due to the force T = 2139 lbs., and torsion due to the force I = 4387 lbs. The force I lies in the plane of the section and has no bending moment with respect to it. Because of the torsion produced by I, which is somewhat special and will be explained later, it will simplify the problem to consider each half of the symmetrical section 3–3 by itself, in which case the forces acting at the spade should be taken as

$$S/2 = 1969 \text{ lbs. and } T/2 = 1069.5 \text{ lbs.}$$

The area of the half section is 2.039 sq. ins. Its center of gravity is given by the intersection of the horizontal and vertical axes XX and YY, the former being at a distance of 3.625 ins. from the lower edge of the section and the latter at a distance of .427 in. from the outer edge. The moment of inertia I_x about the axis XX is 14.431 ins.^{4}, I_y = .643 in.^{4}, and I_p = 15.074 ins.^{4}.

Stresses in Section 3–3 due to the Forces *S* and *T*.—The compression in the half section due to the force $S/2$ is

$$1969/2.039 = 966 \text{ lbs. per sq. in.}$$

The lever arm of $T/2$ with respect to the half section, see Fig. 18, is

$$112.19 - 21.55 = 90.64 \text{ ins.}$$

and that of $S/2$ with respect to the neutral axis XX, see Figs. 18 and 55, is

$$4 + 16 + 3.625 = 23.625 \text{ ins.}$$

The resulting bending moment at the half section is, then,

$$1069.5 \times 90.64 - 1969 \times 23.625 = 50422 \text{ in. lbs.}$$

and the maximum bending stress is

$$S'''y = 50422 \times 3.625/14.431 = 12666 \text{ lbs. per sq. in.}$$

compression at the upper and tension at the lower edge of the section. The shearing stress in the half section due to the force $T/2$ is

$$1069.5/2.039 = 525 \text{ lbs. per sq. in.}$$

Torsional Stress in Section 3–3 caused by the Force *I*.—Referring to Fig. 55, it will be seen that the force I, which is the thrust on the elevating screw, acts midway between the two flasks of the trail. It is transmitted to each flask by two cross transoms riveted thereto which together form a fixed beam with the force applied at its middle. The length of the beam is the distance between the webs of the flasks equal to 14.1 ins. The bending moment at the ends of the transoms under these conditions is given by equation (11) and is

$$M = 4387 \times 14.1/8 = 7732 \text{ in. lbs.}$$

This bending moment will produce a twisting couple of equal value on each flask which will cause a torsional stress in the half section 3–3 and in the sections on each side of it, between it and the axle and between it and the tool box. The axle and the plates forming the top, bottom, and front end of the tool box prevent twisting in the parts of the flasks connected by them so that the right sections of the flasks tangent to the axle at its rear and those at the front end of the tool box may be considered as fixed ends so far as the torsional stress in the flasks is concerned.

The distance between the half section 3–3 and the axle is 20.902 ins. and between it and the front end of the tool box the distance is 15.224 ins., so that the torsional moment in the half section is, from equation (2),

$$M_t = 7732 \times 20.902/36.126 = 4474 \text{ in. lbs.}$$

and the maximum torsional stress, which occurs at the points O, Fig. 55, is

$$rS'''_t = M_t r/I_p = 4474\sqrt{(3.625)^2 + (1.573)^2}/15.074 = 1173 \text{ lbs. per sq. in. shear.}$$

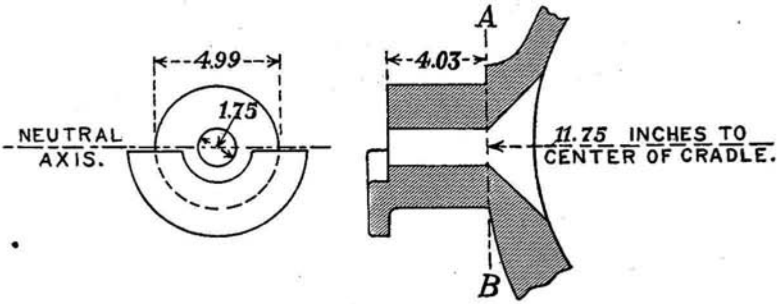
Combination of Compressive, Bending, and Torsional Stresses in Section 3–3. — The compressive stress of 966 lbs. per sq. in. is uniformly distributed over the section; it therefore increases the compression of 12666 lbs. per sq. in. at the upper edge of the section, due to the bending stress, to 13632 lbs. per sq. in., and reduces the tension at the lower edge to 11700 lbs. per sq. in. The compression of 13632 lbs. per sq. in. being the greater stress, it will be combined with the torsional stress of 1173 lbs. per sq. in. at O to obtain the maximum combined stress in the section. From equation (24) this maximum stress is found to be

$$S_{lc} = \frac{13632}{2} + \sqrt{(1173)^2 + \frac{(13632)^2}{4}} = 13732 \text{ lbs. per sq. in.}$$

compression at O on the upper edge of the section. The torsional stress at O is so slight that it will not be necessary to determine how much it is increased by the compression at that point.

STRESSES IN PARTS OF THE 5-INCH BARBETTE CARRIAGE, MODEL OF 1903.

81. Stresses in the Trunnions of the Cradle. — The cradle may be considered as a beam carried in the trunnion beds of the pivot yoke. The diameter of the trunnions of the cradle, see Fig. 56, is .01 in. smaller than the diameter of the trunnion beds and this fact, in connection with the immense depth of the sections of the cradle in a direction parallel to the action lines of the principal forces acting on it, makes of the cradle a beam merely supported at its ends instead of a fixed beam. A fixed beam is one whose ends are held so firmly by the supports that they cannot accommodate themselves to any deflection of the part of the beam between the supports caused by the bending



Area of Section AB = 17.15 sq. ins. I of Section AB = 29.98 ins.^{4}

Fig. 56.

forces acting upon it. A free beam is one whose ends are merely supported in such manner that they can freely accommodate themselves to the deflection of the part of the beam between the supports. By the methods discussed in works on strength of materials it can be shown that the maximum deflection of the cradle, which occurs midway between the trunnion beds, is less than .00002 in. so that it is entirely negligible, and the clearance between the trunnions and their beds is many times greater than it need be to permit the trunnions to conform to the deflection of the rest of the cradle.

One of the trunnions of the cradle is shown in Fig. 56.

The forces acting on each trunnion, see Fig. 21, are the horizontal force $C/2 = 46647$ lbs. and the vertical force $D/2 = 5359$ lbs. The resultant of these forces is

$$\sqrt{(46647)^2 + (5359)^2} = 46954 \text{ lbs.}$$

It is uniformly distributed over the surface of contact between the trunnion and its bed in the pivot yoke. This surface is 4 ins. long, being .03 in. shorter than the trunnion to allow a clearance of .015 in. between the rimbase and the side of the trunnion bed on the inside, and between the half collar on the end of the trunnion and the side of the trunnion bed on the outside.

The bending moment at section AB is greater than at any other section of the trunnion. It may be obtained by considering the resultant force concentrated at the middle of the trunnion length, whence

$$M = 46954 \times 2.015 = 94612 \text{ in. lbs.}$$

and the corresponding maximum bending stress is

$$S'''y = \frac{94612 \times 2.495}{29.98} = 7874 \text{ lbs. per sq. in.}$$

tension at the rear and compression at the front of the section.

The shearing stress in section AB is

$$46954/17.15 = 2738 \text{ lbs. per sq. in.}$$

Stress in the Recoil Cylinder of the Cradle due to the Interior Hydraulic Pressure. — The cylinder liner in which the recoil grooves are cut is made in halves and adds nothing to the strength of the cylinder, which outside of the liner has an inner diameter of 7.5 ins. and an outer diameter of 11 ins. The force $P = 84665$ lbs., see Fig. 21, is the sum of the forces exerted by the piston-rod of the recoil cylinder and the spring rods of the spring cylinders. The force exerted by the spring rods, which is due to the springs, is least when the gun is just commencing to recoil, at which time it has a value of 6540 lbs., so that the corresponding force exerted by the piston-rod of the recoil cylinder is 78145 lbs. The piston of the recoil cylinder has a diameter of 6.49 ins. and the rod a diameter of 3 ins. and consequently the area against which the oil in the cylinder must act to produce a total force of 78145 lbs. is

$$.7854\,\{(6.49)^2 - (3)^2\} = 26.01 \text{ sq. ins.}$$

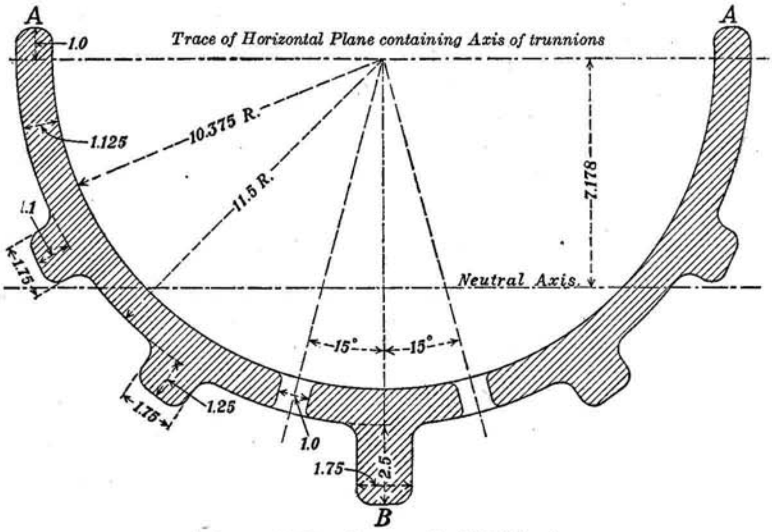
and the corresponding interior pressure per sq. in. in the cylinder is

$$78145/26.01 = 3004 \text{ lbs. per sq. in.}$$

The maximum stress in the cylinder due to this interior pressure is one of tension and occurs at its inner surface. It is as follows:

$$\theta = \frac{2(3.75)^2 + 4(5.5)^2}{3\{(5.5)^2 - (3.75)^2\}} \times 3004 = 9226 \text{ lbs. per sq. in.}$$

Stresses in Section 4–4 of the Cradle. — This section, taken just in front of the spring cylinders, is indicated in Fig. 21 and shown in Fig. 57.



Area = 51.24 sq. ins. $I = 729.85$ ins.^{4}
Section 4–4.
Fig. 57.

There is a bending moment at the section, see Fig. 21, which may be obtained by considering all the forces acting to the right (in rear) of it. The weight of the cradle will be neglected as only a part of it acts to the right of the section and neglecting the weight increases the bending moment slightly. The bending moments of the forces perpendicular to the section must be taken with respect to its neutral axis which is 7.178 ins. below the horizontal plane containing the axis of the trunnions.

Whence

$$M = 84665\,(15.75 - 7.178) + 4662\,(10 - 7.178) - 31084\,(44.875 - 20) + 4311 \cos 14° \,(35 \cos 14° - 20) + 4311 \sin 14° \,(35 \sin 14° - 7.178) = 25433 \text{ in. lbs.}$$

and the corresponding maximum bending stresses are

$$S'''y = 25433 \times 8.178/729.85 = 285 \text{ lbs. per sq. in.}$$

compression at A and A, Fig. 57, and

$$S'''y = 25433 \times 6.822/729.85 = 238 \text{ lbs. per sq. in.}$$

tension at B, Fig. 57.

It will be noted that while the bending moments of the individual forces at this section are large, they nearly neutralize each other so that the resultant bending moment is small and, in connection with the large moment of inertia of the section, produces very little bending stress.

