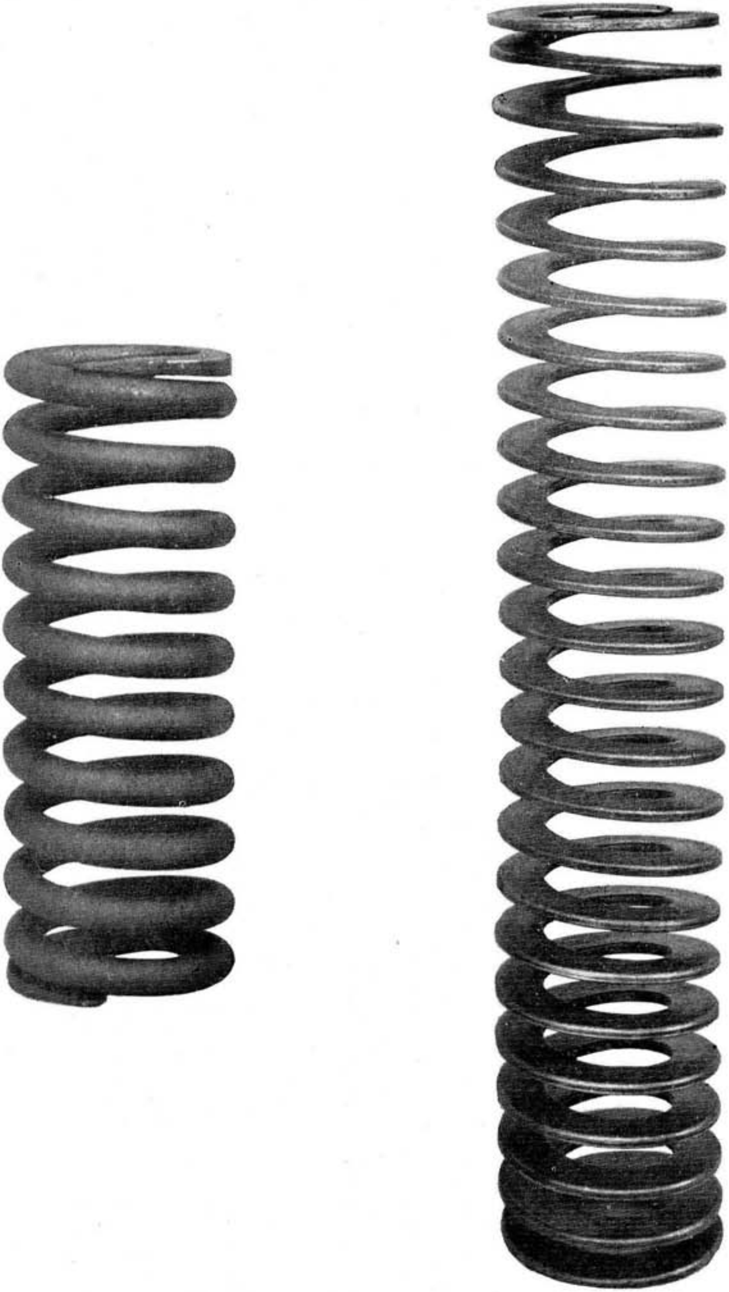


Fig. 93. — Counter-Recoil Springs.

Let *BA*, Fig. 94, be a bar fixed at the end *B* and subjected to a torsional moment caused by the force *F*. Then, as stated in article 71, page 111, the torsional stress per unit of area in the extreme fibre of any section of the bar will be

$$rS_t''' = S = M_t r / I_p \tag{1}$$

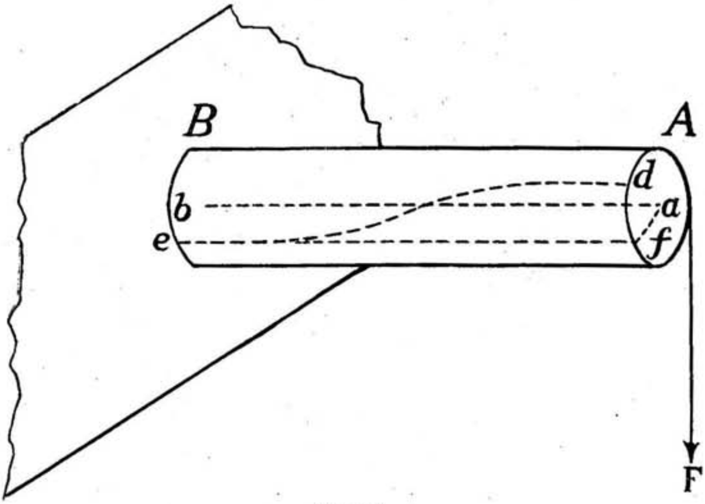



Fig. 94.

in which

$S_t'''$ is the torsional stress per unit of area at a unit's distance from the axis of the bar,

$S$ is the torsional stress per unit of area in the extreme fibre of a section,

$r$ is the distance in inches from the axis of the bar to the extreme fiber of a section,

$M_t$ is the torsional moment in in. lbs., and

$I_p$ is the polar moment of inertia of the section in ins.$^4$

In Fig. 94, which is reproduced from page 37, Fiebeger's Civil Engineering, *ef* is a surface fibre before being distorted by the twisting force *F* and *ed* is the position taken by this fibre when the force acts on the bar. Every cross-section of the bar except that at the fixed end *B* is rotated around the axis *ba* by the twisting force, each section being rotated through a slightly greater angle than the one next it on the side of the fixed end of the bar. The maximum rotation occurs in the section at the free end, whose plane contains the force, and the rotation of any other section is equal to the maximum rotation multiplied by the distance of the section under consideration from the fixed end of the bar divided by the length of the bar. If the force did not act at the end of the bar the maximum rotation would still occur in the section whose plane contains the force, and the rotations of the other sections nearer the fixed end of the bar would bear the same relations to the maximum rotation as before, providing we substitute for the length of the bar the distance from the section whose plane contains the force to the fixed end. All sections between the force and the free end of the bar would have the same rotation as that of the section whose plane contains the force.

Let $L$ be the length of the bar shown in Fig. 94,
$l$, the length of the arc *fd*, and
$E$, the torsional modulus of elasticity, sometimes called also the *coefficient of torsional elasticity;*

then, as shown on page 40, Fiebeger's Civil Engineering,

$$E = \frac{S}{l/L} \qquad (2)$$

or substituting for $S$ its value from equation (1) and solving for $l/L$

$$l/L = \frac{M_t r}{I_p E} \qquad (3)$$

Since $l$ is the length of the arc *fd* and $r$ is the distance of the extreme fibre from the axis of the bar, $l/r$ is the measure of the angle, expressed in radians, through which has rotated the section whose plane contains the twisting force. Solving equation (3) for $l/r$ we have

$$l/r = \frac{M_t L}{I_p E} \qquad (4)$$

**116. Stresses in Helical Springs.**—Let Fig. 95 represent a helical spring resting on a flat surface *JJ* and subjected to compression by the force *C*. Since the action line of the force coincides with the axis of the spring it is symmetrically placed with respect to each section of the bar from which the spring is coiled, and the stresses produced by it in all sections will be the same. The part of the bar between the point of application of the force

