attainment of V_f in free recoil, and E_f the corresponding space that would have been passed over by the gun in free recoil during the time τ; τ and E_f being determined as described in article 163, page 279, Lissak's Ordnance and Gunnery.

Denoting by R_1 the actual resistance to recoil during the time τ when the powder gases are acting on the gun, and making it constant during this time, the velocity of restrained recoil at the end of the time τ will be

$$V_{rr} = V_f - \frac{R_1}{M_r}\tau \qquad (7)$$

M_r being the mass of the gun and the other recoiling parts, cylinder, oil, counter-recoil springs, etc.; and R_1/M_r being the retardation of these parts produced by the resistance R_1.

The space passed over by the gun in restrained recoil during the time τ will be

$$E_{rr} = E_f - \frac{R_1}{2\,M_r}\tau^2 \qquad (8)$$

which is shown in Fig. 13 as the space over which the resistance to the recoil of the gun is constant.

Calling R_2 the value of the resistance when the gun is at its extreme limit of recoil and b the total length of recoil of the gun on the carriage in feet, we may write

$$\frac{R_1 + R_2}{2}\left\{b - E_{rr}\right\} = \frac{M_r V_{rr}^2}{2} \qquad (9)$$

which expresses the fact that the work of the mean resistance $(R_1 + R_2)/2$ over the path $b - E_{rr}$ is equal to the kinetic energy of the recoiling mass at the end of the time τ.

Since the value of the resistance after the space E_{rr} has been passed over by the gun in recoil is to diminish in such manner that the line representing its diminishing values, Fig. 13, is to be parallel to the line representing the diminishing values of the limiting resistance R', we may write the following value for R_2 corresponding to a total distance b recoiled by the gun [see deduction of equation (6)].

$$R_2 = R_1 - 256.54\,(b - E_{rr}) \qquad (10)$$

Substituting this value of R_2 in equation (9) and the values of E_{rr} and V_{rr} from equations (8) and (7), respectively,

$$\{R_1 - 128.27\,[b - E_f + (R_1/2\,M_r)\tau^2]\}\{b - E_f + (R_1/2\,M_r)\tau^2\} = [M_r/2]\,[V_f - (R_1/M_r)\tau]^2 \qquad (11)$$

In equation (11) E_f, V_f, τ, and M_r are known so that only R_1 and b are unknown, either of which may be assumed and the other determined. Knowing R_1 and b, the value of R_2 may be obtained from equation (10) by substituting therein the value of R_1 and of E_{rr} from equation (8). If b is assumed the resulting values of R_1 and R_2 must be less than the corresponding values of the limiting resistances R'_1 and R'_2; and if R_1 is assumed b must not be inconveniently great and the value of R_2 must be less than that of R'_2. In the ordinary case a safe value for R_1 would be assumed and the corresponding values of b and R_2 calculated. If these values were satisfactory they would be accepted, if not another value for R_1 would be assumed and b and R_2 recalculated; or if the design of the carriage was such that acceptable values of R_1, R_2, and b could not be obtained that would satisfy equation (11) the design would have to be changed.

34. Values of the Actual Resistance to Recoil of the 3-inch Field Carriage. — Since in the case of the 3-inch field carriage the length of recoil is 45 ins., equal to 3.75 ft., a value of $b = 3.75$ will be assumed and the corresponding values of R_1 and R_2 calculated from equations (11) and (10). For this carriage

$$V_f = 34.52 \text{ f.s.}; \; E_f = .48 \text{ ft.}; \; \tau = .018 \text{ secs.}; \; M_r = \frac{960}{32.2} = 29.813.$$

Substituting these values and $b = 3.75$ ft. in equation (11) and solving for R_1 we obtain

$$R_1 = 4923 \text{ lbs.} \qquad (12)$$

Substituting this value in equation (7)

$$V_{rr} = V_f - \frac{R_1}{M_r}\tau = 34.52 - \frac{4923 \times .018}{29.813} = 31.55 \text{ f.s.} \qquad (13)$$

From equation (8)

$$E_{rr} = E_f - \frac{R_1}{2M_r}\tau^2 = .48 - \frac{4923 \times (.018)^2}{2 \times 29.813} = .453 \text{ ft.} \qquad (14)$$

From equation (10)

$$R_2 = R_1 - 256.54\,(b - E_{rr}) = 4923 - 256.54\,(3.75 - .453) = 4077 \text{ lbs.} \qquad (15)$$

The actual resistance opposed to the recoil of the gun, therefore, has a value of 4923 lbs. while the gun is recoiling over a distance of .453 ft. and then decreases uniformly to a value of 4077 lbs. at the end of recoil, the length of recoil being 3.75 ft. or 45 ins. After the distance of .453 ft. has been traveled by the gun in recoil, the equation giving the resistance at any point is as follows:

$$R_x = 4923 - 256.54\,(x - .453) \tag{16}$$

in which x is the distance in feet, measured from the origin of movement, over which the gun has recoiled.

Margin of Stability. — Referring to Fig. 13, the difference, called the margin of stability, between the limiting value of the resistance that may be opposed to the recoil of the gun without causing the wheels to rise from the ground, and the actual resistance employed is 1018 lbs. at the beginning of recoil, which decreases to 902 lbs. when the powder gases cease to act on the gun, at which time the gun has recoiled a distance of .453 ft. Thereafter, and until recoil has ceased at a distance of 3.75 ft. = 45 ins., the margin of stability remains constant and equal to 902 lbs.

35. Velocity of Restrained Recoil as a Function of Space. — For substitution in the formula which gives the areas of the throttling orifices it is necessary to determine the velocity of restrained recoil as a function of the distance recoiled. Until the gun has recoiled a distance of .453 ft. the resistance is constant and the velocity of restrained recoil as a function of space must be obtained in the manner described in article 166, pages 283 and 284, Lissak's Ordnance and Gunnery. After the gun has recoiled a distance of .453 ft. the velocity of restrained recoil corresponding to any distance x measured from the origin of movement is obtained by equating the work remaining to be done by the resistance with the energy of the recoiling mass. Thus, calling R_x the value of the resistance at the point x, and V_{rx}, the corresponding value of the velocity of restrained recoil,

$$\frac{R_x + R_2}{2}(b - x) = \frac{M_r V_{rx}^2}{2}$$

or

$$\frac{R_x + 4077}{2}(3.75 - x) = 14.907\, V_{rx}^2 \tag{17}$$

from which the value of V_{rx} corresponding to any point x is obtained. The value of R_x for substitution in equation (17) is obtained from equation (16).

FORCES ON THE CARRIAGE CONSIDERED AS ONE PIECE, GUN AT EXTREME LIMIT OF RECOIL AND AT 15° ELEVATION.

36. The resistance opposed to the recoil of the gun on the carriage having been determined, the forces on the carriage considered as one piece, and also on the various important parts of the carriage may now be determined. In these calculations the gun will be assumed to be at the end of its travel in recoil after having been fired at 15° elevation. For a complete solution of the problem, however, the position of the gun in battery should be considered and other conditions of elevation also assumed. The parts should then be proportioned to resist the maximum stresses brought upon them under any of the assumed conditions. The method of calculating the forces in the case now under consideration is, however, exactly like that to be followed in any of the other cases mentioned.