The forces P, F', and $E \sin 14°$ produce a tensile stress in the section equal to

$$[84665 + 4662 + 4311 \sin 14°]/51.24 = 1764 \text{ lbs. per sq. in.}$$

The forces B and $E \cos 14°$ produce a shearing stress in the section equal to

$$[31084 - 4311 \cos 14°]/51.24 = 525 \text{ lbs. per sq. in.}$$

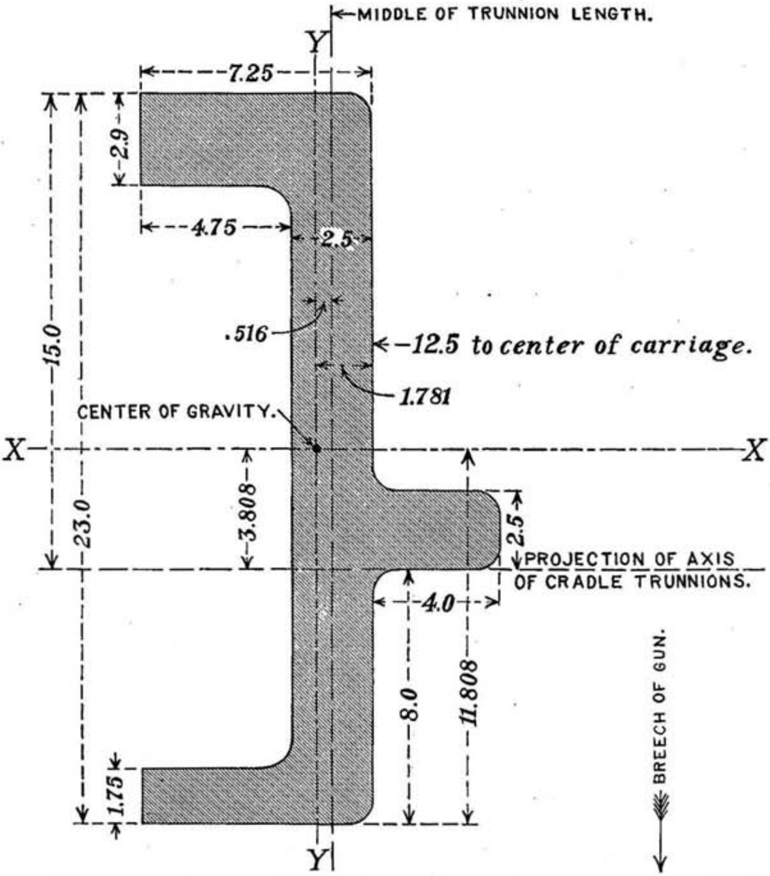
As the force E is applied at a distance of 16.125 ins. to the left of the center of the section, its horizontal component causes a bending stress in the section about a vertical neutral axis, and its vertical component causes a torsional stress. Owing to the large moments of inertia of the section these stresses will be small and will, therefore, not be computed.

The maximum combined stress in the section occurs at B and is 2002 lbs. per sq. in. tension obtained by adding the tension due to the bending stress and that due to the direct action of the forces P, F', and $E \sin 14°$. If the torsion be considered this maximum stress will be increased slightly.

STRESSES IN THE PIVOT YOKE.

82. Stresses in Section 5–5 of the Pivot Yoke. — This section is indicated in Figs. 22 and 23 and shown in Fig. 58.

It is selected rather than one at which the bending moment is greater because the increase in the strength of the sections as we pass further down is greater than the increase of the bending moment.



Area = 89.59 sq. ins. $I_x = 4923.67$ ins.^{4} $I_y = 455.44$ ins.^{4} $I_p = 5379.11$ ins.^{4}

Section 5–5.

Fig. 58.

Forces Producing Stress in Section 5–5. — Neglecting the weight of the pivot yoke, most of which is applied below the section, the forces acting are $C/2$, $D/2$, and E. The whole of the force E is to be considered instead of one-half of it because it is applied, through the elevating and platform brackets, to the left side of the pivot yoke only.

The force $C/2 = 46647$ lbs. produces a bending stress about the axis XX, Fig. 58, a shearing stress, and a torsional stress. Its lever arm in bending, see Fig. 22, is 11.7 ins. Its lever arm in torsion, see Fig. 58, is .516 in. obtained by considering the force concentrated at the middle of the trunnion length and measuring the distance from its action line to a line perpendicular to the plane of the section passing through its center of gravity.

The force $D/2 = 5359$ lbs. produces a bending stress about the axis XX, a stress of compression, and a bending stress about the axis YY, Fig. 58. Its lever arm with respect to the axis XX is 3.808 ins., and with respect to the axis YY it is .516 in., the lever arms being determined by considering the force concentrated at the axis of the trunnions and at the middle of the trunnion length.

The force E acts on the left side of the carriage only, being applied to the teeth of the elevating pinion mounted on a short horizontal shaft carried in bearings in the elevating bracket. The elevating bracket is bolted to the platform bracket and the latter is bolted by two upper and two lower bolts to the rear face of the upright arm of the pivot yoke. E tends to rotate the platform bracket around the lower edge of the surface of contact between it and the pivot yoke. This tendency is resisted by the tension in the two top bolts fastening the bracket to the pivot yoke. The lever arm of E with respect to the edge about which the bracket tends to rotate being 21.61 ins. and that of the tension in the two top bolts being 20 ins., the intensity of the tension, which will be called T_h, is

$$T_h = 4311 \times 21.61/20 = 4658 \text{ lbs.}$$

This tension is transmitted to the pivot yoke as a horizontal force whose point of application may be taken as midway between the centers of the two top bolts fastening the platform bracket to the pivot yoke. This point is 19 ins. to the left of the center of the carriage and 3.25 ins. below the axis of the trunnions.

The bracket is prevented from moving downward under the action of the vertical component of E by the four bolts fastening the bracket to the pivot yoke. As the shearing stress may be considered as uniformly distributed over the four bolts, the total shearing stress in the two top bolts is

$$4311 \cos 14° / 2 = 2092 \text{ lbs.}$$

This shearing stress, which will be called T_v, is transmitted to the pivot yoke as a vertical force, its point of application being the same as that of T_h.

The two bottom bolts and the lower edge of the surface of contact between the bracket and the pivot yoke being below section 5–5, the shearing stress in the bottom bolts and the pressure along the contact edge must not be considered in determining the stresses in the section.

The force $T_h = 4658$ lbs. causes a bending stress about the axis XX, a shearing stress, and a torsional stress. Its point of application being 3.25 ins. below the axis of the trunnions, its lever arm in bending is $11.7 - 3.25 = 8.45$ ins. As its point of application is also 19 ins. to the left of the center of the carriage, its lever arm in torsion, see Fig. 58, is $19 - 12.5 - 1.781 = 4.719$ ins. The torsional stress produced by T_h is opposed to that produced by $C/2$.

The force $T_v = 2092$ lbs. causes a bending stress about the axis XX, a stress of compression, and a bending stress about the axis YY. Its point of application being in the vertical plane containing the rear face of the upright arm of the pivot yoke, its lever arm with respect to the axis XX, see Fig. 58, is 11.808 ins. As its point of application is also 19 ins. to the left of the center of the carriage, its lever arm with respect to the axis YY is $19 - 12.5 - 1.781 = 4.719$ ins.

Bending Stress in Section 5–5 about *XX* as a Neutral Axis. — The bending moment is

$$M = 46647 \times 11.7 + 5359 \times 3.808 + 4658 \times 8.45 + 2092 \times 11.808 = 630239 \text{ in. lbs.}$$

and the corresponding bending stresses are

$$S'''y = 630239 \times 11.808 / 4923.67 = 1511 \text{ lbs. per sq. in.}$$

compression at the rear edge of the section, and

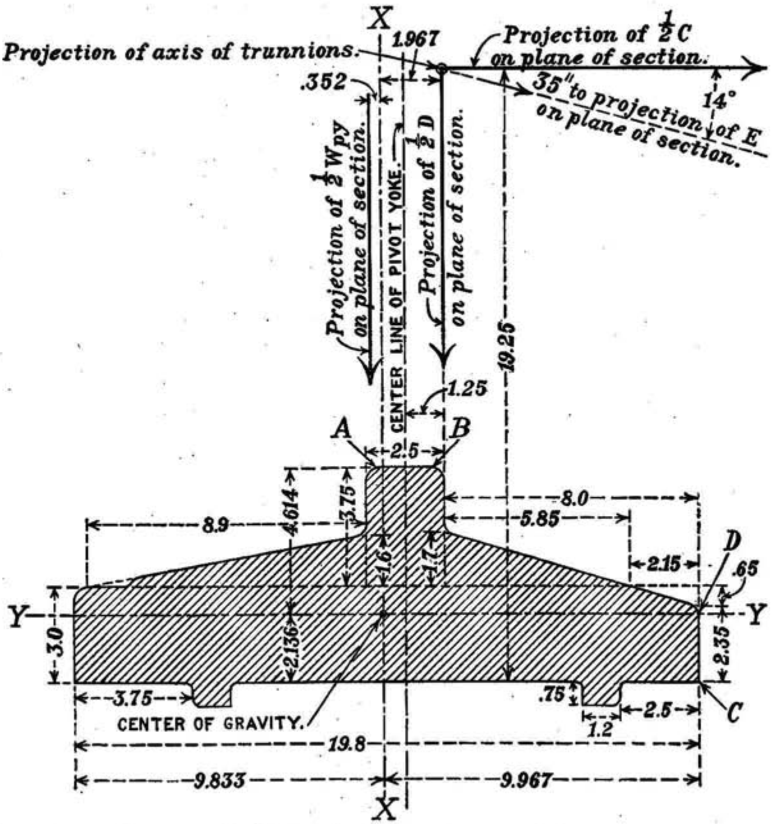
$$S'''y = 630239 \times 11.192 / 4923.67 = 1433 \text{ lbs. per sq. in.}$$

tension at the front edge of the section.

Other Stresses. — The shearing stress is

$$[46647 + 4658]/89.59 = 573 \text{ lbs. per sq. in.}$$

It is apparent by inspection that the compressive stress, the



Area = 81.96 sq. ins. $I_x = 2164.66$ ins.[4] $I_y = 186.91$ ins.[4] $I_p = 2351.57$ ins.[4]

Section 6–6.

Fig. 59.

bending stress about the axis *YY*, and the torsional stress are so small that it is unnecessary to compute them.

Stresses in Section 6–6 of the Pivot Yoke. — This section is indicated in Fig. 23 and shown in Fig. 59.

Forces Producing Stress in Section 6–6. — The force $C/2 = 46647$ lbs., see Figs. 22 and 23, produces in the section a torsional stress, a bending stress about the axis XX, and a shearing stress. Its lever arm in torsion is the perpendicular distance between its action line and the line normal to the plane of the section passing through its center of gravity, which distance, see Fig. 59, is $19.25 - 2.136 = 17.114$ ins. Its lever arm in bending is the perpendicular distance between its action line and the plane of the section equal, see Fig. 23, to 5.75 ins.

The force $D/2 = 5359$ lbs., see Figs. 22 and 23, produces in the section a torsional stress, a bending stress about the axis YY, and a shearing stress. Its lever arm in torsion, see Fig. 59, is $9.967 - 8.0 = 1.967$ ins. Its lever arm in bending, see Fig. 23, is 5.75 ins.

The greater part of the weight of the pivot yoke and shields is applied above this section so that $W_{py}/2 = 7967$ lbs. will be considered as acting upon it. It produces in the section a torsional stress, a bending stress about the axis YY, and a shearing stress. Its lever arm in torsion, see Figs. 23 and 59, is .352 in. Its lever arm in bending, see Fig. 23, may be taken as approximately 11.75 ins.

The force $E \sin 14° = 1043$ lbs. produces in the section a torsional stress, a bending stress about the axis XX, and a shearing stress. Its lever arm in torsion, see Figs. 22 and 59, is $19.25 - 35 \sin 14° - 2.136 = 8.647$ ins. Its lever arm in bending, see Fig. 23, is 8.125 ins.