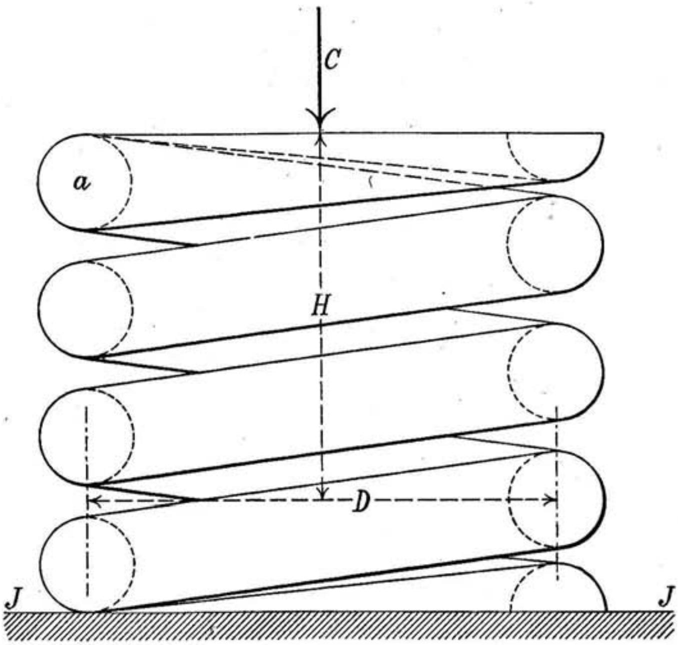



Fig. 95.

and section *a* is not considered in this discussion as it is an end coil that has been flattened and ground to provide an end surface perpendicular to the axis of the spring. End coils are called *ineffective* coils; they are not relied upon to add to the spring effect as a whole but merely to transmit the force to the rest of the spring.

Let *a* be a right section of the bar. As the bar is coiled into a helix this section and every other right section will be slightly inclined from the vertical, and the component of the force *C* perpendicular to the section will cause a slight bending stress and a slight compressive stress therein; but as these stresses are small they are neglected in spring computations, and the plane of a right section of the bar is assumed to be parallel to the axis of the spring and to the action line of the force. Since the bar is coiled around the axis of the spring, the plane of section *a* and of every other right section of the bar, considered as vertical sections, will contain the force which, under the assumption made, can produce no bending or compressive stress in the sections. A shearing stress in the sections is produced by the force but it is slight in comparison with the torsional stress and is on this account also neglected. As the force has a lever arm with respect to the axis of the coiled bar it has a torsional moment with respect to it and the corresponding maximum torsional stress in section *a* is, from equation (1),

$$S = \frac{M_t r}{I_p} = \frac{CDr}{2\,I_p} \qquad (5)$$

in which $C$ is the intensity of the force, $D$ is the mean diameter in inches of the helix formed by the coiled bar, $r$ is the distance in inches from the axis of the bar to the extreme fibre of the section, and $I_p$ is the polar moment of inertia of the section in inches.$^4$ It is evident that the stress in *a* will be transmitted from section to section of the bar until the part resting on the flat surface is reached and, since these cross-sections are all alike, equation (5) gives the maximum torsional stress $S$ that occurs in any part of the spring.

**117. Fundamental Equations Relating to Helical Springs.** — Let $l$ be the length of the arc through which the end of the extreme fibre at section *a* would rotate around the axis of the bar if straight under the action of the force, and let $L$ be the developed length of the portion of the bar between section *a* and the point where it rises above the supporting surface $JJ$, the end coils not being considered. Since a cross-section of the end coil, which rests on the supporting surface and is ground flat where it bears against that surface, cannot rotate under the action of the force, the section at the point where the bar rises above the supporting surface and where the effective part of the spring commences, may be considered as the fixed end of the bar.

The angle $\theta$ through which section *a* would rotate around the axis of the bar if straight under the action of the force is

$$\theta = l/r \text{ radians,}$$

and any other section at a distance $x$ from the fixed end, measured along the developed length of the bar, would rotate around the axis of the bar if straight through an angle

$$\theta' = \frac{l}{r} \times \frac{x}{L} \text{ radians.}$$

A section adjoining the latter and at a distance from it of $dx$, measured along the developed length of the bar toward section $a$, would rotate around the axis of the bar if straight through an angle

$$\theta'' = \frac{l}{r} \times \frac{x + dx}{L} \text{ radians,}$$

and relatively to the latter it would rotate through an angle

$$d\theta = \frac{l dx}{rL} \text{ radians,} \tag{6}$$

and this is also the angle through which the one section would rotate relative to the other under the action of the force where the bar is coiled into a helical spring for the axis of the bar may be considered straight in this case for any infinitesimal length $dx$.

Let $H$, Fig. 95, be the vertical distance from the center of this section to the point of application of the force $C$. Then, since the rotation of any section around the axis of the bar will cause every point of the spring above it to move in the arc of a circle whose center coincides with that of the section or with the projection of this center on the plane of rotation of the point, the displacement of the point of application due to the rotation of the one section under consideration past the other through an angle $d\theta$ around the axis of the bar is

$$ds = \{H^2 + (D/2)^2\}^{\frac{1}{2}} d\theta \tag{7}$$

This displacement may be resolved into horizontal and vertical components $dh$ and $dv$, respectively, as follows:

$$dh = H d\theta \tag{8}$$

and

$$dv = \frac{D}{2} d\theta \tag{9}$$

As each cross-section rotates around the axis of the bar through an angle $d\theta$ with respect to that immediately adjoining it and nearer the supporting surface, the point of application of the force will undergo successively the component displacements given in equations (8) and (9), it being understood in this connection that the distance $H$ is a variable. If we consider only the horizontal components of the displacements it will be seen that, as we pass around one complete coil of the spring from any point on the bar to a corresponding point immediately above it, the horizontal components neutralize each other and we may write

$$\int_0^L H\,d\theta = 0$$

The vertical components of the displacements, however, are seen to be cumulative so that the total displacement of the point of application under the action of the force is in the direction of the axis of the spring and equal to

$$\int_0^L \frac{D\,d\theta}{2} = \int_0^L \frac{Dl\,dx}{2\,rL} = \frac{D}{2}\theta = \frac{Dl}{2\,r} \tag{10}$$

from which we may deduce *that the total displacement of the point of application of the force, or the total compression of the spring caused by the force, is a movement in the direction of the axis of the spring equal to the mean radius of the coil multiplied by the total angle, expressed in radians, through which the section at the free end of the effective portion of the bar rotates under the action of the force.*

Calling the compression of the spring $N'\Delta$, in which $N'$ is the number of effective coils and $\Delta$ is the compression per coil, we have

$$N'\Delta = \frac{Dl}{2\,r}$$

and substituting for $l/r$ its value from equation (4) and for $M_t$ in the latter equation its value $CD/2$

$$N'\Delta = \frac{CD^2L}{4\,EI_p} \tag{11}$$

The developed length of the effective part of the bar is approximately $N'\pi D$, and substituting this value for $L$ in equation (11) and solving for $\Delta$ we have as the compression per coil

$$\Delta = \frac{\pi CD^3}{4\,EI_p} \tag{12}$$

Solving equation (5) for $C$ we have

$$C = \frac{2SI_p}{Dr} \tag{13}$$

Equations (11), (12), and (13) are the fundamental equations relating to helical springs. Equations (11) and (12) give the total compression (or extension) and the compression (or extension) per coil, respectively, caused by a force $C$; and equation (13) gives the force $C$ which will cause a maximum stress $S$ in the fibers of the bar from which the spring is coiled. By giving to $S$ in equation (13) the maximum value permissible in connection with the quality of the material, the maximum force which the spring can support without undergoing a permanent distortion can be determined. The compression corresponding to this maximum force is then obtained from equation (11) or (12).