The gun and carriage are shown diagrammatically in Fig. 14, the gun being at 15° elevation and at its extreme limit of recoil. In this position the resistance opposed to its recoil by the carriage is 4077 lbs., equation (15), which is equal and opposite in direction to the resultant force exerted by the gun on the carriage. The latter force is represented in position and direction by R_2, Fig. 14. The horizontal component of this force tends to move the carriage to the rear, but the movement is prevented by the horizontal force S exerted by the ground on the spade, its action line (center of pressure) being at a distance of four inches below the surface of the ground. The vertical component of the force R_2 and the weight of the system tend to produce vertical motion downward, but this is prevented by the upward pressure of the ground at L and at T, there being a pressure between the wheels and the ground in this case not only because the resistance R_2 is less than the limiting resistance, but also because the gun is now elevated above zero degrees. The forces in this case and in all cases to be discussed hereafter can, because of the symmetrical disposition of the parts, be considered as co-planar, and they will hereafter be so considered without further reference thereto.

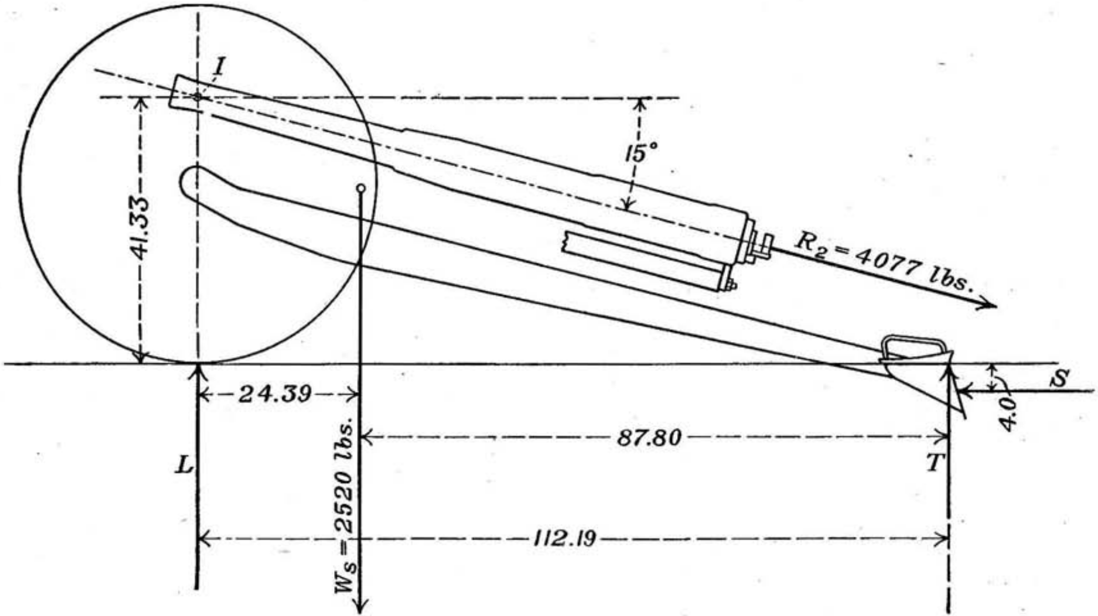

Fig. 14. — Forces on the Carriage considered as one Piece.

Since the carriage is in equilibrium, the sum of the vertical components of the forces must be zero as must the sum of the horizontal components, and also the sum of the moments of the forces about any point in their plane. Whence, considering vertical forces acting downward and horizontal forces acting in a direction opposite to that of recoil as positive,

$$S - 4077 \cos 15° = 0 \tag{18}$$

and

$$4077 \sin 15° + 2520 - L - T = 0 \tag{19}$$

Taking moments about I, the intersection of the action lines of the forces R_2 and L, and considering moments that tend to produce clockwise rotation as positive,

$$2520 \times 24.39 + S \times 45.33 - T \times 112.19 = 0 \tag{20}$$

From equation (18)

$$S = 4077 \cos 15° = 3938 \text{ lbs.} \tag{21}$$

From equation (20)

$$T = 2139 \text{ lbs.} \tag{22}$$

From equation (19)

$$L = 1436+ \text{ lbs.} \tag{23}$$

FORCES ON THE MOST IMPORTANT PARTS OF THE CARRIAGE, GUN AT EXTREME LIMIT OF RECOIL AND AT 15° ELEVATION.

37. Forces on the Gun. — Consider first the gun alone, Fig. 15. The resistance opposed to its recoil is 4077 lbs. and is the resultant of a number of forces acting between the carriage and the gun. The force P is the resistance opposed to the recoil of the gun by the pressure in the recoil cylinder, its action line being parallel to the axis of the gun and at a distance below it of 7.156 ins. As it is not exerted through the center of mass of the gun it tends to rotate it about that center. The weight of the gun, $W_g = 835$ lbs., is also a force acting on it as shown, but as it acts through the center of mass it has no tendency to produce rotation. The rotation of the gun under the action of the force P is prevented by the engagement of the clips on the gun with those of the cradle between which are developed the forces A acting downward on the gun at the front end of the forward gun clip and B acting upward at the rear end of the clip on the cradle. The forces A and B produce friction between the contact surfaces of the clips which is equal to $f \times (A + B)$, f being the coefficient of friction assumed equal to .15. This friction, which

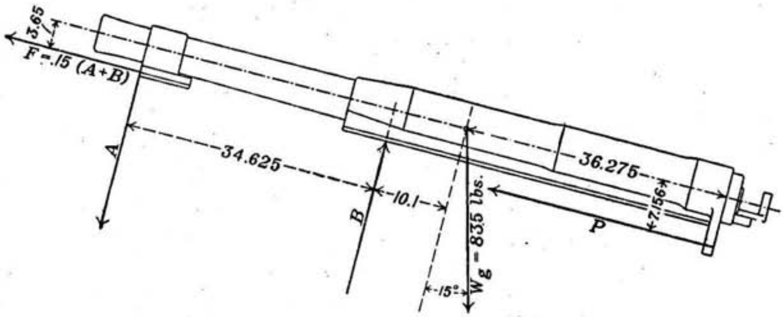

Fig. 15.

will be called F, acts along the contact surfaces of the clips in a direction parallel to the axis of the gun and opposite to that of recoil. Its action line may be taken at a distance of 3.65 ins. below the axis of the gun as shown in Fig. 15, and since it also tends to produce rotation of the gun around its center of mass, it causes additional pressure at A and B to counteract this tendency.

As the resultant of all these forces is a resistance of 4077 lbs. acting along the axis of the gun in a direction opposite to that of recoil, we may write three equations stating

(*a*) that the sum of the components of all the forces parallel to the axis of the gun is equal to 4077 lbs.;

(*b*) that the sum of the components of all the forces in a direction perpendicular to that axis is zero, since the resistance has no component in that direction; and

(*c*) that the sum of the moments of the forces about the center of mass is zero, since the resistance passes through that center and, therefore, has no moment with respect to it.