The force $E \cos 14° = 4183$ lbs. produces in the section a torsional stress, a bending stress about the axis YY, and a shearing stress. Its lever arm in torsion, see Fig. 59, is $35 \cos 14° + 1.967 = 35.927$ ins. Its lever arm in bending, see Fig. 23, is 8.125 ins.

Bending Stress in Section 6–6 about *XX* as a Neutral Axis. — The bending moment is

$$46647 \times 5.75 - 1043 \times 8.125 = 259746 \text{ in. lbs.}$$

and the corresponding maximum bending stress, see Fig. 59, is

$$259746 \times 9.967/2164.66 = 1196 \text{ lbs. per sq. in.}$$

compression occurring at the surface CD.

Bending Stress in Section 6–6 about *YY* as a Neutral Axis. — The bending moment is

$$5359 \times 5.75 + 7967 \times 11.75 + 4183 \times 8.125 = 158413 \text{ in. lbs.}$$

and the corresponding maximum bending stress, see Fig. 59, is

$$158413 \times 4.614/186.91 = 3911 \text{ lbs. per sq. in.}$$

tension occurring at the surface *AB*.

The bending stress at the point *C*, Fig. 59, due to this bending moment is

$$158413 \times 2.136/186.91 = 1810 \text{ lbs. per sq. in. compression.}$$

Torsional Stress in Section 6–6. — The torsional moment is

$$46647 \times 17.114 + 5359 \times 1.967 - 7967 \times .352 - 1043 \times 8.647 + 4183 \times 35.927 = 947318 \text{ in. lbs.}$$

and the corresponding maximum torsional stress, which occurs at *C*, is

$$947318 \sqrt{(9.967)^2 + (2.136)^2}/2351.57 = 4106 \text{ lbs. per sq. in.}$$

Shearing Stress in Section 6–6. — The shearing stress due to the forces $C/2$ and $E \sin 14°$ is

$$[46647 - 1043]/81.96 = 556 \text{ lbs. per sq. in.,}$$

and that due to the forces $D/2$, $W_{py}/2$, and $E \cos 14°$ is

$$[5359 + 7967 + 4183]/81.96 = 214 \text{ lbs. per sq. in.}$$

As these two shearing stresses are due to forces whose action lines are perpendicular to each other, the resultant shearing stress is

$$\sqrt{(556)^2 + (214)^2} = 596 \text{ lbs. per sq. in.}$$

Maximum Resultant Stress in Section 6–6. — The maximum resultant stress in the section occurs at *C*. It is a compressive stress due to combining the torsional stress at this point with the resultant compressive stress there caused by the bending moments about the axes *XX* and *YY*, respectively. The stress due to the bending moment about the axis *XX* being 1196 lbs. per sq. in. compression and that due to the bending moment about *YY* being 1810 lbs. per sq. in. compression, their resultant is 1196 + 1810 = 3006 lbs. per sq. in. compression. Combining this with the torsional stress of 4106 lbs. per sq. in. at C by equation (24) we have

$$S_{lc} = (3006/2) + \sqrt{(4106)^2 + (3006)^2/4} = 5876 \text{ lbs. per sq. in.}$$

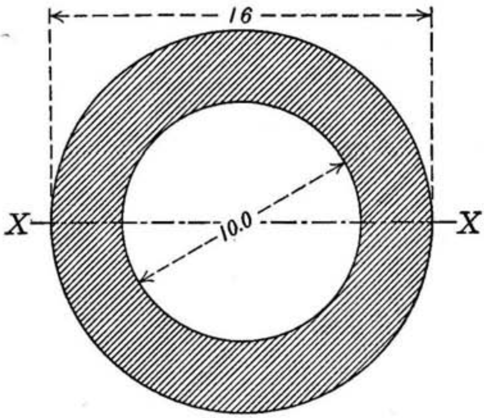
compression as the maximum resultant stress in the section.

The resultant shearing stress at C due to the torsion and the compression is, from equation (25),

$$S_{sc} = \sqrt{(4106)^2 + (3006)^2/4} = 4373 \text{ lbs. per sq. in.}$$

The shearing stress caused by the bending forces (but not the torsional stress) in reality varies from a maximum at the neutral axis to zero at the extreme fiber and is, therefore, negligible at the point C.

Stresses in Section 7–7 of the Stem of the Pivot Yoke.—This section is indicated in Fig. 22 and shown in Fig. 60. Con-



Area = 122.52 sq. ins. I = 2726.79 ins.^{4}

Section 7–7.

Fig. 60.

sidering only the forces applied below the section, the force H = 62807 lbs., see Fig. 22, produces bending and shear in the section, and the force I = 30833 lbs. produces bending and compression. The weight of the pivot yoke is not taken into account as it is considered as applied above this section.

The total bending moment is

$$62807 \times 28 + 30833 \times 7 = 1974427 \text{ in. lbs.}$$

and the corresponding maximum bending stress is, see Fig. 60,

$$1974427 \times 8/2726.79 = 5793 \text{ lbs. per sq. in.}$$

compression at the rear and tension at the front of the section.

The shearing stress due to the force H is

$$62807/122.52 = 513 \text{ lbs. per sq. in.}$$

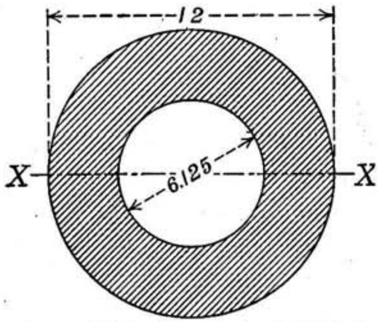
The compression due to the force I is

$$30833/122.52 = 252 \text{ lbs. per sq. in.}$$

The maximum resultant stress, therefore, is one of compression. It occurs at the rear of the section and is equal to

$$5793 + 252 = 6045 \text{ lbs. per sq. in.}$$

Stresses in Section 8–8 of the Stem of the Pivot Yoke.—This section is indicated in Fig. 22 and shown in Fig. 61.



Area = 83.63 sq. ins. I = 949.01 ins.^{4}

Section 8–8.

Fig. 61.

The force H, see Fig. 22, produces bending and shear in the section, and the force I produces bending and compression.

The total bending moment is

$$62807 \times 12 + 30833 \times 7 = 969515 \text{ in. lbs.}$$

and the corresponding maximum bending stress is, see Fig. 61,

$$969515 \times 6/949.01 = 6130 \text{ lbs. per sq. in.}$$

compression at the rear and tension at the front of the section.

The shearing stress due to the force H is

$$62807/83.63 = 751 \text{ lbs. per sq. in.}$$

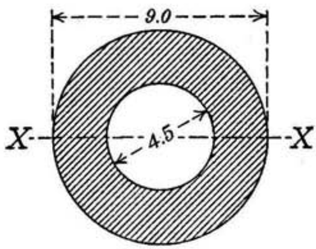
The compression due to the force I is

$$30833/83.63 = 369 \text{ lbs. per sq. in.}$$

The maximum resultant stress, therefore, is one of compression. It occurs at the rear of the section and is equal to

$$6130 + 369 = 6499 \text{ lbs. per sq. in.}$$

Stresses in Section 9–9 of the Stem of the Pivot Yoke.— This section is indicated in Fig. 22 and shown in Fig. 62.



Area = 47.71 sq. ins. I = 302.01 ins.[4]

Section 9–9.

Fig. 62.

The force H, see Fig. 22, produces bending and shear in the section. The force I which acts on sections 7–7 and 8–8 cannot be considered as acting in this case for its point of application is above the section, and only the forces whose points of application are below it are being considered, in accordance with the general principles already explained.

The bending moment at the section is

$$62807 \times 7.875 = 494605 \text{ in. lbs.}$$

and the corresponding maximum bending stress is, see Fig. 62,

$$494605 \times 4.5/302.01 = 7370 \text{ lbs. per sq. in.}$$

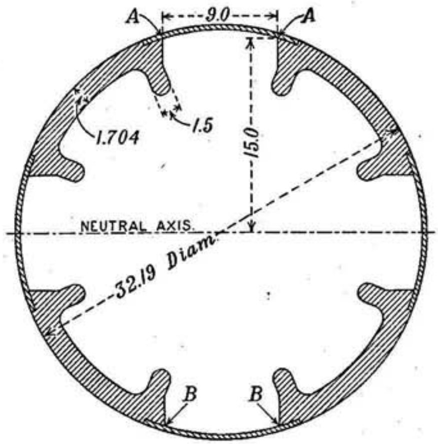
compression at the rear and tension at the front of the section.

The shearing stress is

$$62807/47.71 = 1316 \text{ lbs. per sq. in.}$$

STRESSES IN THE PEDESTAL

83. Stresses in Section 10–10 of the Pedestal. — This section is indicated in Fig. 24 and shown in Fig. 63. It is a horizontal section taken through the centers of the four hand-holes in the pedestal. It is assumed that the axis of the gun when fired is immediately above the line joining the centers of two of the hand-holes; and the cover plates over the holes are neglected. Neg-



Area = 134.28 sq. ins. $I = 14196$ ins.^{4}

Section 10–10.

Fig. 63.

lecting the weight of the pedestal as the bulk of it is below this section, and considering the forces whose points of application are above the section, the only force acting, see Fig. 24, is $G = 155057$ lbs. This produces bending and shear in the section.

Its bending moment is

$$155057 \times 16.37 = 2538283 \text{ in. lbs.}$$

and the corresponding maximum bending stress is

$$2538283 \times 15/14196 = 2682 \text{ lbs. per sq. in.}$$

tension at A and compression at B.

The shearing stress in the section is

$$155057/134.28 = 1155 \text{ lbs. per sq. in.}$$

Stresses in the Foundation Bolts. — There are sixteen foundation bolts fastening the pedestal to the gun platform, each of which is 1.75 ins. in diameter at the top of the threads and 1.564 ins. in diameter at the bottom of the threads. The positions of these bolts are shown in Fig. 64.

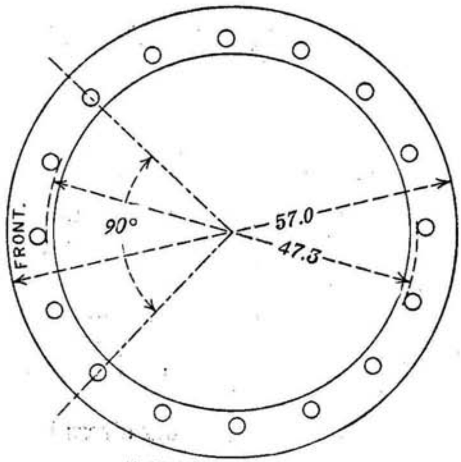


Fig. 64.

Assume that the holding down force L = 69441 lbs., see Fig. 24, is exerted through the five bolts included in the quadrant at the front of the pedestal as indicated in Fig. 64. Then, the area of each bolt at the bottom of the threads being 1.921 sq. ins., the stress in each is

$$69441 / [5 \times 1.921] = 7230 \text{ lbs. per sq. in. tension.}$$

Stresses in the Flange at the Rear of the Pedestal. — The stresses in this part of the flange are larger than those which occur at the front owing to the greater bending moment at the rear.