Equations (11), (12), and (13) have been deduced under the supposition that the force compresses the spring but they are equally applicable when the force extends it. In the latter case special means have to be provided for holding the spring at one end and to enable the force to be applied in the line of the axis of the spring at the other end. Supposing these means to have been provided for the spring shown in Fig. 95, it is evident that if the force $C$ acts upward it will extend the spring producing in the bar from which it is coiled a torsional stress whose intensity will be the same as when the force acts downward. The angle $l/r$ through which section $a$ rotates under the action of the force when it acts upward will also have the same numerical value as when the force acts downward, but the direction of rotation of the section around the axis of the bar will be opposite in the two cases.

**118. Helical Springs Coiled from Bars of Circular Cross-Section.** — For a circular cross-section the value of $I_p$ is $\pi d^4/32$ ins.$^4$, $d$ being the diameter of the bar. Substituting this value of $I_p$ in equation (13) it becomes

$$C = \frac{.3927\, Sd^3}{D} \tag{14}$$

Substituting the value of $I_p$ in equations (11) and (12) they become, respectively,

$$N'\Delta = \frac{8\,CD^2L}{E\pi d^4} \tag{15}$$

and

$$\Delta = \frac{8\,CD^3}{Ed^4} \tag{16}$$

Equations (14), (15), and (16) are the formulas generally used in computing the strength and compression or extension of helical springs coiled from bars of circular cross-section.

**119. Helical Springs Coiled from Bars of Rectangular Cross-Section.** — As first shown by Saint-Venant, an eminent French investigator, a plane section whose axes are unequal becomes a warped surface when subjected to great torsional strain, and its polar moment of inertia is not then equal to the sum of its moments of inertia about two axes in its plane perpendicular to each other passing through its center of gravity. Reuleaux states that the polar moment of inertia of a rectangle when subjected to great torsional strain is

$$I_p = \frac{h^3b^3}{3\,(h^2+b^2)} \text{ ins.}^4$$

and that the distance from the center of gravity to the point of the section most distant from it is

$$r = \frac{hb}{\sqrt{h^2+b^2}} \text{ ins.}$$

in which $h$ and $b$ are the sides of the rectangle expressed in inches. Substituting these expressions for $I_p$ and $r$ in equation (13), we have

$$C = \frac{2\,Sh^3b^3\sqrt{h^2+b^2}}{D\,3\,(h^2+b^2)\,hb} = \frac{2\,S}{D} \times \frac{h^2b^2}{3\sqrt{h^2+b^2}} \tag{17}$$

and substituting the expression for $I_p$ in equations (11) and (12) they become, respectively,

$$N'\Delta = \frac{3\,CD^2L}{4\,E} \times \frac{h^2+b^2}{h^3b^3} \tag{18}$$

and

$$\Delta = \frac{3\,\pi CD^3}{4\,E} \times \frac{h^2+b^2}{h^3b^3} \tag{19}$$

Equations (17), (18), and (19) are Reuleaux's formulas for helical springs coiled from bars of rectangular cross-section.

Equations (14) to (19), inclusive, are used by the Ordnance Department, U. S. Army, in the design of counter-recoil springs. Since $D$ is the mean diameter of the helix, the outer diameter of a helical spring coiled from a bar of circular cross-section is $D + d$ and its inner diameter is $D - d$. The outer diameter of a helical spring coiled from a bar of rectangular cross-section is $D + b$ and its inner diameter is $D - b$, $b$ being the side of the rectangle that is perpendicular to the axis of the spring.

**120. Requirements to be Fulfilled by a Counter-Recoil Spring. — Nomenclature.** — In the design of a counter-recoil spring the principal requirements to be fulfilled are the following:

(*a*) The spring must have sufficient power to return the gun into battery with certainty at any elevation for which the carriage is designed.

(*b*) Its length when the gun is in battery, called its assembled height, must not be inconveniently great.

(*c*) It must be capable of sustaining indefinitely a compression corresponding to its assembled height and of being repeatedly compressed through a further distance somewhat greater than the length of recoil of the gun, without suffering permanent deformation.

(*d*) The outer diameter of the coiled spring must not be inconveniently great.

(*e*) Its inner diameter must be large enough to permit the spring to pass over the spring piston-rod, hydraulic recoil cylinder, or other part on which the spring is assembled.

Let

$C$ be the maximum capacity of the spring in pounds, that is, the maximum force which it can exert or which can act against it without causing the maximum stress in it to exceed a permissible amount.

$T$ be its capacity in pounds at assembled height.

$\Delta$ be the compression per coil in inches.

$\Delta_r$ be the remaining compression per coil at assembled height in inches.

$N'$ be the number of effective coils in the spring.

$N$ be the total number of coils in the spring.
$N'\Delta$ be the total compression of the spring in inches.
$N'\Delta_r$ be the total remaining compression at assembled height in inches.
$l$ be the length of recoil in inches.
$A$ be the assembled height of the spring in inches.
$H$ be the solid height of the spring in inches.
$F$ be the free height of the spring in inches.
$P$ be the number of sections of the spring.
$S$ be the maximum permissible stress, which will be taken as 100000 lbs. per sq. in.
$E$ be the torsional modulus of elasticity, equal to 12600000 lbs. per sq. in.
$\alpha$ be the maximum angle of elevation of the gun.
$W$ be the weight in pounds of the recoiling parts.
$B$ be the friction in pounds of the packing around the piston-rod of the hydraulic cylinder.
$f$ be the coefficient of starting friction.

**121. Design of Counter-Recoil Springs Coiled from Bars of Circular Cross-Section.** — In order that the spring may be capable of returning the gun into battery with certainty after it has been fired at the maximum angle of elevation $\alpha$, the force $T$ which it must be capable of exerting at assembled height must be at least equal to the component of the weight of the recoiling parts parallel to the surface of a plane inclined at an angle $\alpha$ with the horizontal plus the friction due to the component of the weight of the recoiling parts normal to this plane plus the friction of the packing around the piston-rod, or

$$T = W \sin \alpha + fW \cos \alpha + B \tag{20}$$

The force which a counter-recoil spring is capable of sustaining or exerting varies directly with its compression so that the force to which it is subjected at the end of recoil is necessarily greater than that to which it is subjected at assembled height; and by varying the design of the spring the ratio $C/T$ can be varied within wide limits. In order, however, that the assembled height of the spring shall be as small as possible it is necessary that it shall be compressed at the end of recoil until the coils are nearly in contact, when the force to which it is subjected will be nearly the maximum which it is capable of sustaining without injury.

The force $T$ which the spring must exert as its assembled height having been determined from equation (20), it remains to determine the force $C$, or maximum capacity of the spring. The movement of the spring required between the loads $T$ and $C$ will be fixed and somewhat greater than the length of recoil. We will place it equal to $l + e$ where $e$ may be 1 inch or more. With the fixed movement, fixed load $T$ at assembled height, and fixed value of $D$, the mean diameter of the coils, we will now determine the relation between $C$ and $T$ that will result in a minimum solid height and therefore a minimum assembled height of the spring.

It is important to have the solid and assembled heights a minimum for the reason that a saving in weight and cost of spring cylinders, piston rods, etc., is thus effected.