The action lines and location of all the forces are shown in Fig. 15. Forces acting to oppose the recoil of the gun and those acting downward at right angles to the axis of the gun are considered positive. Those acting in opposite directions are considered negative. Moments that tend to produce clockwise rotation are considered positive and those tending to produce counter-clockwise rotation, negative. Therefore,

$$P - W_g \sin 15° + .15\,(A + B) = 4077 \qquad (24)$$

$$A - B + W_g \cos 15° = 0 \qquad (25)$$

$$P \times 7.156 + .15\,(A+B) \times 3.65 + B \times 10.1 - A \times 44.725 = 0 \qquad (26)$$

Substituting in equations (24) and (26) the value of B obtained by solving equation (25) for B and replacing $W_g \cos 15°$ by its value 835 cos 15° = 807 lbs. and $W_g \sin 15°$ by its value of 216 lbs. and reducing, we have

$$P + .3\,A = 4172 \qquad (27)$$

$$P \times 7.156 - 33.53\,A = -8593 \qquad (28)$$

Multiplying both members of equation (27) by 7.156 and subtracting from equation (28)

$$-35.6768\,A = -38{,}448 \qquad (29)$$

or $A = 1078$ lbs. (30)

From equation (25) $B = 1885$ lbs. (31)

From equation (27) $P = 4172 - .3 \times 1078 = 3849$ lbs. (32)

$$F = .15\,(A + B) = .15\,(1078 + 1885) = 444 \text{ lbs.} \qquad (33)$$

38. Forces on the Cradle. — The forces acting between the cradle and the gun when the latter is fired act on the cradle in a direction opposite to that in which they act on the gun. They are shown in position, direction, and magnitude in Fig. 16. W_c = 409 lbs., the weight of the cradle, acts on it vertically through its center of mass. The forces P, F, and $W_c \sin 15°$ tend to move the cradle in a direction parallel to that of the recoil of the gun, but are prevented from doing so by the pintle of the cradle bearing against the pintle socket of the rocker, giving rise to the force C between the contact surfaces which acts in the opposite direction but parallel to the force P. Rotation of the cradle about its center of mass under the action of the forces A, B, C, and F is prevented by the engagement of the clips of the pintle on the cradle with those of the pintle socket of the rocker and by the pressure

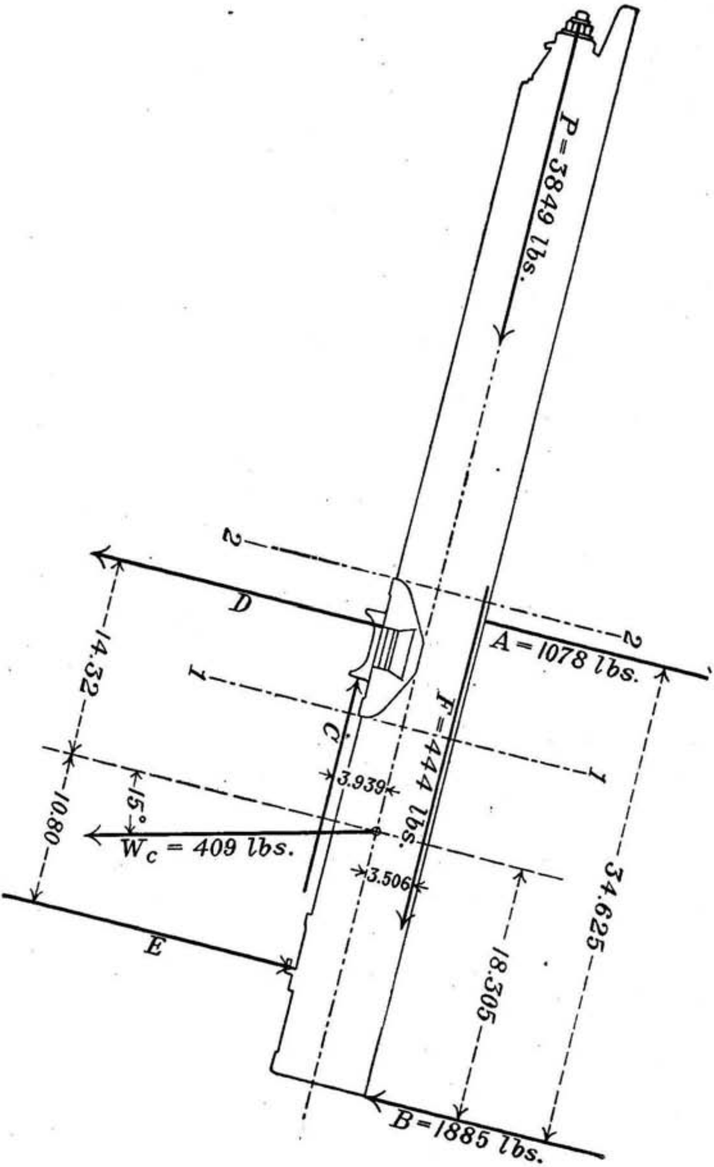

Fig. 16. — Forces on the Cradle.

between the surfaces of the rear cradle clip and the part of the rocker above the connection for the elevating screw, giving rise to a force D acting downward on the cradle at the clips of the pintle and a force E acting upward against the rear cradle clip, both forces being perpendicular to the axis of the cradle. The location of these forces is shown in Fig. 16.

As the cradle is in equilibrium under the forces acting upon it, the sum of all the components of the forces in a direction parallel to the axis of the cradle must be zero as well as the sum of all the components of the forces in a direction perpendicular to that axis. The sum of the moments of the forces about any point in their plane must also be zero.

Therefore, $$C - 444 - 3849 - 409 \sin 15° = 0 \qquad (34)$$

$$1885 + 409 \cos 15° + D - E - 1078 = 0 \qquad (35)$$

and taking moments about the center of mass of the cradle,

$$1078 \times 16.32 + 1885 \times 18.305 + 444 \times 3.506 + C \times 3.939 - D \times 14.32 - E \times 10.80 = 0 \qquad (36)$$

From equation (34) $$C = 4399 \text{ lbs.} \qquad (37)$$

Multiplying both members of equation (35) by 10.80 and subtracting from equation (36)

$$25.12\, D = 58{,}001. \qquad (38)$$

Whence $$D = 2309 \text{ lbs.} \qquad (39)$$

From equation (35) $$E = 3511 \text{ lbs.} \qquad (40)$$

39. Forces on the Rocker. — The forces acting between the cradle and the rocker act on the latter in a direction opposite to that in which they act on the cradle. They are shown in position, direction, and magnitude in Fig. 17. $W_r = 56$ lbs., the weight of the rocker, acts on it vertically through its center of mass. Movement of the rocker under the action of these forces is prevented by its engagement on the axle and by the support afforded by the elevating screw, giving rise to a force between the axle and the rocker and one between the elevating screw and the rocker. The direction of the force exerted by the axle on the rocker is not known but it must be normal to the surfaces in contact and must, therefore, intersect the axis of the