Although the force T is shown in Fig. 24 as acting at the extreme rear point of the flange of the pedestal it is in reality distributed over the rear half of the flange,* the intensity of the pressure per unit of area varying from a maximum at the extreme

* Part of the force T is also distributed over the bottom of the pedestal inside the flange but in its vicinity.

rear point to zero at the diameter perpendicular to the axis of the gun. Under the circumstances, however, it will be safe to assume that the force is uniformly distributed over the portion of the flange within the quadrant at the rear and outside the dangerous section thereof, which occurs along the line where the fillet, see Fig. 24, joins the upper surface of the flange, this line being the circumference of a circle 47.3 ins. in diameter tangent to the foundation bolt holes on the inside. The portion of the flange under consideration may be taken as a cantilever whose dangerous section is $\pi \times 47.3/4 = 37.15$ ins. wide, and, since the flange is 2 ins. thick, the dangerous section has a moment of inertia about its neutral axis of 24.77 ins.^{4} and an area of 74.3 sq. ins. The outer radius of the flange being 28.5 ins. and the radius of the dangerous section 23.65 ins., the cantilever can be taken as $28.5 - 23.65 = 4.85$ ins. long, and the bending moment of the force T uniformly distributed over it is, from equation (19),

$$\frac{103897 \times (4.85)^2}{4.85 \times 2} = 251950 \text{ in. lbs.}$$

The corresponding maximum bending stress is

$$251950 \times 1/24.77 = 10172 \text{ lbs. per sq. in.}$$

compression at the upper and tension at the lower surface of the flange.

The shearing stress is

$$103897/74.3 = 1398 \text{ lbs. per sq. in.}$$

The compression due to the force S, see Fig. 24, is

$$92250/74.3 = 1242 \text{ lbs. per sq. in.}$$

The maximum resultant stress, which occurs at the upper edge of the section is, therefore,

$$10172 + 1242 = 11414 \text{ lbs. per sq. in. compression.}$$

STRESSES IN PARTS OF THE 6-INCH DISAPPEARING CARRIAGE, MODEL OF 1905 MI.

84. Stresses in Section 11–11 of the Gun Levers. — The gun-lever axle and the part of the gun levers above it are shown in Fig. 65, in which are also indicated the forces (in full lines)

Mass of Part above Sec. 11 – 11 = 40.
Σmr^2 of Mass above Sec. 11 – 11 = 230.
(Σmr^2 is expressed in mass units and feet.)
The dot and dash force lines indicate internal stresses (total stresses) in Section 11 – 11.

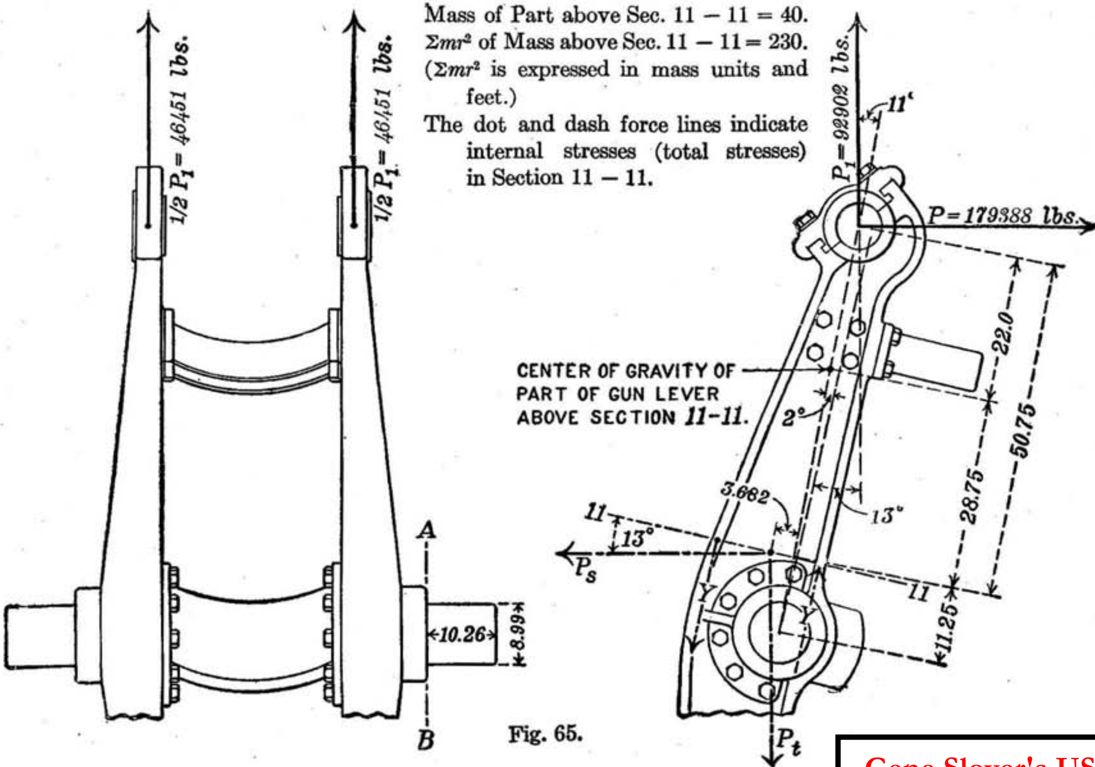
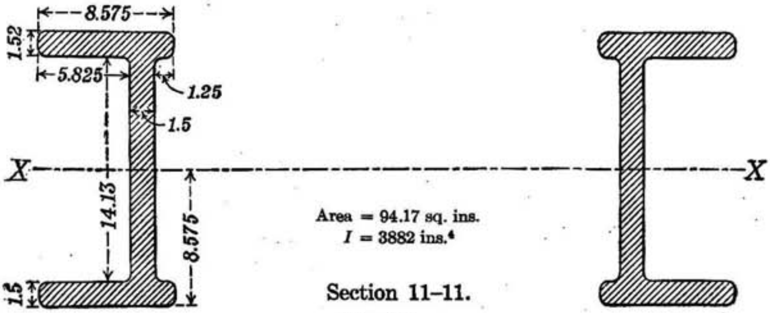


Fig. 65.

exerted on the levers by the gun and the position of section 11–11. The values of the forces P and P_1 were determined in Chapter III, see equations (1‴) and (2‴), page 89, to be 179338 lbs. and 92902 lbs., respectively.

Section 11–11 is shown in Fig. 66.



Section 11–11.

Fig. 66.

Method of Computing Stresses in Parts not in Equilibrium. — The stresses in this section cannot be obtained in the same way as were the stresses in the parts of the 3-inch field carriage and the 5-inch barbette carriage because those parts were in equilibrium under the forces acting upon them; while the gun levers have linear accelerations in the vertical and horizontal directions and an angular acceleration about their center of mass, and a part of each of the forces acting on them is taken up in producing these accelerations without causing internal stresses in the parts. If, however, a section such as 11–11, Fig. 65, be taken through any part of a body not in equilibrium, the internal stresses acting in that section must be such that, in connection with the external forces acting on the part of the body on one side (either side) of the section, they will produce in that part the accelerations of translation and rotation possessed by it. Therefore, considering the part of the gun levers above section 11–11, we may form three equations between the known external forces and the unknown stresses in section 11–11 stating: (a) that the sum of the components of both forces and stresses in a vertical direction is equal to the mass of the part of the gun levers above section 11–11 multiplied by its acceleration in that direction; (b) that the sum of the components in the horizontal direction is equal to the mass multiplied by its acceleration in the horizontal direction; and (*c*) that the sum of the moments of both forces and stresses with respect to the center of mass of the part of the gun levers above section 11–11 is equal to the moment of inertia of that part, with respect to an axis passing through its center of mass perpendicular to the plane of the forces, multiplied by the angular acceleration of the part about that axis. If these three equations do not contain more than three unknown quantities, the internal stresses in section 11–11 can be determined.

Resolution of the Total Stresses in Section 11–11 into Unknown Horizontal and Vertical Components P_s and P_t and an Unknown Couple *YY*.—Referring to Fig. 65, the total stresses in section 11–11 can be resolved into unknown horizontal and vertical components P_s and P_t, respectively, and an unknown couple *YY* as shown by the dot and dash force lines, these being the most general assumptions that can be made with regard to any set of unknown co-planar forces or stresses acting there. The point of application of P_s and P_t can be taken at the center of the section. They will be assumed to act in opposite directions to the external forces P and P_1, and if this assumption is not correct it will be shown by a negative sign opposite the numerical value of one or both of the stresses obtained from the equations. The assumption of wrong directions for the component stresses will not affect the correctness of the numerical results. The sum of the components of P_s and P_t parallel to section 11–11 is the total shearing stress in the section, and the sum of their components normal to the section is the total stress of tension or compression therein. Since P_s and P_t are applied at the center of the section they can have no tendency to rotate it and will, therefore, have no effect on the bending moment at the section. The couple *YY* can produce neither shear nor simple tension or compression in the section since the individual forces of a couple are equal in intensity but opposite in direction. It does, however, tend to rotate the section about the neutral axis *XX*, Fig. 66, and it is, therefore, the bending moment at the section.

Determination of the Values of P_s, P_t, and *YY*.—Writing the three equations expressing the relations between the known forces, $P = 179338$ lbs. and $P_1 = 92902$ lbs., and the unknown total stresses in section 11–11, and noting that as *YY* is a couple it need only appear in the equation of moments, we have, since $M = 40$ and $\Sigma mr^2 = 230$, see Fig. 65,

$$179338 - P_s = 40\, d^2x/dt^2 \tag{41}$$

$$92902 - P_t = 40\, d^2y/dt^2 \tag{42}$$

$$179338 \times \frac{22 \cos 11°}{12} + P_s \times \left\{\frac{28.75 \cos 11° - 3.682 \sin 13°}{12}\right\}$$
$$-92902 \times \frac{22 \sin 11°}{12} - P_t \times \left\{\frac{28.75 \sin 11° + 3.682 \cos 13°}{12}\right\}$$
$$- YY = 230\, d^2\phi/dt^2 \tag{43}$$

d^2x/dt^2 and d^2y/dt^2 being the accelerations of the center of mass of the part of the gun levers above section 11–11 in the horizontal and vertical directions, respectively, and $d^2\phi/dt^2$ being the angular acceleration of that part about the axis through its center of mass perpendicular to the plane of the forces. Since the unit of force is the pound, the linear accelerations must be expressed in feet per second per second and the moment of inertia in mass units and feet.

The horizontal acceleration in equation (41) may be obtained from equation (43), page 82, by neglecting the term containing $(d\phi/dt)^2$ as being too small to be considered; by substituting for $d^2\phi/dt^2$ its value of 224.98 radians per sec. per sec. from equation (7^{IV}), page 89; by substituting for a, ϕ, and $(\phi - \beta)$ their values of 4.3333 ft., 13°, and 11°,* respectively, from table 3, page 85; and by replacing c, the distance of the axis of the gun trunnions from the axis of the trunnions of the gun-lever axle, by 40/12 ft., the distance of the center of mass of the part of the gun-levers above section 11–11 from the axis of the trunnions of the gun-lever axle; whence

$$d^2x/dt^2 = (4.3333 \cos 13° + \frac{40}{12} \cos 11°)\, 224.1$$
$$= 1679 \text{ ft. per sec. per sec.}$$

and

$$M\, d^2x/dt^2 = 40 \times 1679.5 = 67180.$$

* See article 56, page 86.