From equation (16) we have for the load $C$,

$$N'\Delta = \frac{8\,N'CD^3}{Ed^4},$$

and for the load $T$,

$$N'(\Delta - \Delta_r) = \frac{8\,N'TD^3}{Ed^4}.$$

Subtracting the latter equation from the former, we have,

$$N'\Delta_r = \frac{8\,N'dD^3}{Ed^5}(C - T).$$

Considering all coils effective, we have,

$$N'\Delta_r = l + e = \text{a constant.}$$
$$N'd = H = \text{solid height of spring.}$$

Substituting these values and the value of $d^5$ obtained from equation (14) in the above equation, we obtain

$$H = \frac{E\,(l + e)}{\pi\,S^{\frac{5}{3}}\,D^{\frac{4}{3}}}\,\frac{C^{\frac{5}{3}}}{C - T}.$$

To get the values of $C$ for which $H$ is a minimum we differentiate $H$ with respect to $C$ and place the differential coefficient equal to 0. This gives, finally, neglecting the constant coefficient,

$$\frac{dH}{dC} = \frac{5/3\, C^{\frac{2}{3}}\,(C-T) - C^{\frac{5}{3}}}{(C-T)^2} = 0$$

which makes $C = 2.5\,T$. This is the condition that should be assumed if it is desired to keep the mean diameter of the coils a fixed amount.

By similar reasoning it may be shown that if instead of assuming $D$ constant, we assume $D + d$, or the outside diameter of the spring constant, the solid height will be a minimum when $C = 2\,T$.

This is the condition that must be assumed when it is desired to design a spring to work inside a cylinder whose diameter has been fixed by other considerations.

If we assume $D - d$, or the inside diameter of the spring constant we obtain $C = 3\,T$, as the condition for minimum solid height. This is the condition that must be assumed when it is desired to design a spring to work around a rod whose diameter has been fixed by other considerations.*

Assuming the first of the above conditions we have,

$$C = 2.5\,T = 2.5\,(W \sin\alpha + fW \cos\alpha + B) \tag{21}$$

Since the force which a spring is capable of exerting varies directly as its compression we may write

$$\Delta : \Delta - \Delta_r :: C : T$$

or

$$\Delta_r = \Delta - \frac{T}{C}\Delta = .6\,\Delta \tag{22}$$

In order that the spring shall not be compressed at the end of recoil to its solid height, that is, until one coil bears solidly against the next, the total remaining compression at assembled height $N'\Delta_r$ should be somewhat greater than the length of recoil. The excess of $N'\Delta_r$ over the length of recoil has varied somewhat in different designs but it will be sufficient under ordinary circumstances to take it equal to one inch. In order, however, that

* The above demonstration as to the proper ratio $\frac{C}{T}$ under various conditions was prepared by Lt. Col. W. H. Tschappat, Ord. Dept., U. S. A.

the equations may be of general application this excess will be represented by $e$, and we may write

$$N'\Delta_r = .6\,N'\Delta = l + e$$

or

$$N' = \frac{5\,(l + e)}{3\,\Delta}. \qquad (23)$$

**122. Solid Height of Spring Column.** — The solid height of a spring is its length when it is compressed until each coil bears solidly against those adjoining it. It has already been explained that the end coils of a counter-coil spring are considered to be ineffective. By examination of Fig. 95 it will be seen that in order to obtain surfaces at the ends of the spring which are truly perpendicular to its axis it is necessary, in addition to closing the ends of the end coils into contact with the adjoining coils, to grind away part of each end coil, the amount ground away being dependent on the amount the end coil is flattened down and on the ratio of the distance between the coils at assembled height to the dimension of the bar parallel to the axis of the spring. While no general rule can be given to cover every case, it will be assumed in this discussion that one-half of each end coil is ground away, commencing with a very light cut at the section adjoining the effective coil and increasing the cut gradually until the end of the end coil is reduced to a comparatively thin edge. Under this assumption the space taken up by each end coil when the spring is compressed to its solid height is only about one-half of that taken up by an effective coil, and the effect of the end coils on the solid height of the spring may be obtained by considering that only half a coil at each end is ineffective but that its volume is equal to that of an effective half coil.

Because of the difficulty of manufacturing very long helical springs it is frequently necessary to make a counter-recoil spring in two or more sections that are placed end to end in the spring cylinder with pieces of metal called separators between them.

If $P$ be the number of sections of the spring, $d$ the diameter of the coiled bar, and $t$ the thickness of each separator, the solid height of the spring column, under the assumption that one-half a coil at each end of a spring section is ineffective and that the volume of an ineffective half coil is the same as that of an effective half coil, will be

$$H = (N' + P)\,d + (P - 1)\,t \qquad (24)$$

**123. Assembled Height of Spring Column.** — Since the total remaining compression of the spring column at assembled height is to be $e$ inches greater than the length of recoil we may write

$$A = (N' + P)\,d + (P - 1)\,t + l + e \qquad (25)$$

**124. Free Height of Spring Column.** — The free height of the spring column is its solid height plus the total compression which it is capable of undergoing, or

$$F = (N' + P)\,d + (P - 1)\,t + N'\Delta$$

and substituting for $N'$ in the last term of this expression its value from equation (23)

$$F = (N' + P)\,d + (P - 1)\,t + \frac{5\,(l + e)}{3} \qquad (26)$$

**125. Introduction of Values of Constants in Equations (14) and (23).** — Replacing $S$ in equation (14) by its value, 100000 lbs. per sq. in., and solving for $d$, we have

$$d = [8.46864 - 10]\,C^{\frac{1}{3}}D^{\frac{1}{3}} \qquad (27)$$

Substituting for $\Delta$ in equation (23) its value from equation (16) and for $E$ in the latter equation its value, 12600000 lbs. per sq. in.

$$N' = \frac{[6.41913]\,(l + e)\,d^4}{CD^3} \qquad (28)$$

the figures in brackets in equations (27) and (28) being the logarithms of the numbers.

**126. Order of Procedure.** — In designing a counter-recoil spring column the length of recoil and the value of $C$, equation (21), are fixed by the construction of the carriage, which also restricts within comparatively narrow limits the outer diameter of the spring and, consequently, its mean diameter. Moreover the assembled height of the spring column must not be inconveniently great. In the solution of the problem a convenient value of $D$ is first selected for trial. With this value of $D$ and the fixed value of $C$, the diameter of the bar from which the spring is to be coiled is obtained from equation (27), and this in connection with the assumed value of $D$ determines the inner and outer diameters of the spring. If these are satisfactory the required number of effective coils is next obtained from equation (28) after substituting therein the fixed values of $C$ and $l$, the desired value of $e$, the assumed value of $D$, and the resulting value of $d$. The number of effective coils will determine how many spring sections are required in the spring column, and this having been decided and a suitable value for the thickness of the separators assumed, the solid height of the column is given by equation (24), the assembled height by equation (25), and the free height by equation (26).

If the assembled height thus determined is too great another value for $D$ is selected and the assembled height again determined. The assembled height depends directly on the solid height and the latter upon the diameter of the bar and the number of coils. By reference to equation (27) it is seen that the value of $d$ increases directly as $D^{\frac{1}{3}}$, and by substituting for $d$ in equation (28) its value from equation (27) it will be found that the value of $N'$ varies inversely as $D^{\frac{5}{3}}$. The product $N'd$, which is practically the solid height, therefore varies inversely as $D^{\frac{4}{3}}$. The assembled height for a given length of recoil consequently decreases rapidly with the increase in the mean diameter of the spring.