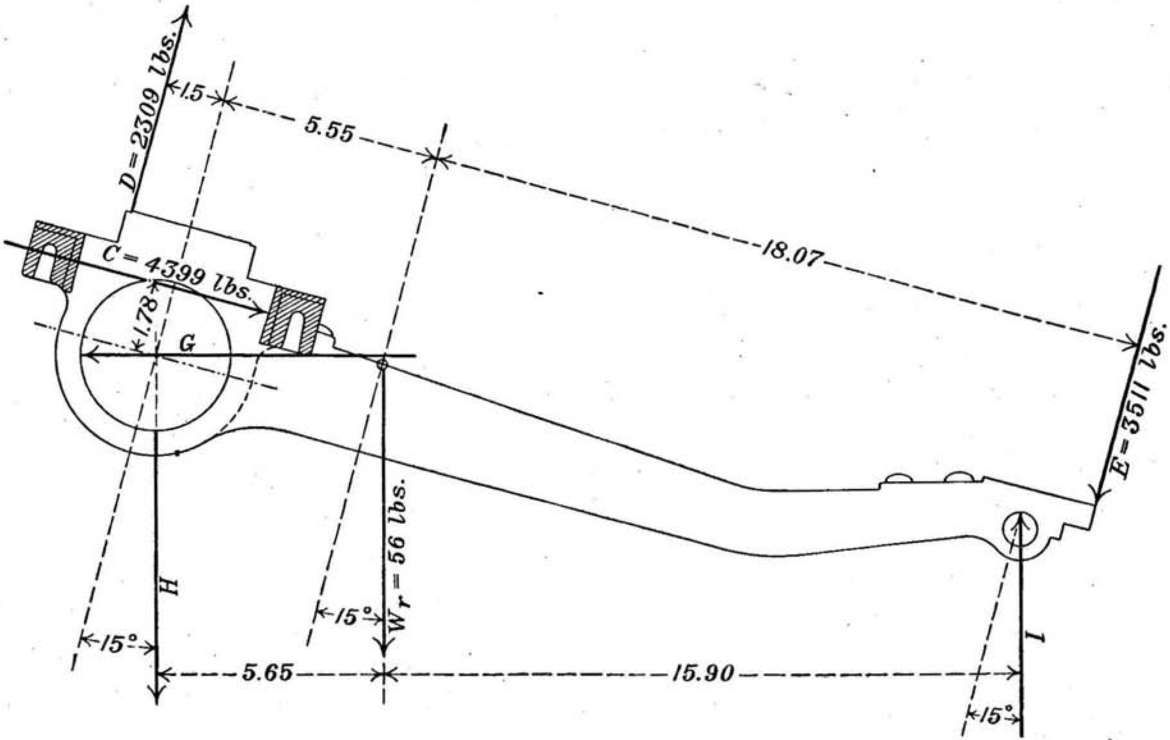

Fig. 17. — Forces on the Rocker.

axle. This force may, however, be resolved into two components, one horizontal and the other vertical, and when the intensities of the components are determined the direction of the resultant can be obtained if desired. These components are shown in Fig. 17, designated as G and H, respectively. Since the elevating screw is pivoted both to the rocker and the trail and is, therefore, free to rotate about its connections thereto, the action line of the force I, Fig. 17, exerted by it on the rocker must coincide with the axis of the screw, which is vertical when the gun is at an angle of elevation of 15°.

As the rocker is in equilibrium under the forces acting on it, we may write three equations of equilibrium as follows:

$$G - 4399 \cos 15° - 2309 \sin 15° + 3511 \sin 15° = 0 \qquad (41)$$

$$3511 \cos 15° - I + 56 + H + 4399 \sin 15° - 2309 \cos 15° = 0 \qquad (42)$$

Taking moments around the point of intersection of the forces G and H,

$$2309 \times 1.5 + 4399 \times 1.78 + 3511 \times 23.62 - I \times 21.55 + 56 \times 5.65 = 0 \qquad (43)$$

From equation (41) $\qquad G = 3938$ lbs. $\qquad$ (44)

From equation (43) $\qquad I = 4387$ lbs. $\qquad$ (45)

From equation (42) $\qquad H = 2031$ lbs. $\qquad$ (46)

40. Forces on the Axle, Wheels, Flasks, and Other Parts Below the Rocker, Considered as One Piece. — The forces acting between the rocker and the parts below it, act on the parts below in a direction opposite to that in which they act on the rocker. They are shown in position, direction, and magnitude in Fig. 18. $W_f = 1220$ lbs., the weight of all the parts below the rocker, acts on them through their center of mass. All movement of the parts of the carriage below the rocker under the action of these forces is prevented by the horizontal and vertical reactions of the ground which give rise to a vertical force L acting upward on the wheel, another T acting upward on the float of the spade, and a horizontal force S acting against the spade in a direction opposed to recoil. T is assumed to act on the float at its rear point of contact with the ground. The point of application of S is at the center of pressure of the earth on the spade, a distance of

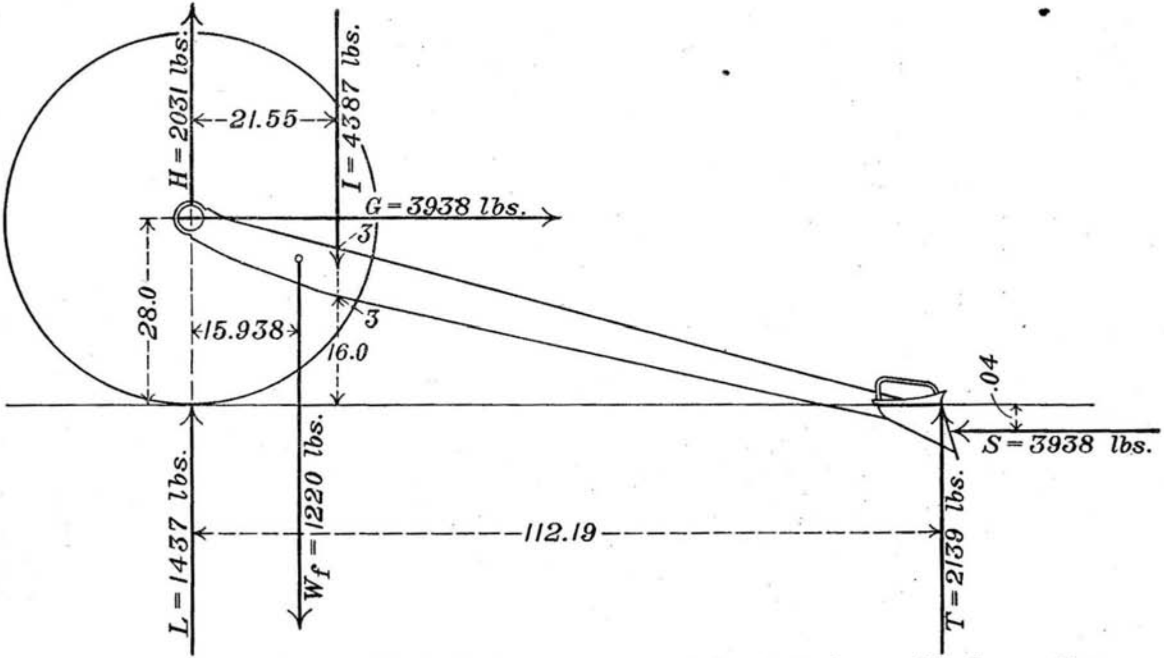

Fig. 18. — Forces on the Axle, Wheels, Flasks, and other Parts below the Rocker, considered as one Piece

four inches below the surface of the ground. The forces L, T, and S are shown in position, direction, and magnitude in Fig. 18.

Writing the ordinary equations of equilibrium

$$4387 - T + 1220 - L - 2031 = 0 \quad (47)$$

$$S - 3938 = 0 \quad (48)$$

Taking moments about the center of the axle, the point of intersection of the forces G, H, and L,

$$4387 \times 21.55 + S \times 32.0 - T \times 112.19 + 1220 \times 15.938 = 0 \quad (49)$$

From equation (48) $S = 3938$ lbs. (50)

From equation (49) $T = 2139$ lbs. (51)

From equation (47) $L = 1437$ lbs. (52)

Comparing the values of S, T, and L given in equations (50), (51), and (52) with those given in equations (21), (22), and (23) deduced by considering the carriage as one piece, it will be seen that the values of S and T deduced by the two methods are identical and the values of L differ by less than one pound, which proves the correctness of the values of all the forces determined above. The very slight difference in the values of L deduced by the two methods is due to not carrying out the values of the forces on the various parts beyond the decimal point.