By making the same omission and substitutions in equation (48), page 83, and substituting for α its value of 1° 20′ from table 3, we obtain as the vertical acceleration in equation (42)

$$d^2y/dt^2 = (4.3333 \tan 1° 20' \cos 13° - \frac{40}{12} \sin 11°)\, 224.08$$
$$= -120.5 \text{ ft. per sec. per sec.}$$

and $$M\, d^2y/dt^2 = 40\,(-120.5) = -4820$$

The angular acceleration of the part of the gun levers above section 11–11 about an axis passing through its center of mass perpendicular to the plane of the forces is the same as that of the gun levers, complete, about the axis of the gun-lever pins, and is 224.08 radians per sec. per sec., whence

$$\Sigma mr^2 \times d^2\phi/dt^2 = 230 \times 224.08 = 51543$$

Substituting these values of $M\, d^2x/dt^2$, $M\, d^2y/dt^2$, and $\Sigma mr^2\, d^2\phi/dt^2$ in equations (41), (42), and (43), respectively, the only unknown quantities remaining therein are P_s, P_t, and YY which may be determined to have the following values:

$$P_s = 112158 \text{ lbs.}$$
$$P_t = 97722 \text{ lbs.}$$
$$YY = 420839 \text{ ft. lbs.} = 5050068 \text{ in. lbs.}$$

Shearing Stress in Section 11–11. — The total shearing stress in section 11–11 is the algebraic sum of the components of P_s and P_t parallel to the section. It is equal to

$$112158 \cos 13° - 97922 \sin 13° = 87300 \text{ lbs.}$$

and the shearing stress per unit of area is, see Fig. 66,

$$87300/94.17 = 927 \text{ lbs. per sq. in.}$$

Tensile Stress in Section 11–11. — The total longitudinal stress in this section is the sum of the components of P_s and P_t perpendicular to the section; and, since these components act away from the section, the stress is one of tension. It is equal to

$$112158 \sin 13° + 97922 \cos 13° = 120446 \text{ lbs.}$$

and the tensile stress per unit of area is

$$120446/94.17 = 1279 \text{ lbs. per sq. in.}$$

Bending Stress in Section 11–11. — The bending moment YY is 5050068 in. lbs. and the corresponding maximum bending stress is, see Fig. 66,

$$5050068 \times 8.575/3882 = 11155 \text{ lbs. per sq. in.}$$

tension at the front and compression at the rear of the section.

Maximum Resultant Stress in Section 11–11. — The maximum resultant stress in the section is one of tension; it occurs at the front of the section and is equal to

$$1279 + 11155 = 12434 \text{ lbs. per sq. in.}$$

85. Stresses in the Trunnions of the Gun-Lever Axle. — The gun levers, see Fig. 65, are shrunk and bolted to the gun-lever axle, and the whole may be regarded as a beam carried in bearings in the top carriage and subjected to bending forces applied to each end of the gun levers, as shown in Fig. 31, Chapter III. Because of the great stiffness of the beam formed by the gun levers and the gun-lever axle in connection with the fact that the trunnions are .01 in. smaller in diameter than their bearings in the top carriage, the levers and axle may, like the cradle of the 5-inch barbette carriage, model of 1903, be considered as a beam that is merely supported at its ends.

The forces acting on each trunnion, see equations (4‴) and (5‴), page 89, are the horizontal force $P_5/2 = 39111.5$ lbs. and the vertical force $P_4/2 = 67874$ lbs. The resultant of these forces is

$$\sqrt{(39111.5)^2 + (67874)^2} = 78328 \text{ lbs.}$$

and it is uniformly distributed over the surface of contact between the trunnion and its bed in the top carriage. This surface is 10 ins. long, being .26 in. shorter than the trunnion to provide a clearance of .01 in. on the inside and to allow the trunnion to project through for a distance of .25 in. on the outside.

The bending moment at section AB, Fig. 65, is greater than at any other section of the trunnion. It may be obtained by considering the resultant force concentrated at the middle of the length of the surface of contact between the trunnion and its bed, whence

$$M = 78328 \times 5.01 = 392423 \text{ in. lbs.}$$

The diameter of the trunnion being 8.99 ins., the area of any right section thereof is 63.48 sq. ins. and the moment of inertia of the section with respect to its neutral axis is 320.72 ins.[4] The maximum bending stress in the section is, therefore,

$$392423 \times 4.495 / 320.72 = 5500 \text{ lbs. per sq. in.}$$

tension at the rear and compression at the front of the section.

The shearing stress in section *AB* is

$$78328 / 63.48 = 1234 \text{ lbs. per sq. in.}$$

86. Stresses in the Elevating Arm and Other Parts of the 6-inch Disappearing Carriage, Model of 1905 MI. — The stresses in any section of the elevating arm, which rotates about an axis on the rear transom of the carriage, must be determined by the same method as the stresses in section 11–11 of the gun levers. The method of computing the stresses in the remaining parts of the carriage is the same in principle as that followed throughout this text in the determination of the stresses in parts of the 3-inch field carriage and of the 5-inch barbette carriage.

CHAPTER V.

TOOTHED GEARING.

87. Definition. — Ratio of Angular Velocities. — In order to transmit rotary motion from one shaft to another of a gun carriage, or of any other machine, toothed gearing is employed. This consists of a toothed wheel fastened to one shaft which when the shaft rotates moves with it, and by the engagement of its teeth with those of a second wheel fastened to a second shaft causes a rotation of the second wheel and shaft.

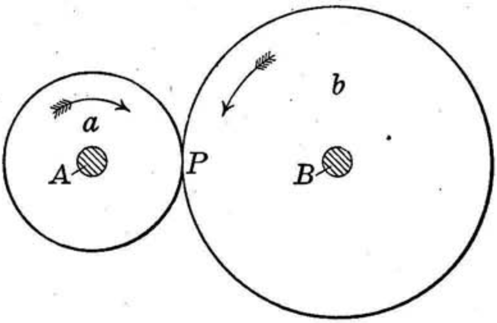


Fig. 67.

Let *a* and *b*, Fig. 67, be two wheels mounted on parallel shafts *A* and *B* and pressed together with considerable force so that the friction at their line of contact projected on *P* is sufficient to make either wheel turn if the other is turned. Suppose the shaft *B* be rotated counter-clockwise as shown by the arrow. Then *b* will rotate in the same direction and because of the friction at *P* the wheel *a* and the shaft *A* will be forced to rotate in the opposite direction as shown by the arrow on *a*.

Let V be the surface velocity of the wheel *b*,
r_b the radius of the wheel *b*,
ω_b the angular velocity of the wheel *b*,

N_b the number of revolutions per minute of the wheel b,
r_a the radius of the wheel a,
ω_a the angular velocity of the wheel a, and
N_a the number of revolutions per minute of the wheel a.

Assuming that there is no slipping at the line of contact, the surface velocities of a and b must be the same and, consequently,

$$V = r_b\omega_b = r_a\omega_a$$

or

$$\omega_a/\omega_b = r_b/r_a$$

That is, *the angular velocities of the two wheels are inversely as their radii.*

Since the number of revolutions per minute of a wheel varies directly as its angular velocity we may write

$$N_a/N_b = \omega_a/\omega_b = r_b/r_a$$

or *the numbers of revolutions per minute of the two wheels are inversely as their radii.*

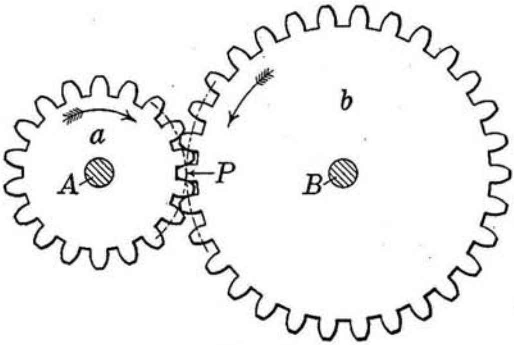


Fig. 68.

88. Necessity for Teeth. — Tooth Surfaces Must Have Certain Definite Forms. — The wheels a and b are called friction wheels and are used to a limited extent in machinery to transmit rotary motion from one shaft to another. If any great amount of power has to be transmitted, however, the wheels will slip as the greatest force that can be transmitted between their surfaces is equal to the normal pressure there multiplied by the coefficient of friction. To prevent slipping the surface of each wheel is provided with projections called teeth which engage with corresponding teeth on the other wheel as shown in Fig. 68.

It is evident whatever be the shape of the teeth, provided they are of the same uniform size and uniformly spaced on both wheels and the recesses between them are large enough for them to enter, that the numbers of revolutions per minute wil be inversely proportional to the radii of the wheels or to the numbers of the teeth, since these must vary directly as the radii; and it may not, therefore, be apparent at first why the shape of the teeth should not be arbitrarily selected. It is necessary, however, that the machinery shall run smoothly, without jerks, and in most cases it is very essential that the velocity of one moving part of a machine shall at all times have definite relations to the velocities of the other moving parts. Therefore, not only must the numbers of revolutions per minute of the wheels have a definite relation to each other but the angular velocities at each instant of time must have this same definite ratio. This is possible only when the tooth surfaces have certain definite forms.

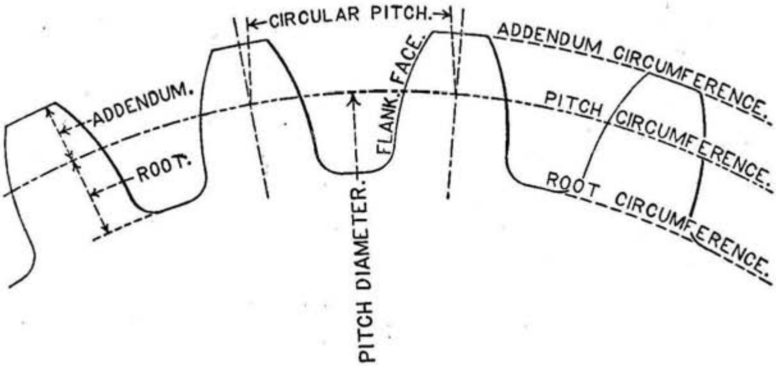


Fig. 69.

89. Outline of Gear-Teeth. — Pitch Circumference. — Circular Pitch. — Diametral Pitch. — Fig. 69 shows the outline of several gear-teeth.

The *pitch circumference* * shown in the figure is the projection of the imaginary cylindrical surface which, rolling on a similar

* The circumferences of the pitch, addendum, and root circles are generally referred to as circles, not circumferences.

cylindrical surface of another gear-wheel, would cause the two wheels to rotate together with the same relative angular velocities as would be caused by the engagement of the teeth of the wheels; or the pitch circumference may be considered as the projection of the cylindrical surface of a friction wheel such as shown in Fig. 67 on which teeth have been fastened as shown in Fig. 68 to make the motion more reliable, it being understood that the ratio between the angular velocities of the wheels in both figures is the same.

The circle drawn through the outer ends of the teeth is called the *addendum circle* and that drawn through their inner ends is called the *root circle*. The distance between the pitch and addendum circumferences measured on a radial line is called the *addendum* and that between the pitch and root circumferences measured on a radial line is called the *root*. The term *addendum* is also frequently used to designate the part of the tooth outside the pitch surface, and the term *root* to designate the part of the tooth inside that surface. The working surface of the portion of the tooth outside the pitch surface is called the *face* of the tooth and the working surface inside the pitch surface is called the *flank* of the tooth. The distance between corresponding points of two adjacent teeth measured on the pitch circumference is called the *circular pitch*. The *diametral pitch* is the number of teeth per inch of diameter of the pitch circle; thus if p_1 represent the diametral pitch, the number of teeth of a gear-wheel whose pitch diameter is d is p_1d. From the definition of circular pitch the number of teeth of the same wheel expressed in terms of the circular pitch, which will be represented by p, is $\pi d/p$. Since $p_1d = \pi d/p$ we have $p_1 = \pi/p$, or the diametral pitch is equal to π divided by the circular pitch. The diametral pitch is the one more frequently used to express the pitch of the teeth of gear-wheels.