**127. Design of Counter-Recoil Springs Coiled from Bars of Rectangular Cross-Section.** — Springs coiled from bars of rectangular cross-section are capable of greater compression for a given solid height than those coiled from bars of circular cross-section and, therefore, when the former are used $C$ is taken equal to $2\,T$, whence

$$C = 2\,(W \sin \alpha + fW \cos \alpha + B) \tag{29}$$

and

$$N' \Delta_r = .5\, N' \Delta = l + e$$

or

$$N' = \frac{2\,(l + e)}{\Delta} \tag{30}$$

The dimension of the rectangular cross-section parallel to the axis of the spring being $h$, the solid height of the spring column, under the assumption made in article 122 as to the ineffective coils, is

$$H = (N' + P)\,h + (P - 1)\,t, \tag{31}$$

the assembled height is

$$A = (N' + P)\,h + (P - 1)\,t + l + e, \tag{32}$$

and the free height is

$$F = (N' + P)\,h + (P - 1)\,t + 2\,(l + e) \tag{33}$$

Let $r$ equal the ratio $b/h$, whence $b = rh$. Substituting this value of $b$ in equation (17), replacing $S$ by its value, 100000 lbs. per sq. in., and solving for $h$, we have

$$h = [8.39203 - 10]\,(1 + r^2)^{\frac{1}{6}}\,r^{-\frac{2}{3}}\,C^{\frac{1}{3}}D^{\frac{1}{3}} \tag{34}$$

Substituting for $\Delta$ in equation (30) its value from equation (19) and for $E$ and $b$ in the latter equation their values of 12600000 lbs. per sq. in. and $rh$, respectively,

$$N' = \frac{[7.02919]\,(l + e)\,h^4}{CD^3} \times \frac{r^3}{1 + r^2} \tag{35}$$

For a given value of $r$ the product $N'h$, like the product $N'd$ for springs coiled from bars of circular cross-section, varies inversely as $D^{\frac{4}{3}}$.

By substituting in equation (35) the value of $h$ from equation (34) it will be seen that $N'$ varies approximately inversely as $r^{\frac{1}{3}}$ and the product $N'h$ approximately inversely as $r^{\frac{2}{3}}$. The solid and assembled heights for a given length of recoil will, therefore, decrease rapidly as $r$ increases, and if a value of $r = 4$ or $r = 5$ be assumed the assembled height will be much less than it would for a value of $r = 1$. When $r = 1$, that is, when the bar is of square cross-section, the power and compressibility of the spring are about the same as for one coiled from a bar of circular cross-section. A spring coiled from a bar of rectangular cross-section with a value of $r$ from 4 to 5 has, therefore, a very decided advantage over one coiled from a bar of circular cross-section in that its assembled height for a given length of recoil is much less than that of the latter spring. The disadvantage of springs coiled from bars of rectangular cross-section has been the difficulty of their manufacture and until within a comparatively few years much trouble has been experienced in getting satisfactory springs of this type from manufacturers. The practical limit for $r$ in connection with a suitable mean diameter of the spring seems at present to be about 5, this value being used in the counter-recoil springs of the 3-inch field carriage.

The method of designing counter-recoil springs to be coiled from bars of rectangular cross-section is identical in principle with that described for springs coiled from bars of circular cross-section, the only difference being that a value for $r$ must first be selected. Ordinarily this value will be taken as large as practicable, at present not exceeding about 5.

**128. Telescoping Springs.** — To reduce the assembled height of the spring column when the length of recoil is great, particularly if it is desired to use springs coiled from bars of circular cross-section, telescoping springs are used, as in the case of the 5-inch barbette carriage, model of 1903, the 4.7-inch siege carriage, model of 1906, the 6-inch siege howitzer carriage, model of 1908, etc.

The principle of the telescoping spring is shown in Fig. 96. $A$ is the spring cylinder attached to a non-recoiling part of the

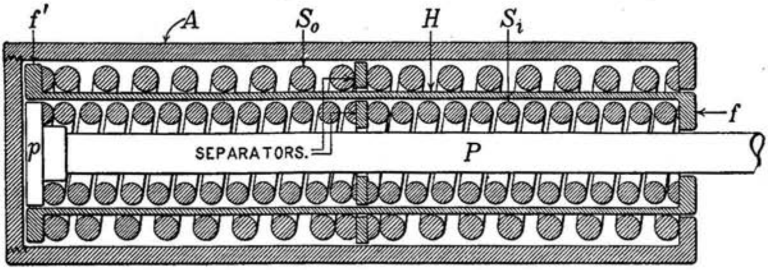



Fig. 96.

carriage, $P$ is the spring piston-rod attached to the gun, $S_o$ is the outer and $S_i$ the inner spring, or spring column in case there is more than one spring section, and $H$ is the stirrup connecting the inner and outer spring columns. The stirrup is a hollow cylinder of steel with an inward projecting flange $f$ at its rear end and an outward projecting flange $f'$ at its front end. When the gun recoils it draws the spring piston-rod with it, compressing the spring column $S_i$ between the spring piston $p$ and the inner flange $f$ of the stirrup. At the same time the pressure on the flange $f$ is communicated through the stirrup and its outer flange $f'$ to the outer spring column $S_o$, compressing it also. Owing to the compression of the outer spring column the rear ends of the stirrup and inner spring column will move out of the opening provided for the purpose in the spring cylinder, but the rear end of the outer column rests against the end of the spring cylinder and cannot move during recoil. The sum of the compressions of the two columns is equal to the length of recoil of the gun. It is evident that the number of spring columns connected by stirrups can be increased to three or more, if desired, with consequent decrease in the assembled height for a given length of recoil. The forces exerted by the spring columns must be equal to each other at all times.

In a special case such as that of the 5-inch barbette carriage, model of 1903, to be described later, it may not be advisable to permit the rear ends of the stirrup and inner spring column to be drawn through an opening in the spring cylinder during recoil. When this is so the assembled height of the inner column must be such that it can, when the gun is in battery, be held by the stirrup away from the rear end of the spring cylinder at a distance at least equal to the amount by which the outer column is compressed during recoil plus the thickness of the rear flange of the stirrup, for otherwise the end of the stirrup would strike the rear end of the cylinder during recoil.

**129. Design of Telescoping Springs.** — As in the case of non-telescoping springs the mean diameters of the springs are first selected for trial, keeping in view the limitations imposed by the design of the carriage. A convenient mean diameter for the outer spring would be selected and the mean diameter of the inner spring made as great as possible under the circumstances, taking into consideration the necessary thickness of wall of the stirrup and the clearances required between it and the springs. To allow for variations in the diameters of the springs during manufacture and for the bulging outward of the springs, which is likely to occur when they are compressed nearly to their solid height, it is well to allow a clearance of about .3 in. between the outer diameter of the inner spring and the inner diameter of the stirrup and between the outer diameter of the outer spring and the inner diameter of the spring cylinder. The thickness of wall of the stirrup and the thickness of its flanges are determined in accordance with the principles discussed in Chapter IV. The mean diameters of the springs having been selected, the diameters of the bars from which they must be coiled are next obtained from equation (27).