PROBLEMS.

41. Figs. 19 to 24, inclusive, indicate diagrammatically the location, direction, and magnitude of the forces on the 5-inch barbette carriage, model 1903, when the gun has been fired at 0° elevation and is at its extreme limit of recoil. This carriage is in all respects similar to the 6-inch barbette carriage described in article 195, pages 335 to 337, Lissak's Ordnance and Gunnery. It was designed to present a resistance of 92,250 lbs. to the recoil of the gun.

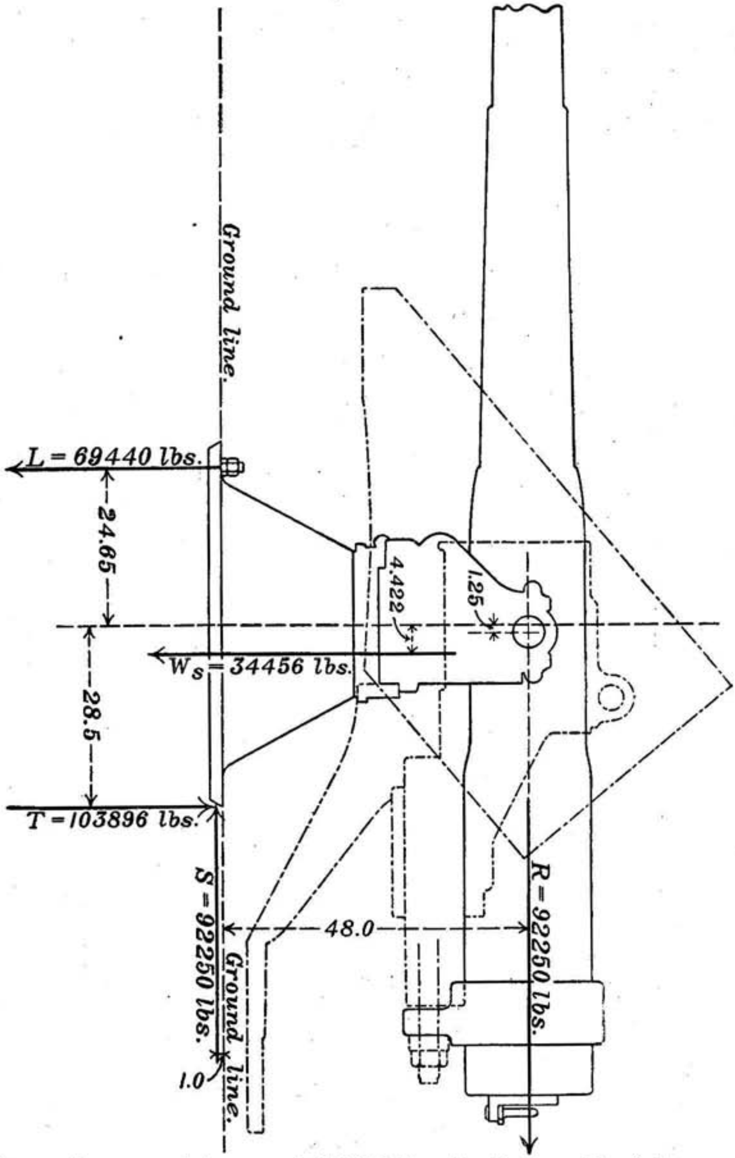

Fig. 19. — Forces on the Carriage considered as one Piece

1. Assuming a resistance of 92,250 lbs. to the recoil of the gun, determine the values of the forces S, T, and L shown in Fig. 19. Explain why these forces are developed at the points indicated.

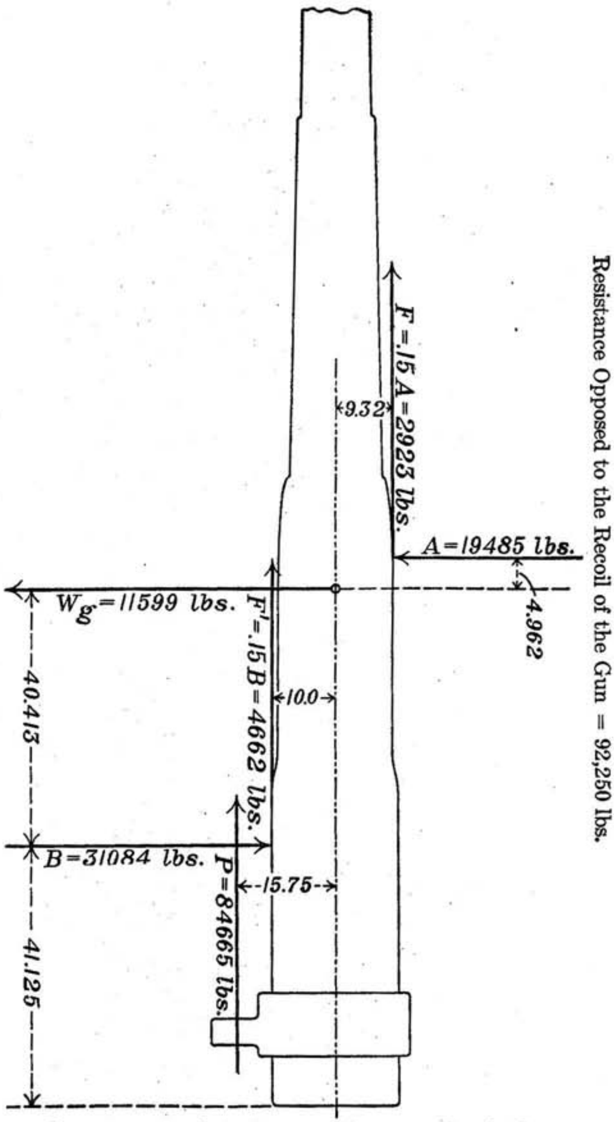

Fig. 20. — Forces on the Gun.

2. Assume the same resistance to the recoil of the gun as in problem 1, and determine the values of the forces A, B, P, F, and F' shown in Fig. 20. Explain why these forces are developed at the points indicated.