90. Angular Velocity of Rotating Wheel. — Let the wheel a shown in Fig. 70 be rotating about its center O and let the velocity of any point b on its circumference be represented in magnitude and direction by the line marked V. Then if r be the radius of the wheel its angular velocity ω will be V/r. Resolve the velocity V into two components normal to each other, V_n and V'_n as shown. Let α be the angle between the directions of V and V_n and let r_n

be the perpendicular distance from the center of the wheel to the direction line of V_n. Then

$$V_n = V \cos \alpha$$

and

$$r_n = r \cos \alpha$$

whence

$$\frac{V_n}{r_n} = \frac{V \cos \alpha}{r \cos \alpha} = \frac{V}{r} = \omega$$

and, similarly

$$V'_n/r'_n = \omega.$$

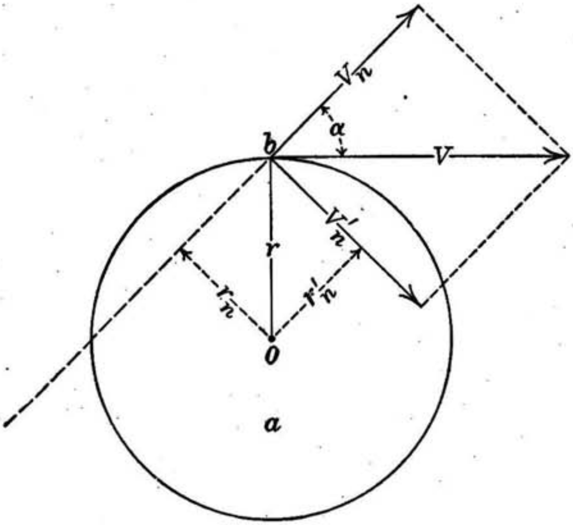


Fig. 70.

If the wheel be increased in size so that the point b no longer lies on its circumference, the relations just established will still hold for the velocity at the point b, r in this case, however, will be the perpendicular distance from the center of the wheel to the direction line of V; whence we may write

If the velocity of any point of a wheel rotating about its center be resolved into two components normal to each other, the angular velocity of the wheel will be equal to either component divided by the perpendicular distance from the center of the wheel to the direction line of that component.

91. Condition to be Fulfilled by Tooth Curves. — Let T_a and T_b, Fig. 71, be two teeth in contact at the point m, and C and C_1 be the centers of the wheels to which T_a and T_b belong. Since the tooth curves are tangent at m they have at this point a common normal NN_1 which intersects the line of centers CC_1 at some point P. Suppose the wheel B to be rotating counter-clockwise

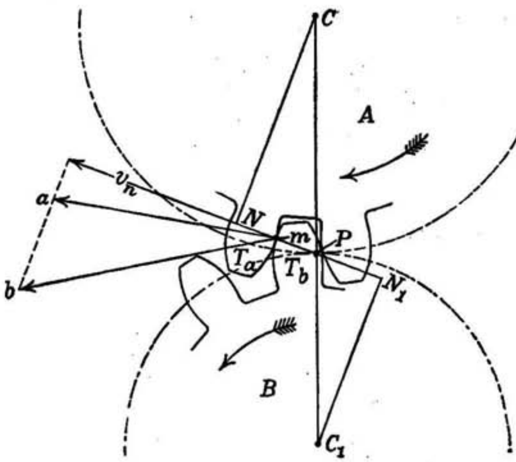


Fig. 71.

around its center C_1 driving the wheel A around its center C in a clockwise direction; then the velocity of the point m on the tooth T_b may be represented by the line mb drawn tangent to the circle described through m about the center C_1, and, similarly, the velocity of the point m on the tooth T_a may be represented by the line ma drawn tangent to the circle described through m about the center C. Let the point m on the tooth T_b be called m_b and the point m on the tooth T_a be called m_a, and resolve the velocities of m_b and m_a into components parallel and perpendicular to the common normal NN_1.

While the velocities of m_b and m_a vary in direction and in magnitude, the components of these velocities in the direction of their common normal must be equal during the time m_b and m_a are in contact, for otherwise the teeth would separate or one surface would penetrate the other. Representing by v_n the equal component volecities of m_b and m_a in the direction of the common normal, and dropping the perpendiculars C_1N_1 and CN on this normal, the angular velocities ω_b and ω_a of the wheels B and A, respectively, are (see article 90)

$$\omega_b = v_n/C_1N_1 \text{ and } \omega_a = v_n/CN$$

whence $$\omega_b/\omega_a = CN/C_1N_1 = CP/C_1P$$

or

The angular velocities of the two wheels are inversely as the segments into which the line of centers is cut by the common normal to the tooth curves at their point of contact.

The condition to be fulfilled by tooth curves in order that the ratio of the angular velocities of the wheels shall be constant at every instant is, therefore, *that the common normal to the tooth curves at their point of contact shall always pass through a fixed point on the line of centers.*

In order that the wheels shall have the same angular velocities as their pitch circles the *common normal to the tooth curves at their point of contact must intersect the line of centers at the pitch point, which is the point of tangency of the two pitch circles.* Any number of curves will fulfill this condition but the two generally adopted for tooth outlines are the involute curve and the cycloidal curve.

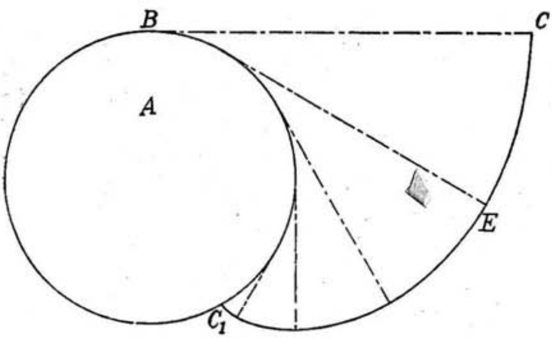


Fig. 72.

92. The Involute Curve. — If a cord be wound around a circle and then unwound, keeping the unwound portion straight, a point of the cord will describe an involute to the circle. Let BC, Fig. 72, be a portion of a cord unwound from the circle A. The point

C of the cord before it was unwound was at the point C_1 of the circle, and the curve C_1EC is the involute to the circle traced by the point C. At each instant C is rotating about the point of tangency of the cord to the circle and, consequently, its direction at that instant is at right angles to the tangent line drawn from it to the circle. Any normal to the involute is, therefore, tangent to the circle.

The Involute System of Gear-Teeth. — Let APB and CPD, Fig. 73, be the pitch circles of two wheels on which it is desired to construct teeth of such profile that the ratio of their angular velocities when running in gear shall at each instant be exactly the same as the ratio of the angular velocities of the pitch circles when running together without teeth. Through the pitch point P draw a line NN_1 at an angle α with the common tangent to the pitch circles, and draw circles EF and GH tangent to NN_1 about the centers O and O_1, respectively, of the pitch circles. Let the profiles of the teeth on APB be involutes to the circle EF and the profiles of those on CPD be involutes to the circle GH. Now let APB be rotated in the direction of the arrow until one of its teeth is brought in contact with a tooth on CPD. At the point of contact the curves on the two teeth will have a common normal which must be tangent to the circle EF because the profile of the tooth on APB is an involute to that circle, and which must also be tangent to the circle GH because the profile of the tooth on CPD is an involute to the circle GH. From the figure it is evident that NN_1 is the only line tangent to the circles EF and GH that can pass through the point of contact of the teeth, and this line must, therefore, be the common normal to the tooth curves at their point of contact. If, however, the wheel APB be rotated in the opposite direction the point of contact will change to the other sides of the teeth and the common normal to the tooth curves at their point of contact will be the line $N'N'_1$, which is also a common tangent to the circles EF and GH passing through the pitch point P. The point of contact of the teeth must, therefore, always lie on one or the other of the lines NN_1 and $N'N'_1$, both of which are common normals to the tooth curves passing through the pitch point. It is evident, therefore, that the involute to a circle fulfills strictly the condition required for tooth profiles of gear-wheels.

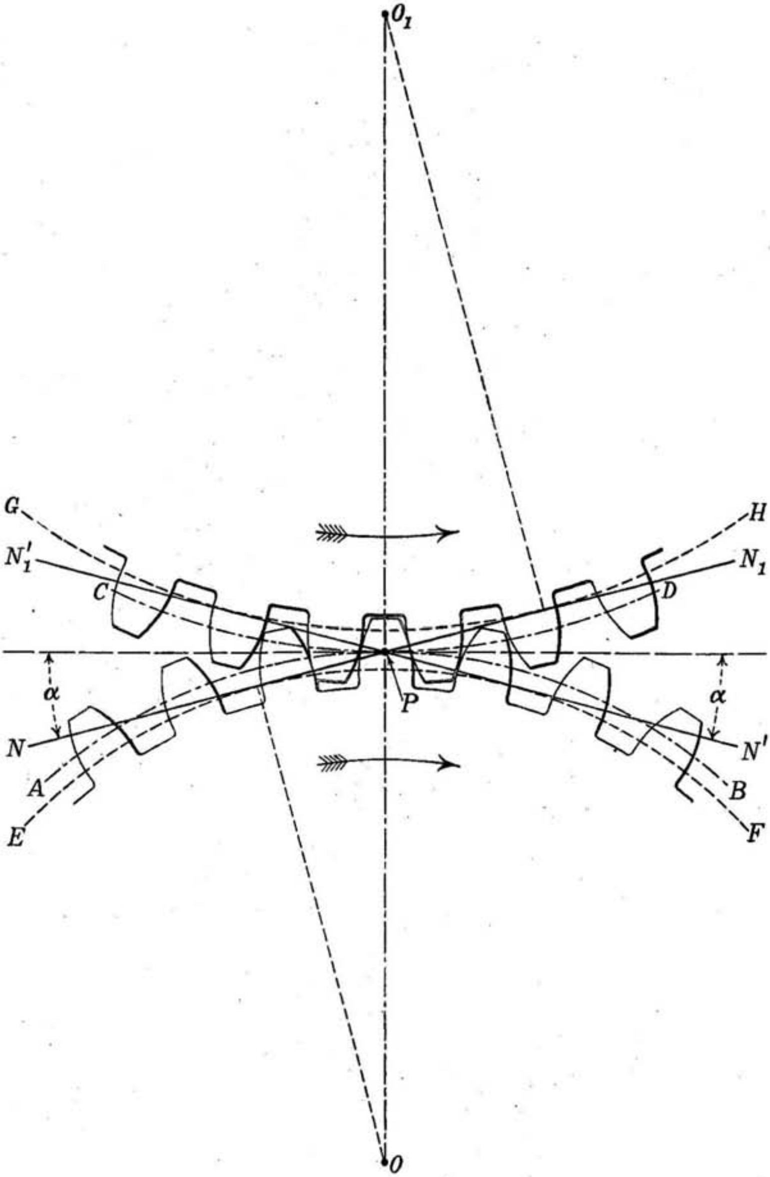


Fig. 73

The circles EF and GH are called base circles. Since the point of contact of the teeth is always on one or the other of the lines NN_1 and $N'N'_1$, these lines are called the lines of action. Neglecting friction the line of action is the direction of the pressure between the teeth. The angle α between the line of action and the common tangent to the pitch circles is called the angle of obliquity. In order that gears with involute teeth shall be interchangeable they must have the same circular pitch and the same angle of obliquity. The standard angle of obliquity is 15°.

The arc of the pitch circle that subtends the angle through which a wheel rotates from the time when one of its teeth comes in contact with a tooth of the other wheel until the point of contact reaches the pitch point P is called the arc of approach; and the arc of the pitch circle that subtends the angle through which the wheel rotates from the time the point of contact leaves the pitch point until the teeth separate is called the arc of recess. The sum of these arcs is called the arc of action. In order that there shall always be contact between the teeth, the arc of action should be at least equal to the circular pitch. If it is desired that there shall always be contact between two pairs of teeth, the arc of action should be at least equal to twice the circular pitch. The arc of action may be increased by lengthening the teeth up to a certain limit. Fig. 74 shows the profile of the involute tooth.

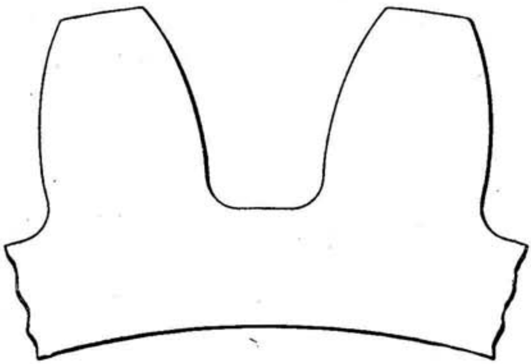


Fig. 74.