In the ordinary case the assembled heights of the inner and outer spring columns will be equal. Let the subscript 1 be used to distinguish the symbols pertaining to the outer spring column and the subscript 2 to distinguish those pertaining to the inner column, and let $l_1 + e/2$ and $l_2 + e/2$ be the total remaining compressions at assembled height of the outer and inner spring columns, respectively. Then we may write from equation (25)

$$(N'_1 + P_1)\,d_1 + (P_1 - 1)\,t_1 + l_1 + e/2 = (N'_2 + P_2)\,d_2 + (P_2 - 1)\,t_2 + l_2 + e/2$$

or, since $P$ and $t$ will ordinarily be the same in the outer and inner spring columns,

$$(N'_1 + P)\,d_1 + l_1 = (N'_2 + P)\,d_2 + l_2 \qquad (36)$$

From equation (28), noting that $l + e$ becomes in this case $l_1 + e/2$ for the outer spring column and $l_2 + e/2$ for the inner,

$$N'_1 = \frac{[6.41913]\,(l_1 + e/2)\,d_1^4}{CD_1^3}$$

and

$$N'_2 = \frac{[6.41913]\,(l_2 + e/2)\,d_2^4}{CD_2^3}$$

and substituting these values in equation (36) it becomes

$$\left\{\frac{[6.41913]\,(l_1 + e/2)d_1^4}{CD_1^3} + P\right\}d_1 + l_1 = \left\{\frac{[6.41913]\,(l_2 + e/2)d_2^4}{CD_2^3} + P\right\}d_2 + l_2 \qquad (37)$$

We also have

$$l_1 + l_2 = l \qquad (38)$$

In equations (37) and (38) $C$ and $l$ are known from the construction of the carriage, $P$ and $e$ are also known, their values having been decided upon earlier, $D_1$ and $D_2$ have been assumed and $d_1$ and $d_2$ calculated from equation (27). The only unknown quantities, therefore, are $l_1$ and $l_2$ and their values can be obtained by the solution of the equations. Having obtained the values of $l_1$ and $l_2$ the corresponding values of $N'_1$ and $N'_2$ are obtained from equation (28) and the solid, assembled, and free heights from equations (24), (25), and (26), respectively. As a check on the accuracy of the work the assembled heights should be equal. If the assembled height of the spring columns thus determined is not satisfactory other values for $D_1$ and $D_2$ would be selected and the assembled height re-determined and so on. It is of course not essential that the assembled heights of the inner and outer columns shall be equal, and the total required compression may if desired be divided between the columns in any arbitrary manner. For a given length of recoil, however, the least assembled height is obtained when it is the same for both columns.

**130. Design of Non-Telescoping Springs Assembled One Within the Other.** — If it is desired to decrease the assembled height somewhat without increasing the outer diameter of the spring, the amount of the desired decrease not being so considerable as to require the use of telescoping springs, it can be done by placing one spring within the other as shown in Fig. 97,

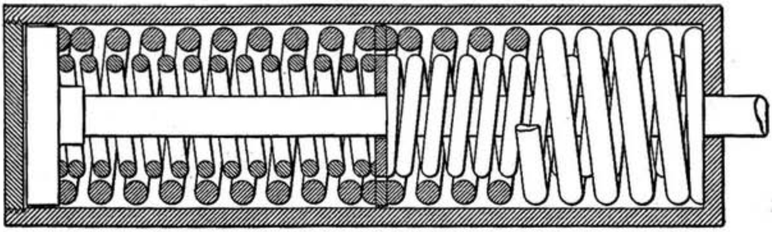


Fig. 97.

in which case each spring column sustains a part only of the total load. When springs are assembled in this manner the inner and outer springs must be coiled in opposite directions so that if one breaks the broken bar will not in untwisting be caught between the coils of the other.

In the equations that follow the symbols relating to the outer spring column will be distinguished by the subscript 1 and those relating to the inner column by the subscript 2. From Fig. 97 it is seen that the assembled heights of the inner and outer spring columns must be equal, and, if a minimum assembled height is desired, the total remaining compressions at assembled height and the solid heights of the columns should also be equal. Since the solid heights are to be equal we may write from equation (24)

$$(N'_1 + P_1)\, d_1 + (P_1 - 1)\, t_1 = (N'_2 + P_2)\, d_2 + (P_2 - 1)\, t_2$$

In this equation we may neglect the terms $P_1d_1$, $P_2d_2$, $(P_1 - 1)\, t_1$, and $(P_2 - 1)\, t_2$ for $P$ may be taken the same in both columns and the slight difference in assembled height due to the difference in the diameter of the bars in the end coils of the two columns can be made up by the difference in thickness of the separators in the columns, if any are used, or by the manufacturer in a number of ways, by increasing slightly the number of coils in the inner spring or by increasing slightly the pitch of the coils at assembled height, it being understood that the manufacturer is not limited by the specifications to the number of coils but only to the strength of the springs at assembled and solid heights, to inner and outer diameters, and to maximum solid heights which must not be exceeded.

Making these omissions, we have

$$N'_1\, d_1 = N'_2\, d_2$$

and substituting for $N'_1$ and $N'_2$ their values from equation (28) and for $d_1$ and $d_2$ in the resulting expression their values from equation (27) we have

$$\frac{[6.41913]\,(l_1 + e)\,\{[8.46864 - 10]\,C_1^{\frac{1}{3}}\,D_1^{\frac{1}{3}}\}^5}{C_1D_1^3} =$$

$$\frac{[6.41913]\,(l_2 + e)\,\{[8.46864 - 10]\,C_2^{\frac{1}{3}}\,D_2^{\frac{1}{3}}\}^5}{C_2D_2^3}$$

and reducing, since $l_1 + e = l_2 + e$,

$$C_1^{\frac{2}{3}}/D_1^{\frac{4}{3}} = C_2^{\frac{2}{3}}/D_2^{\frac{4}{3}} \text{ or } C_1/D_1^2 = C_2/D_2^2 \qquad (39)$$

We also have

$$C_1 + C_2 = C \qquad (40)$$

For any required value of $C$ the values of $C_1$ and $C_2$ can be obtained from equations (39) and (40) after the values of $D_1$ and $D_2$ have been decided upon. $D_1$ is first selected and $D_2$ is made as nearly equal to it as possible keeping in mind a required clearance of about .3 in. between the inner diameter of the outer spring and the outer diameter of the inner spring.

If there are three or more springs assembled one within the other we may write

$$C_1/D_1^2 = C_2/D_2^2 = C_3/D_3^2, \text{ etc.}$$

and

$$C_1 + C_2 + C_3 + \text{etc.} = C$$

from which the values of $C_1$, $C_2$, $C_3$, etc., can be determined after the values of $D_1$, $D_2$, $D_3$, etc., have been decided upon.

Having obtained the values of $C_1$, $C_2$, etc., the diameter of the bar from which each spring is to be coiled, the number of effective coils, the solid height, assembled height, and free height of each spring or spring column can be determined from equations (27), (28), (24), (25), and (26), respectively. Any adjustment of the thicknesses of the separators or other measures to make the assembled heights of the spring columns exactly the same can now be decided upon.

**131. Measures for Decreasing Assembled Height also Applicable to Springs Coiled from Bars of Rectangular Cross-Section.** — In the discussions of telescoping springs, and of non-telescoping springs assembled one within the other, it has been assumed that the bars from which the springs are coiled are of circular cross-section because special measures for decreasing the assembled heights of such springs are more often necessary than for springs coiled from bars of rectangular cross-section. The principles and methods discussed are, however, equally applicable to springs of the latter type.

**132. Counter-Recoil Springs for the 5-Inch Barbette Carriage, Model of 1903.** — The 5-inch barbette carriage, model of 1903, is provided with two spring cylinders and two sets of springs acting together on the gun through the spring piston-rods and a spring yoke, which is a cross piece bearing against the rear of the recoil-band lug and carried on the recoil piston-rod. A spring piston-rod is fastened to each end of the spring yoke. With this arrangement each set of springs has to exert but one-half of the force required to return the gun into battery after recoil has ended.