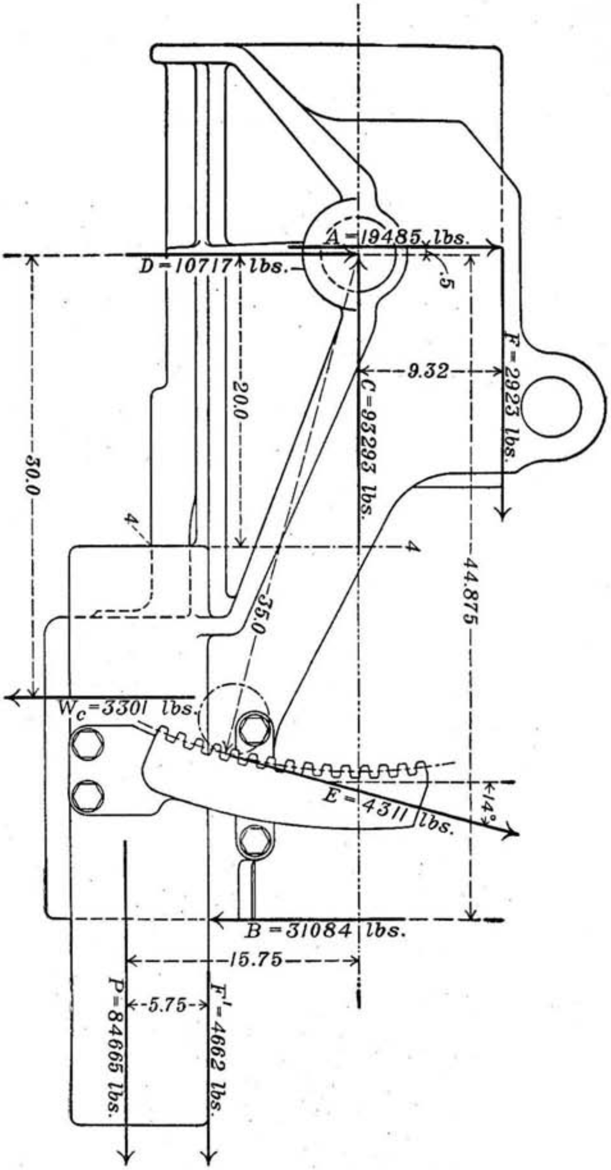

Fig. 21. — Forces on the Cradle.

3. Assume the values of the forces A, B, P, F, F', and W_c, and determine the values of the forces C, D, and E shown in Fig. 21. Explain why these forces are developed at the points indicated.

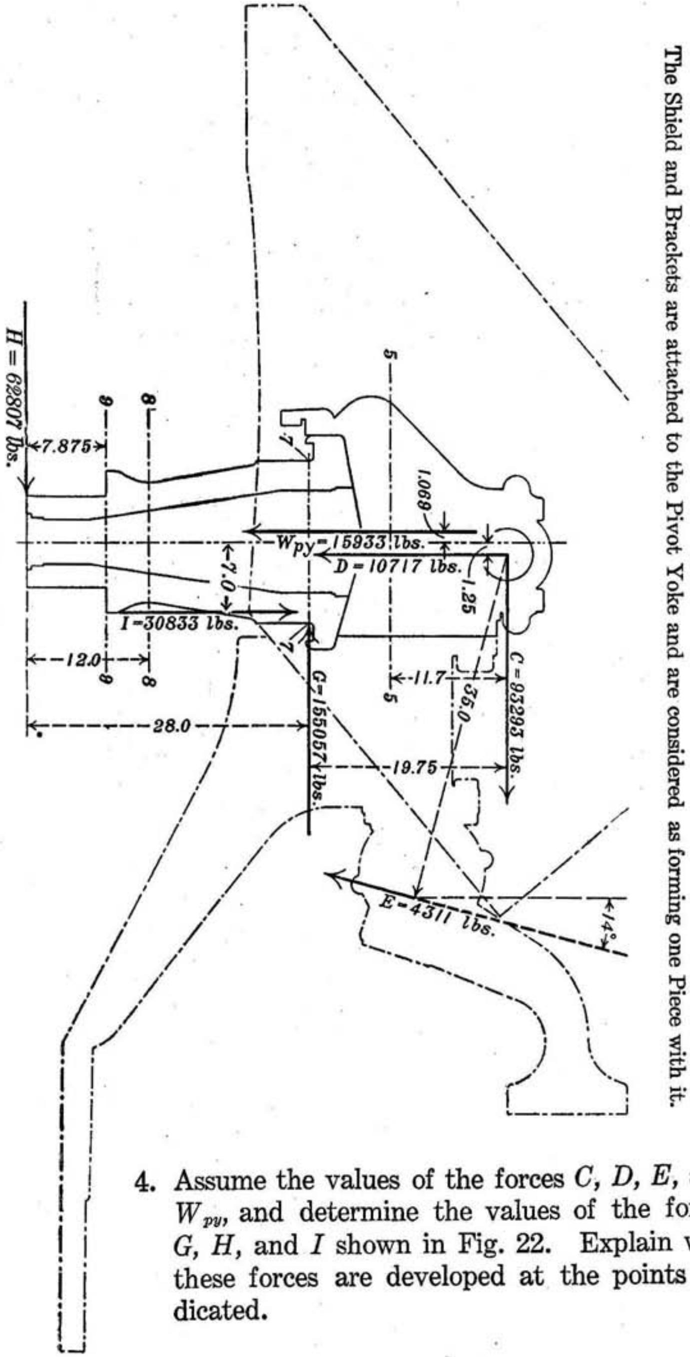

Fig. 22. — Forces on the Pivot Yoke.

4. Assume the values of the forces C, D, E, and W_{py}, and determine the values of the forces G, H, and I shown in Fig. 22. Explain why these forces are developed at the points indicated.

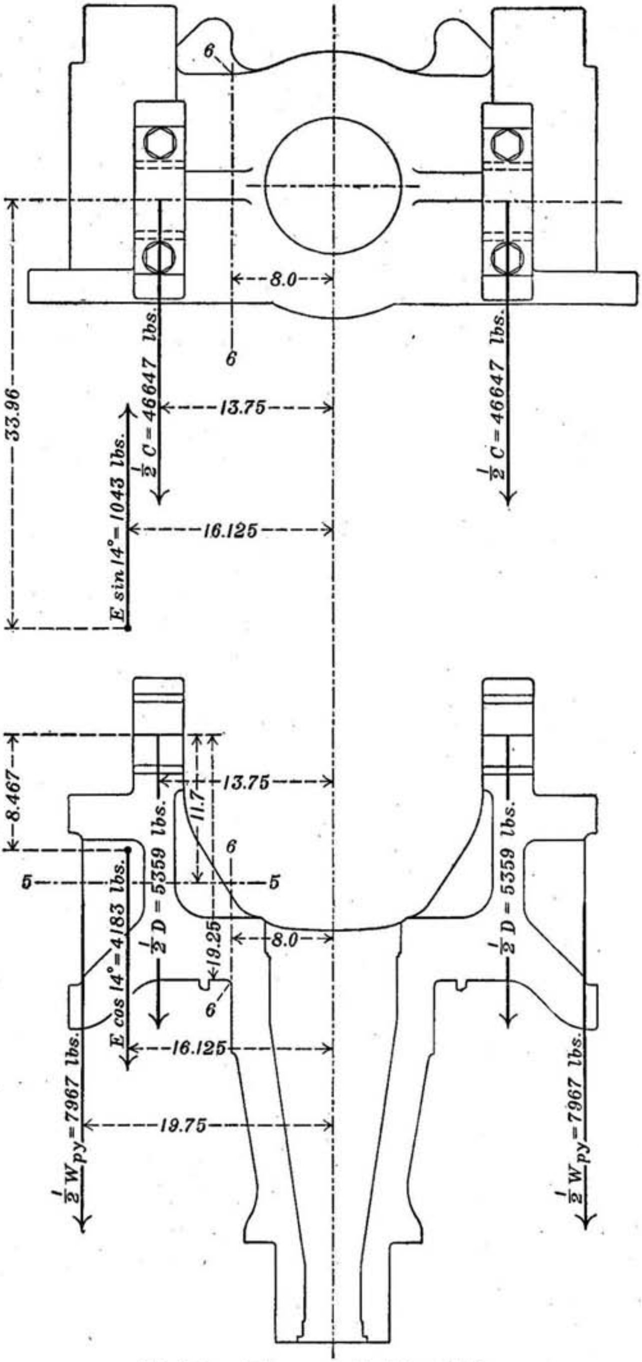

Fig. 23. — Forces on the Pivot Yoke.