93. The Cycloid. — Epicycloid. — Hypocycloid. — If a circle be rolled upon a straight line a point in its circumference will trace a curve called a cycloid. If the circle be rolled upon the

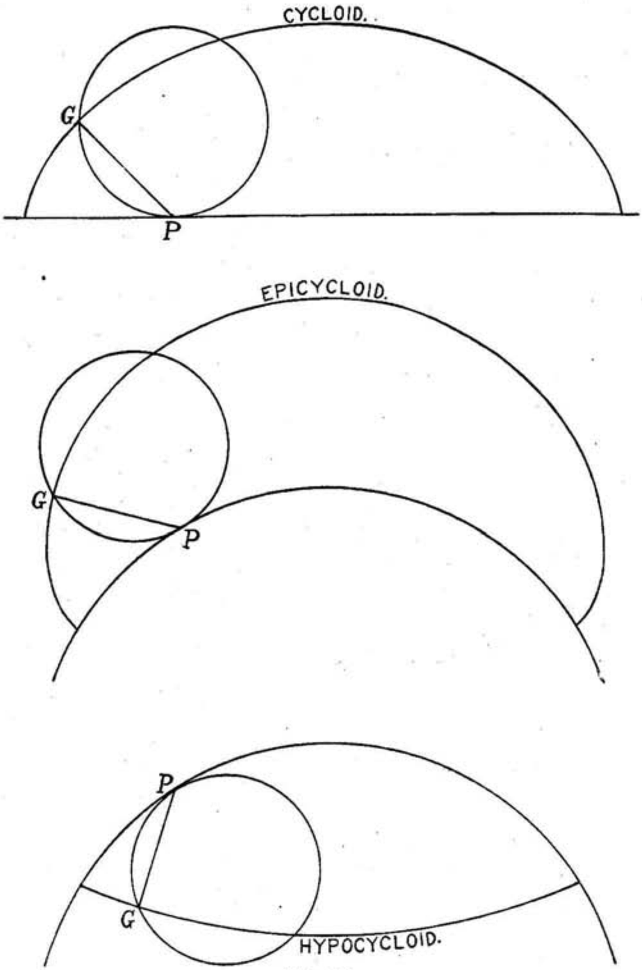


Fig. 75.

convex side of another circle a point in the circumference of the former circle will trace a curve called an epicycloid; and if rolled upon the concave side of the circle the curve traced is called a hypocycloid. These curves are shown in Fig. 75.

Let *G* be the point on the generating circle that traces each of the three curves shown in Fig. 75. Then, since the motion of the generating circle and the point *G* is at each instant a rotation about the point of tangency *P* of the generating circle and the line upon which it rolls, the direction of motion of *G* is at right angles to the line joining it to *P*, and this line *GP* is a normal to the cycloid, epicycloid, or hypocycloid as the case may be.

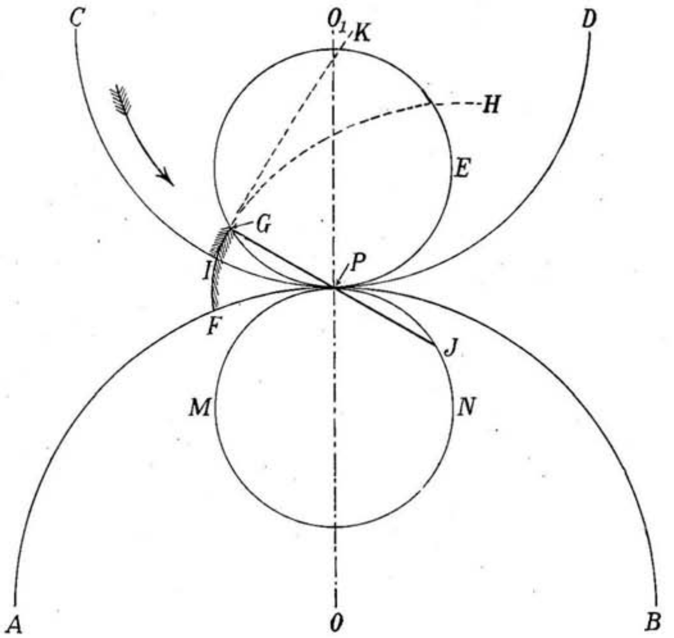


Fig. 76.

The Cycloidal System of Gear-Teeth. — Let *APB* and *CPD*, Fig. 76, be the pitch circles of two wheels on which it is desired to construct teeth of such profile that the ratio of their angular velocities when running in gear shall at each instant be exactly the same as that of the pitch circles when running together without teeth. Suppose the generating circle *GPE* be rolled on the outside of *APB* and that the point *G* traces the epicycloid *FGH*; suppose also that the generating circle be rolled on the inside of *CPD*, the point *G* tracing the hypocycloid *IGK*. Let the portion

FG of the epicycloid be taken as the profile of the part of a tooth on *APB* outside the pitch circle, that is, as the profile of the face of a tooth on *APB* (see Fig. 69), and the portion *IG* of the hypocycloid be taken as the profile of the flank of a tooth on *CPD*; and let *FG* and *IG* be brought into contact as shown. Then the common normal to the tooth curves at their point of contact must pass through the point at which the generating circle was tangent to the pitch circle *APB* at the instant the point *G* of the epicycloid was being traced, and it must also pass through the point at which the generating circle was tangent to the pitch circle *CPD* at the instant the point *G* of the hypocycloid was being traced; and further, since the point of contact *G* is a point of both the epicycloid and the hypocycloid, the generating circle must have been tangent to both of the pitch circles at the instant the point *G* was being traced. The common normal to the tooth curves at their point of contact must, therefore, pass through the pitch point *P*. As this is true for any point of contact of the tooth curves *FG* and *IG*, it follows that when the profile of the face of a tooth on one gear is part of the epicycloid traced by a point of the generating circle rolling on the outside of the pitch circle of that gear, and the profile of the flank of the tooth on the other gear is part of the hypocycloid traced by a point of the *same generating circle* rolling on the inside of the pitch circle of the second gear, the tooth curves fulfill strictly the condition required for tooth profiles of gear-wheels.

Referring again to Fig. 76, the generating circle *GPE* rolling on the outside of the pitch circle *APB* can only generate the profile for the faces of the teeth on the lower wheel, and rolling on the inside of *CPD* it can only generate the profile for the flanks of the teeth on the upper wheel. To generate the profile for the flanks of the teeth on the lower wheel a second generating circle *MN* must be rolled on the inside of the pitch circle *APB*, and to generate the profile for the faces of the teeth on the upper wheel the same circle *MN* must be rolled on the outside of *CPD*. For two wheels to gear together the circular pitch must be the same in each and the generating circle for the profile of the faces of the teeth on the one must be the same size as the generating circle for the profile of the flanks of the teeth on the other, but it is not necessary that the same generating circle be used for the profiles of the faces and the flanks of the teeth on the same wheel. It is very important, however, that all gear-wheels of the same circular pitch shall be interchangeable, and to accomplish this the same generating circle must be used for the profiles of the faces and flanks of the teeth on all the wheels.

If the diameter of the generating circle is larger than the radius of the pitch circle the profile of the flank of the tooth will curve rapidly towards the center of the tooth at the root circumference, thereby diminishing its thickness at the root and weakening the tooth. For a set of wheels of the same circular pitch required to be interchangeable the size of the generating circle, which must be the same for all, is limited by the fact that, in order not to unduly weaken the teeth of the smallest wheel of the set, the diameter of the generating circle must not be greater than the pitch radius of that wheel. The number of teeth in the smallest wheel of the cycloidal system is taken as twelve, and for all wheels of the same circular pitch the diameter of the generating circle is made equal to the pitch radius of the twelve-tooth wheel.

Referring to Fig. 76, suppose the upper wheel be rotating counter-clockwise in the direction of the arrow. It will then rotate the lower wheel clockwise and, if *G* be the first point of contact between the flank of the tooth on the upper wheel and the face of the tooth on the lower, the line of pressure between the teeth at that instant, neglecting friction, will be *GP*. As the wheels advance the point of contact of the teeth will slide along the arc *GP* of the generating circle *GE*, and the direction of the line of pressure will continue to change until when the point of contact reaches *P* this line will be tangent to both pitch circles. After the point *P* has been reached the face of the tooth on the upper wheel will come in contact with the flank of the tooth on the lower, and the point of contact will slide along the arc *PJ* of the generating circle *MN*, the direction of the line of pressure between the teeth changing continually until the teeth are about to separate at the point *J*, when the line of pressure is *PJ*. The greatest angle between the common tangent to the pitch circles and the line of pressure between the teeth occurs when contact is just commencing or ending. This angle should ordinarily not be permitted to be greater than 30°.

As in the case of the involute system, the arc of the pitch circle that subtends the angle through which a wheel rotates from the time when one of its teeth comes in contact with a tooth of the other wheel until the point of contact reaches the pitch point P is called the arc of approach; and the arc of the pitch circle that subtends the angle through which the wheel rotates from the time the point of contact leaves the pitch point until the teeth separate is called the arc of recess. The sum of these arcs is called the arc of action. In order that there shall always be contact between the teeth the arc of action should be at least equal to the circular pitch. If it is desired that there shall always be contact between two pairs of teeth, the arc of action should be at least equal to twice the circular pitch. The arc of action may be increased by increasing the length of the teeth up to a certain limit.

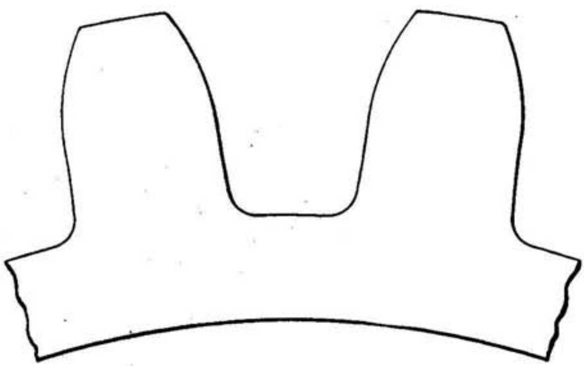


Fig. 77.

Fig. 77 shows the outline of the cycloidal tooth.

94. Spur Gears. — The gear-wheels shown in Fig. 68 are used to transmit rotary motion between parallel shafts. The pitch surfaces of the wheels are cylindrical and the tooth surfaces can be generated by moving the involute or cycloidal profile in a direction parallel to the axis of the shaft. Gear-wheels used to transmit rotary motion between parallel shafts are called *spur gears*. The wheel that imparts motion to the other is called the driver and the driven wheel is called the follower. Either one of a pair of spur gears in mesh may be the driver.