The spring cylinders were designed primarily for springs coiled from bars of rectangular cross-section but as difficulty had been experienced in obtaining satisfactory springs of this type it was decided to allow springs coiled from bars of circular cross-section to be used if desired. As the assembled height of a spring column of the latter springs assembled in the ordinary way is greater than the length of the spring cylinder it was decided to provide for telescoping springs; and further, as it was desirable that the spring cylinder be capable of receiving either type of spring, an opening could not be provided in the cylinder to allow the rear ends of the stirrup and inner telescoping spring to be drawn through it during recoil as is ordinarily the case when telescoping springs are used. It was, consequently, necessary, in order to prevent the stirrup from striking the rear end of the cylinder during recoil, to shorten it and the inner spring column so as to leave a space between them and the rear end of the spring cylinder when the gun is in battery somewhat greater than the amount by which the outer spring is compressed during recoil.

Fig. 98 shows a spring cylinder of this carriage containing springs coiled from bars of rectangular cross-section and also a cylinder containing telescoping springs coiled from bars of circular cross-section.

**133. Example 1.**— Let it be required to design a counter-recoil spring coiled from a bar of rectangular cross-section for use in the 5-inch barbette carriage, model of 1903. The following data are known from the construction of the carriage, viz.:

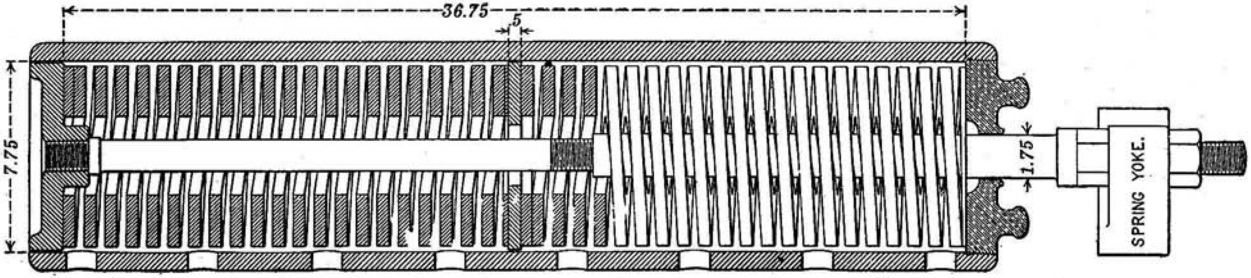
Weight of recoiling parts = 12632 lbs.
Friction of packing around recoil piston-rod = 220 lbs.
Length of recoil = 13 ins.
Maximum angle of elevation = 15°.
Coefficient of starting friction = .25.
Number of spring cylinders = 2.
Assume values of $e = 1$ in., $r = 4.25$, and $D = 5.225$ ins.

From equation (29), since there are two spring cylinders,

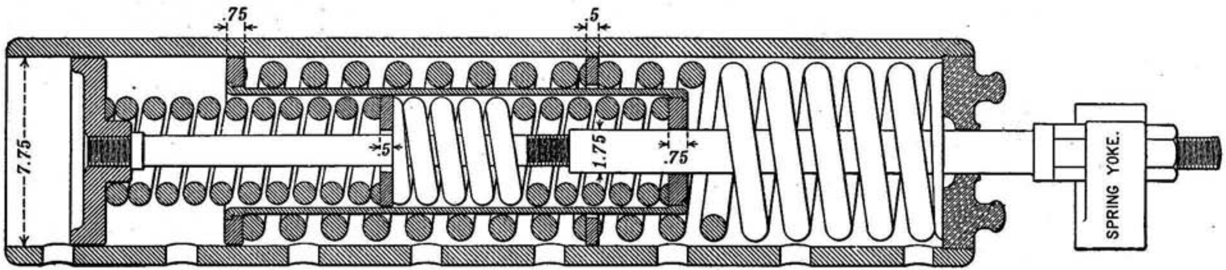
$$C = \frac{2}{2}(12632 \sin 15° + .25 \times 12632 \cos 15° + 220) = 6540 \text{ lbs.}$$

From equation (34)

$$h = [8.39203 - 10]\{1 + (4.25)^2\}^{\frac{1}{6}}(4.25)^{-\frac{2}{3}}(6540)^{\frac{1}{3}}(5.225)^{\frac{1}{3}} = .499 \text{ in.}$$




Springs Coiled from Bars of Rectangular Cross-Section.




Telescoping Springs Coiled from Bars of Circular Cross-Section.

Fig. 98.—Spring Cylinders and Springs of 5-Inch Barbette Carriage, Model of 1903.

From equation (35)

$$N' = \frac{[7.02919]\,14\,(.499)^4}{6540\,(5.225)^3} \times \frac{(4.25)^3}{1 + (4.25)^2} = 40.07 \text{ effective coils.}$$

On account of the number of coils the spring will be divided into two sections and the thickness of the separator will be taken as .5 in. The solid height is, then, from equation (31),

$$H = (40.07 + 2) \times .499 + .5 = 21.49 \text{ ins.}$$

The assembled height is, from equation (32),

$$A = 21.49 + 14 = 35.49 \text{ ins.,}$$

and, since the length of the space available for the spring column in the cylinder is 36.75 ins., the assembled height is satisfactory. The free height is, from equation (33),

$$F = 21.49 + 2 \times 14 = 49.49 \text{ ins.}$$

The dimension of the bar perpendicular to the axis of the spring is

$$b = rh = 4.25 \times .499 = 2.121 \text{ ins.}$$

The outer diameter of the coil is

$$D + b = 5.225 + 2.121 = 7.346 \text{ ins.}$$

and, since the inner diameter of the spring cylinder is 7.75 ins., the clearance is sufficient. The inner diameter of the coil is

$$D - b = 5.225 - 2.121 = 3.104 \text{ ins.}$$

and, since the larger diameter of the spring piston-rod is only 1.75 ins., the inner diameter of the spring is satisfactory.

Compare these results with the data given in the specifications for the 5-inch barbette carriage, model of 1903, noting that allowance is therein made for a total remaining compression of the spring column at assembled height equal to 15 ins. In order that the cross-section of the coiled bar should conform to a commercial size the spring just designed would in practice be made of a bar having a cross-section of .5 in. × 2.125 ins.

**134. Example 2.**—Let it be required to determine the assembled height of a column of counter-recoil springs for the 5-inch barbette carriage, model of 1903, the springs being coiled from bars of circular cross-section and having such an outer diameter that they can be used in the existing spring cylinder provided the assembled height is not too great.

From equation (21), since there are two spring cylinders,

$$C = \frac{2.5}{2}(12632 \sin 15° + .25 \times 12632 \cos 15° + 220) = 8175 \text{ lbs.}$$

Assuming $D = 6.29$ ins. we have, from equation (27),

$$d = [8.46864 - 10]\,(8175)^{\frac{1}{3}}(6.29)^{\frac{1}{3}} = 1.095 \text{ ins.}$$

The outer diameter of the spring is, therefore,

$$6.29 + 1.095 = 7.385 \text{ ins.}$$

and, since the inner diameter of the cylinder is 7.75 ins., the clearance is sufficient.

From equation (28), taking $e$ as 1 in.,

$$N' = \frac{[6.41913]\,14\,(1.095)^4}{8175\,(6.29)^3} = 25.97 \text{ effective coils.}$$

Dividing the spring into two sections and taking the thickness of the separator as .5 in., the assembled height is, from equation (25),

$$A = (25.97 + 2)\,1.095 + .5 + 14 = 45.13 \text{ ins.}$$

As the length of the space available for the spring column in the cylinder is only 36.75 ins. springs coiled from bars of circular cross-section cannot be used therein unless special measures are taken to diminish their assembled height.