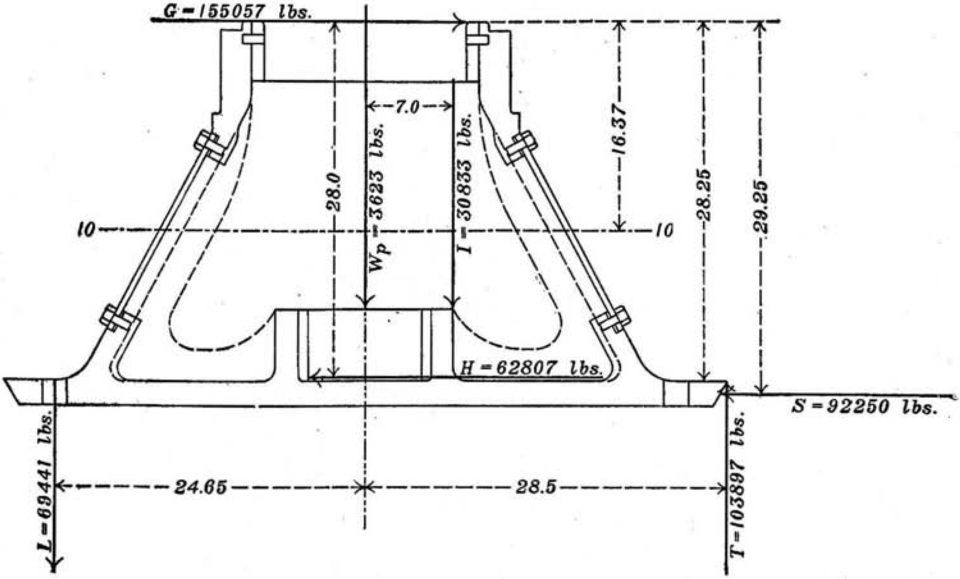

Fig. 24. — Forces on the Pedestal.

5. Assume the values of the forces G, H, I, and W_p, and determine the values of the forces S, T, and L shown in Fig. 24. Explain why these forces are developed at the points indicated.

CHAPTER III.

DETERMINATION OF THE FORCES BROUGHT UPON THE PRINCIPAL PARTS OF A DISAPPEARING GUN CARRIAGE BY THE DISCHARGE OF THE GUN.

42. The 6-inch disappearing carriage model of 1905 MI is chosen to illustrate the subject; and the forces brought upon the gun, gun levers, top carriage, and elevating arm will be determined.

The first step is to obtain the curve showing the velocity of free recoil of the gun as a function of time.

Velocity of the Projectile in the Bore. — Fig. 25 shows the velocity of the projectile in the bore as a function of the travel of the projectile in inches.

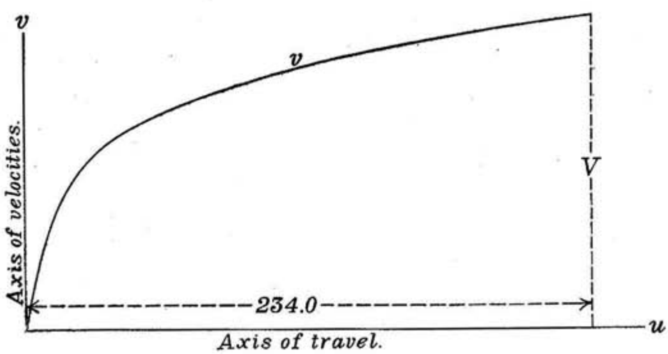

Fig. 25.

The ordinates of this curve corresponding to various values of the travel of the projectile calculated by the formulas of interior ballistics are given in Table 1.

TABLE 1.

Travel of projectile in inches.	Velocity in bore, feet per second.	Travel of projectile in inches.	Velocity in bore, feet per second.
2	223.24	54	1776.61
4	411.60	60	1829.30
6	577.41	72	1924.05
8	714.57	84	2004.32
10	835.79	96	2076.75
12	940.95	108	2142.55
15	1074.25	120	2202.73
18	1184.05	132	2259.86
21	1275.70	144	2309.20
24	1353.46	156	2356.67
27	1419.15	168	2400.89
30	1477.17	180	2442.23
33	1527.95	192	2481.01
36	1573.20	204	2517.49
42	1651.56	216	2551.89
48	1718.15	234	2600.02

Curve of Reciprocals. — Velocity and Travel of Gun in Free Recoil while the Projectile is in the Bore. — Fig. 26 shows the reciprocals of the velocities of the projectile in the bore as a function of the travel of the projectile in inches.

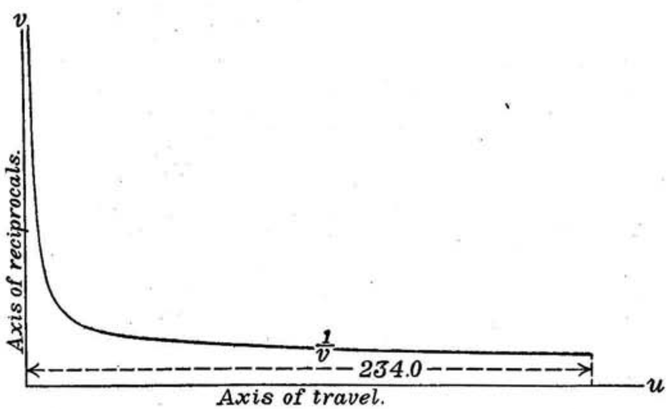

Fig. 26.

Table 2 gives the values of the reciprocals of the velocities of the projectile in the bore corresponding to various values of the travel of the projectile, the corresponding times obtained by measuring the areas under the curve of Fig. 26 up to the corresponding limiting ordinates, the corresponding values of the velocities of free recoil of the gun, and the corresponding distances traveled by it in free recoil. The velocity of free recoil of the gun while the projectile is in the bore is obtained from equation (2), page 275, Lissak's Ordnance and Gunnery, which is as follows:

$$v_f = \frac{w + \frac{1}{2}\tilde{\omega}}{W} v$$

in which w, the weight of the projectile, is 106 lbs.; $\tilde{\omega}$, the weight of the powder charge, is 30 lbs.; W, the weight of the gun, is 12,764 lbs., and v_f is the velocity of free recoil of the gun corre-

TABLE 2.

Travel of projectile in inches.	Reciprocal of velocity in bore.	Time in seconds.	Velocity of free recoil of gun, feet per second.	Length of free recoil in feet.
2	.0044796	.0014932	2.116	.00158
4	.0024296	.0020187	3.902	.00316
6	.0017359	.0023557	5.474	.00474
8	.0013995	.0026157	6.774	.00632
10	.0011965	.0028308	7.923	.00790
12	.0010628	.0030184	8.920	.00948
15	.0009309	.0032665	10.181	.01185
18	.0008446	.0034879	11.225	.01422
21	.0007839	.0036912	12.094	.01659
24	.0007389	.0038814	12.831	.01896
27	.0007047	.0040617	13.454	.02133
30	.0006770	.0042343	14.003	.02370
33	.0006545	.0044007	14.485	.02607
36	.0006356	.0045619	14.914	.02844
42	.0006055	.0048720	15.657	.03318
48	.0005820	.0051687	16.288	.03792
54	.0005629	.0054548	16.842	.04266
60	.0005467	.0057196	17.342	.04740
72	.0005194	.0062595	18.240	.05688
84	.0004989	.0067616	19.001	.06636
96	.0004815	.0072517	19.688	.07584
108	.0004667	.0077257	20.312	.08532
120	.0004540	.0081860	20.882	.09480
132	.0004425	.0086342	21.424	.10428
144	.0004331	.0090712	21.892	.11376
156	.0004243	.0094994	22.341	.12324
168	.0004651	.0099198	22.760	.13272
180	.0004095	.010333	23.152	.14220
192	.0004031	.010739	23.520	.15168
204	.0003972	.011139	23.866	.16116
216	.0003919	.011534	24.192	.17064
234	.0003846	.012116	24.648	.18486

τ = .0606 second. E = 1.6786 feet.

sponding to a velocity v of the projectile. The distance traveled by the gun in free recoil while the projectile is in the bore is given by the formula

$$u_f = \frac{w + \frac{1}{2}\tilde{\omega}}{W}\frac{u}{12}$$

in which w, $\tilde{\omega}$, and W have the same meanings as before, u is the travel of the projectile in inches, and u_f is the distance in feet traveled by the gun in free recoil.

Curve of Free Recoil. — From the data given in the third and fourth columns of table 2 the curve showing the velocity of free recoil of the gun as a function of time may be plotted up to the time when the projectile leaves the bore of the gun. The remainder of this curve up to the time when the maximum velocity of free recoil is reached and the powder gases cease to act on the gun is obtained as outlined in paragraph 163, page 278, Lissak's Ordnance and Gunnery. The maximum velocity of free recoil computed from equation (4), page 275, Lissak's Ordnance and Gunnery, is

$$V_f = (wV + 4700\tilde{\omega})/W = 32.64 \text{ f.s.}$$

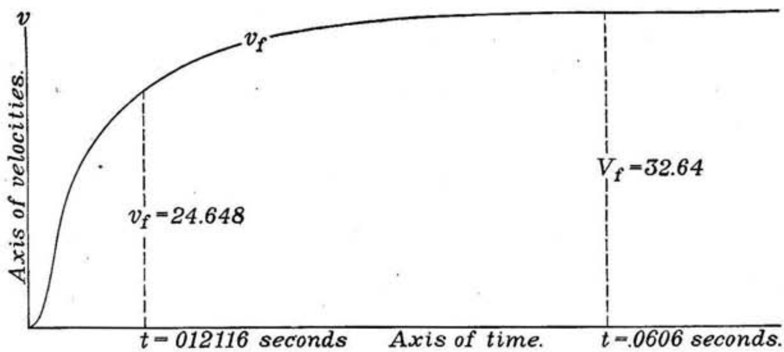

Fig. 27.

With this value as an ordinate to the proper scale a line is drawn parallel to the axis of t and the curve obtained from table 2 is continued with its general rate of change in curvature until it becomes tangent to this line. Fig. 27 shows the curve so obtained, representing the velocity of free recoil of the gun as a function of time.

From Fig. 27 the time when the maximum velocity of free recoil is reached is .0606 secs. This time is designated by τ.

To obtain to a close degree of approximation the distance traveled by the gun in free recoil after the projectile has left the bore and until the powder gases have ceased to act, the velocities of free recoil corresponding to .016, .020, .028, .038, and .048 secs. are measured from the curve shown in Fig. 27. We also have .012116 and .0606 as the times when the projectile left the bore and when the powder gases ceased to act, respectively. The velocities corresponding to these times in sequence are 24.648, 27.103, 28.764, 30.715, 31.881, 32.442, and 32.64 f. s., respectively. Assuming that between any two of these times the velocity increases uniformly, the space passed over by the gun in free recoil is equal to the product of the difference in time by the mean velocity during the interval.

For the first interval

$$\frac{24.648 + 27.103}{2} \times (.016 - .012116) = .10050 \text{ ft.}$$

For the second interval

$$\frac{27.103 + 28.764}{2} \times (.020 - .016) = .11173 \text{ ft.}$$

and similarly the spaces passed over by the gun during each of the succeeding intervals are found to be .237196, .31298, .321615, and .41001 ft., respectively. The sum of these spaces, 1.49374 ft., is the distance traveled by the gun in free recoil after the projectile has left the muzzle and until the powder gases have ceased to act. Adding to this distance .18486 ft., the space over which the gun traveled in free recoil while the projectile was in the bore, we have 1.6786 ft. as the total distance traveled by the gun in free recoil up to the time when the powder gases ceased to act on it. This distance is designated by E.

When a planimeter is available the distances recoiled may be more accurately obtained by measuring the areas under the curve, Fig. 27. The use of data given by the curve of free recoil will appear later.

43. Length and Lettering of Parts. — Reference to Coordinate Axes. — The parts upon which the forces are to be determined are shown in Fig. 28.

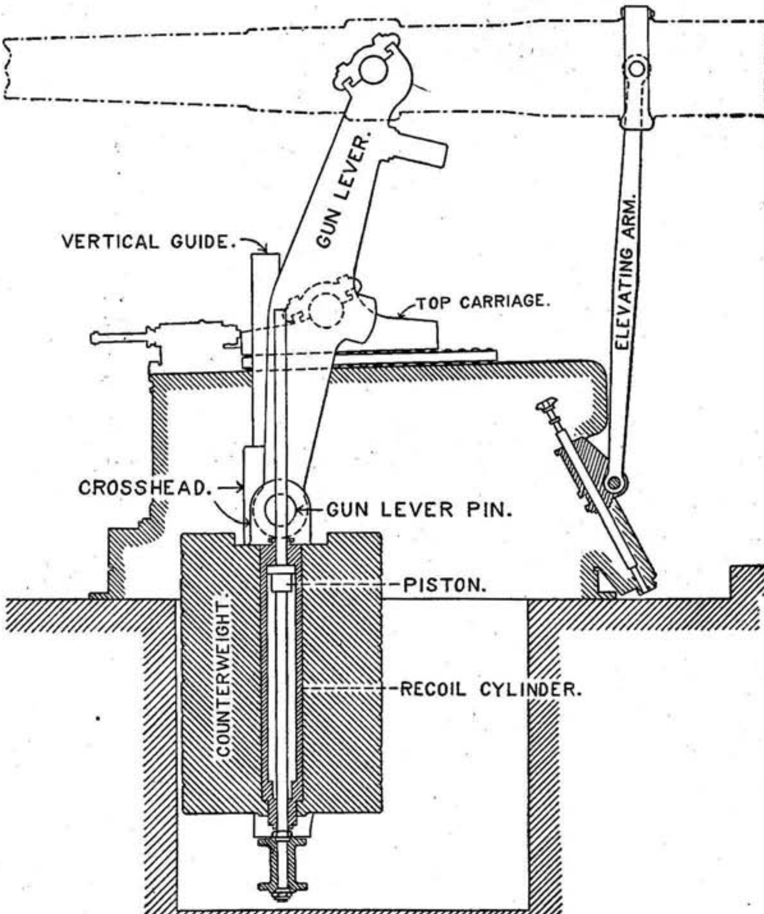

Fig. 28.