**135. Example 3.** — Let it be required to determine the assembled height of a column of counter-recoil springs for this carriage formed by placing springs of smaller diameter inside the larger springs, the springs being coiled from bars of circular cross-section and acting directly on the spring piston.

Assume the mean diameter of the outer spring to be 6.45 ins., from which it follows that to allow for proper clearance between the inner and outer springs the mean diameter of the inner should not exceed about 4.45 ins.

From equation (39)

$$C_2 = \left(\frac{4.45}{6.45}\right)^2 C_1 = .476\,C_1$$

and since $C$ = 8175 lbs. as before, we have, from equation (40),

$$C_1 + C_2 = 1.476\ C_1 = C = 8175 \text{ lbs.}$$

or

$$C_1 = 5539 \text{ lbs.}$$

From equation (27)

$$d_1 = [8.46864 - 10]\ (5539)^{\frac{1}{3}}(6.45)^{\frac{1}{3}} = .969 \text{ in.}$$

and from equation (28), taking $e$ = 1 in.,

$$N'_1 = \frac{[6.41913]\ 14\ (.969)^4}{5539\ (6.45)^3} = 21.80 \text{ effective coils.}$$

Dividing the spring column into two sections and taking the thickness of the separator as .5 in., the assembled height of the outer column (and of the inner column also) is, from equation (25),

$$A_1 = (21.80 + 2) \times .969 + .5 + 14 = 37.56 \text{ ins.}$$

By comparison with the assembled height of a column of springs coiled from bars of circular cross-section and assembled in the ordinary way it will be seen that a material reduction in assembled height has been made, but this height is still greater than that of a column of springs coiled from bars of rectangular cross-section and assembled in the ordinary way. Although the assembled height is greater than the length of the space available for the spring column in the cylinder, the difference is so small that by using springs coiled from bars of slightly smaller diameters and assuming a slightly higher value for the permissible torsional stress, springs could be obtained that, when assembled in this way, would fit in the spring cylinder of this carriage and function satisfactorily. Such springs would, however, require greater care in their production by the manufacturer.

**136. Example 4.** — Let it be required to design telescoping springs for this carriage to be coiled from bars of circular cross-section, and with the condition that the spring cylinder must not be altered in any respect to the end that either springs coiled from bars of rectangular cross-section or telescoping springs coiled from bars of circular cross-section may be used therein at will.

Since the spring cylinder must not be altered, provision cannot be made for drawing the rear ends of the stirrup and the inner spring column through an opening in the cylinder during recoil, and a space must be left between the rear end of the stirrup and the rear end of the cylinder, when the gun is in battery, that is at least equal to the total remaining compression of the outer spring column at assembled height. As the inner spring is smaller and, therefore, less expensive than the outer, the inner spring column will be given the greatest possible assembled height consistent with the conditions of the problem.

Let the total remaining compression at assembled height of the outer spring column be $l_1 + .5$ and the total remaining compression at assembled height of the inner spring column be $l_2 + .5$, and let

$$l_1 + .5 + l_2 + .5 = l + 1 = 14 \text{ ins.}$$

Then the assembled height of the inner spring column is, from equation (25),

$$A_2 = (N'_2 + P_2)\, d_2 + (P_2 - 1)\, t_2 + l_2 + .5$$

and this height plus $l_1 + .5$ plus the thickness of the flange at the rear end of the stirrup must equal the length of the space available for the spring column in the cylinder, which is 36.75 ins. Taking the thickness of the flanges of the stirrup as .75 in., we may write

$$(N'_2 + P_2)\, d_2 + (P_2 - 1)\, t_2 + l_2 + .5 + l_1 + .5 + .75 = 36.75 \text{ ins.}$$

or

$$(N'_2 + P_2)\, d_2 + (P_2 - 1)\, t_2 = 36.75 - 14 - .75 = 22 \text{ ins.}$$

which expresses the fact that the solid height of the longest inner spring column that can be used in the cylinder under the assumed conditions is 22 ins.

The mean diameter of the outer springs and the diameter of the bars from which they are coiled will be taken as 6.29 ins. and 1.095 ins., respectively, as before; and to allow for a proper thickness of the wall of the stirrup and for the proper clearances between the springs and the stirrup, the mean diameter of the inner springs will be taken as 3.4 ins., which will also make the inner diameter of these springs sufficiently large to enable them to pass easily over the spring piston-rod.

From equation (27)

$$d_2 = [8.46864 - 10]\,(8175)^{\frac{1}{3}}\,(3.4)^{\frac{1}{3}} = .891 \text{ in.}$$

and taking $P_2 = P_1 = 2$ and $t_2 = t_1 = .5$ we have

$$(N'_2 + 2) \times .891 + .5 = 22 \text{ ins.}$$

or

$$N'_2 = 22.13 \text{ effective coils.}$$

From equation (28)

$$l_2 + .5 = \frac{22.13 \times 8175 \times (3.4)^3}{[6.41913]\,(.891)^4} = 4.298 \text{ ins.}$$

The amount of compression to be provided by the outer spring column is, therefore,

$$14 - 4.298 = l_1 + .5 = 9.702 \text{ ins.}$$

and the number of effective coils in the outer column is, from equation (28),

$$N'_1 = \frac{[6.41913]\,(9.702)\,(1.095)^4}{8175\,(6.29)^3} = 18.0$$

The assembled height of the outer column is, therefore, from equation (25),

$$A_1 = 20 \times 1.095 + .5 + 9.702 = 32.10 \text{ ins.}$$

and since this is less than the length of the available space in the spring cylinder diminished by the thickness of the front flange of the stirrup, equal to 36.75 − .75 = 36 ins., the design of telescoping springs just made is satisfactory for use in the spring cylinder of the 5-inch barbette carriage, model of 1903, designed primarily for counter-recoil springs coiled from bars of rectangular cross-section.

**137. Example 5.**—Let it be required to determine the assembled height of a set of telescoping springs for this carriage under the assumptions that the assembled heights of the inner and outer columns are equal, that the rear ends of the stirrup and inner spring column may be drawn through an opening in the spring cylinder during recoil, and that the diameter of the cylinder is the same as at present.

The values of $D$ and $d$ will be taken the same as for the telescoping springs already designed which are as follows:

$$D_1 = 6.29 \text{ ins.} \qquad d_1 = 1.095 \text{ ins.}$$
$$D_2 = 3.40 \text{ ins.} \qquad d_2 = .891 \text{ in.}$$

Let the total remaining compression at assembled height of the outer spring column be $l_1 + .5$ and of the inner spring column $l_2 + .5$, and let $l_1 + l_2 = 13$ ins., the length of recoil required. Then from equation (37), assuming $P = 2$,

$$3.0311\, l_1 + 3.2056 = 5.5878\, l_2 + 4.076$$

and substituting in this equation for $l_2$ its value $13 - l_1$, and solving for $l_1$ we have

$$l_1 = 8.53 \text{ ins. and } l_2 = 4.47 \text{ ins.}$$

From equation (28)

$$N'_1 = \frac{[6.41913]\,(9.03)\,(1.095)^4}{8175\,(6.29)^3} = 16.75 \text{ effective coils.}$$

and

$$N'_2 = \frac{[6.41913]\,(4.97)\,(.891)^4}{8175\,(3.40)^3} = 25.59 \text{ effective coils.}$$

From equation (25)

$$A_1 = 18.75 \times 1.095 + .5 + 9.03 = 30.06 \text{ ins.}$$

and

$$A_2 = 27.59 \times .891 + .5 + 4.97 = 30.05 \text{ ins.}$$