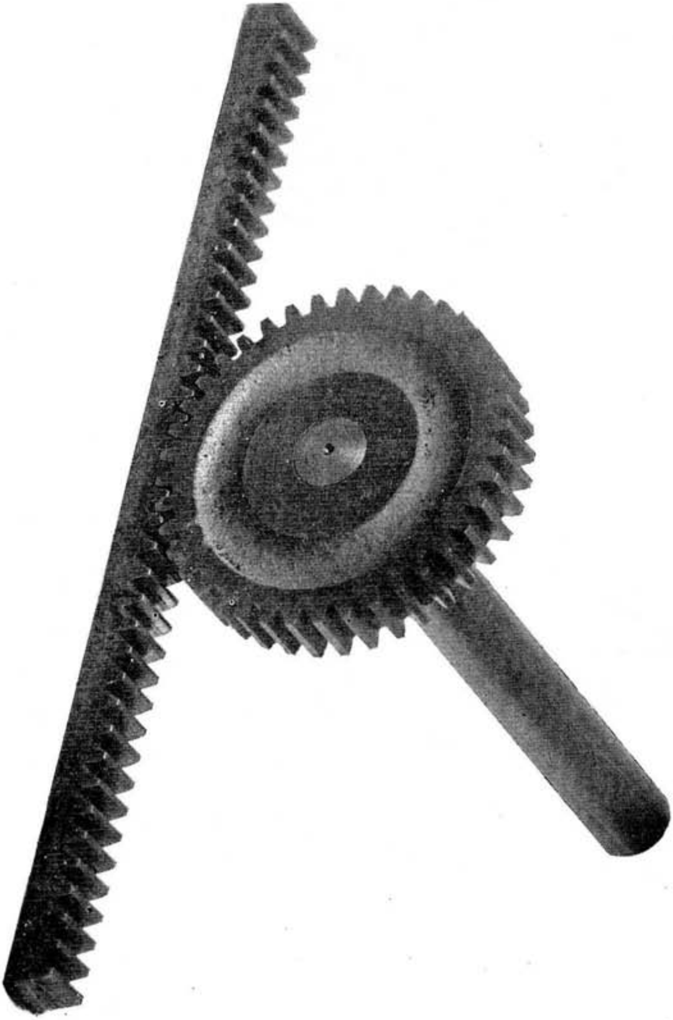


Fig. 78. — Rack and Spur Gear.

Rack and Spur Gear. — If the radius of one of the spur gears shown in Fig. 68 be increased until it becomes infinite, the spur gear becomes a rack and its pitch circumference a straight line whose linear velocity is the same as that of a point on the pitch circumference of the gear with which it meshes. A rack and spur gear are generally used to change a reciprocating rotary motion into a reciprocating motion of translation or vice versa.

Either the involute or the cycloidal system may be used for the teeth of a rack and spur gear. In the involute system the profile of the rack tooth is a straight line perpendicular to the line of action to which the base circle of the spur gear is tangent. In the cycloidal system the profile of the face of the rack tooth is a cycloid traced by a point on the generating circle while rolling on the outside of the pitch line of the rack. The profile of the flank of the rack tooth is also a cycloid traced by a point of the generating circle while rolling on the inside of the pitch line of the rack. For interchangeability the diameter of the generating circle is made equal to the pitch radius of the twelve-tooth gear of the same circular pitch. Either the spur gear or the rack may be the driver.

Fig. 78 shows a rack and spur gear.

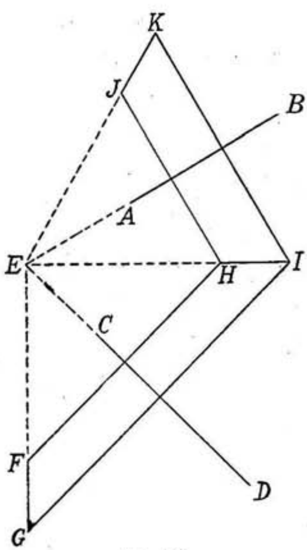


Fig. 79.

95. Bevel Gears. — If the shafts to be connected by gear-wheels would intersect if prolonged, the pitch surfaces cannot be cylindrical but must be the surfaces of frustums of cones whose apexes are at the point of intersection of the shafts, as shown in Fig. 79, in which *AB* and *CD* are the shafts which if prolonged would intersect at *E*, and *FGHI* and *HIJK* are the frustums of cones whose apexes are at *E* and whose surfaces are the pitch surfaces of gear-wheels transmitting rotary motion between the shafts. Gear-wheels with conical pitch surfaces transmitting rotary motion between shafts that would intersect if prolonged are called *bevel gears*. The angular velocities of a pair of bevel gears in mesh are inversely proportional to the sines of the angles between the elements of the pitch surfaces in contact and the shafts to which the gears are attached or, more simply, inversely proportional to the numbers of the teeth on the gears. Either one of a pair of bevel gears in mesh may be the driver.

Both the involute and the cycloidal systems are used for the teeth of bevel gears but the tooth surfaces, while having rectilinear elements, cannot be formed by sliding a profile of the tooth along a straight line. For the involute system the tooth surfaces are obtained by rolling a plane on a base cone whose apex is at the point of intersection of the shafts, and for the cycloidal system the tooth surfaces are obtained by rolling generating cones on the inside and outside of the pitch cones. The tooth profile of a bevel gear at any section at right angles to the axis of the shaft can of course be obtained by rolling a line on the base circle cut by the section from the base cone, or by rolling a generating circle on the inside and outside of the pitch circle cut by the section from the pitch cone.

Fig. 80 shows a pair of bevel gears in mesh.

96. Screw Gearing. — Worm and Worm-Wheel. — Referring to Fig. 78, showing a rack and spur gear, suppose the teeth of the rack instead of being parallel to the axis of the gear be given an inclination to that axis as shown in Fig. 81. In order that the teeth of the gear shall mesh properly with the inclined teeth of the rack the former must be inclined also as shown in elevation in Fig. 81, what were formerly rectilinear elements of the teeth now becoming helices on the cylindrical surface of the gear. The inclination of the teeth will in no way affect the working of the rack and gear and a longitudinal movement of the rack at right angles to the axis of the gear will cause rotation of the latter as before. Owing to the inclination of the teeth, however, a movement of the rack at right angles to the axis of the gear is not the only movement of the rack that will rotate the gear, for an examination of Fig. 81 shows at once that a movement of the rack in a direction parallel to the axis of the gear, that is, perpendicular to the plane of the paper, will also cause rotation of the gear which will, however, be rather limited because of the comparatively limited width of the rack.

In the figure the distance *ab* is the pitch of the rack teeth,

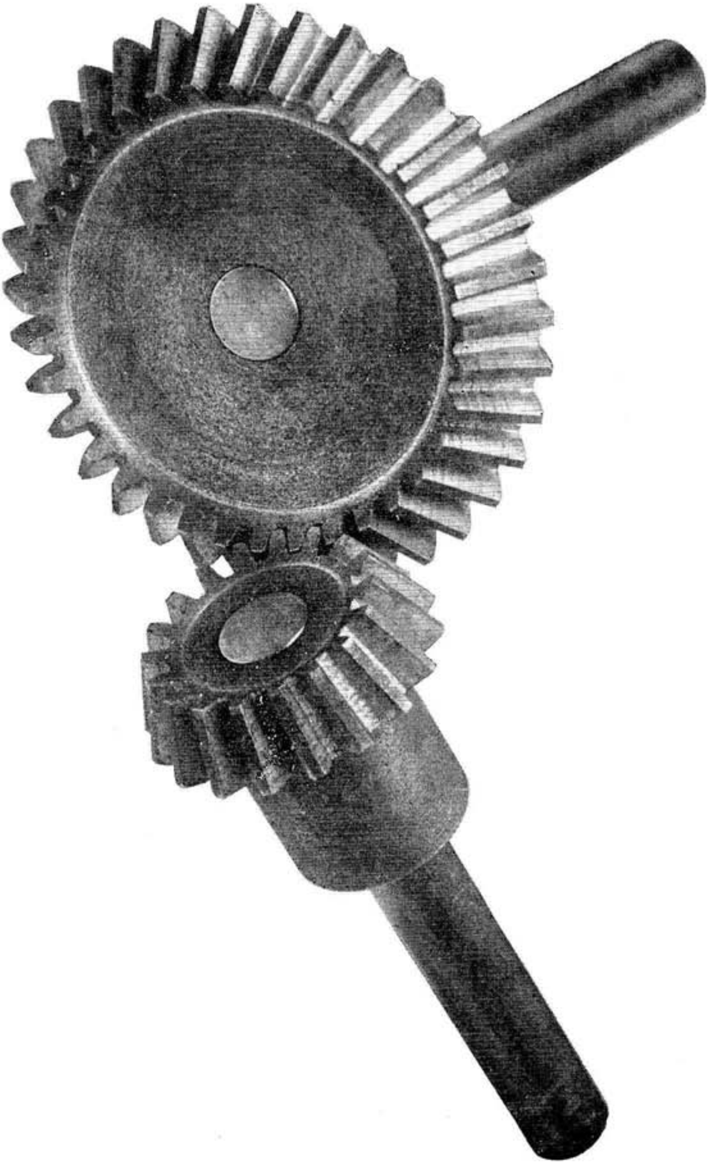


Fig. 80. — Bevel Gears.

equal to the circular pitch of the teeth of the gear, and *ac* is equal to the length of the arc measured on its pitch circumference through which the gear would be rotated by a movement of the rack over a distance equal to its width in a direction parallel to

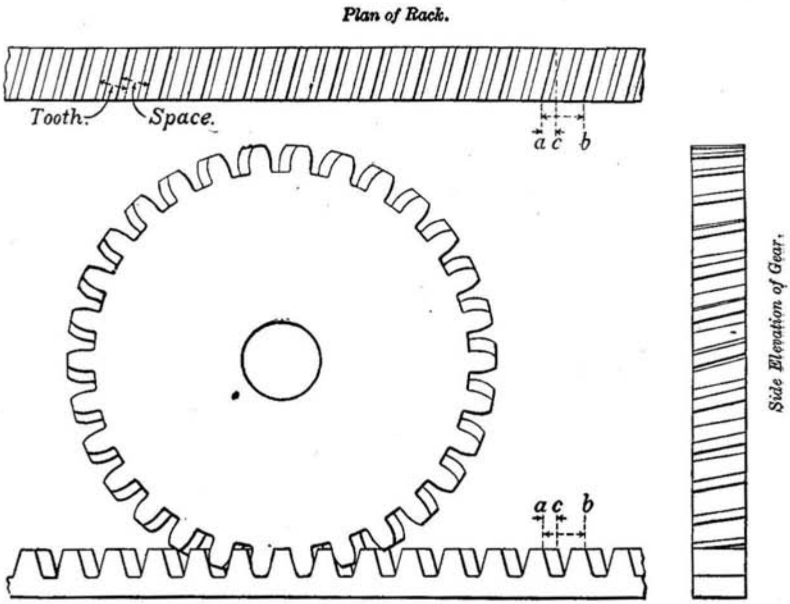


Fig. 81.

the axis of the gear. Now suppose the rack be rolled into a cylinder whose axis is parallel to the longest dimension of the original rack and whose circumference is equal to the width of the rack; then a tooth on the rack will become one complete turn of a helical projection or tooth on the cylinder, and the pitch of the helix so formed will be equal to the distance *ac*. If all teeth but one be removed from the cylinder and the latter be placed with its axis perpendicular to that of the gear and so that one end of the helix formed by the remaining tooth engages in a tooth space of the gear, the cylinder can be rotated about its axis and its helical tooth during one complete revolution will cause the gear to rotate through an arc measured on its pitch circumference equal to the distance *ac*, Fig. 81, in the same manner as did the inclined tooth on the rack when the latter was moved over a distance equal to its width parallel to the axis of the gear. If the rack is of the width shown in the figure, it is evident that after one revolution of the cylinder formed therefrom further motion will be prevented by the end of its helical tooth coming against the end of one of the teeth on the gear, for the movement of the latter measured on its pitch circumference has been less than its circular pitch. If, however, the width of the rack be increased so that the distance *ac*, Fig. 81, which is the amount one end of an inclined tooth is in advance of the other end, is just equal to *ab*, the circular pitch of the gear, and the new rack be rolled into a cylinder, it will be seen that, when the helical tooth on the cylinder is engaged in a tooth space of the gear as before and the cylinder is rotated once about its axis, the gear will be rotated through an arc measured on its pitch circumference equal to its circular pitch. After one complete revolution of the cylinder the end of its helical tooth will occupy exactly the same position as when it was first engaged with a tooth space on the gear, but that space will have moved through an arc equal to the circular pitch of the gear so that the helical tooth can no longer engage with it; but as the distance between the centers of the tooth spaces of the gear is equal to its circular pitch, the space in rear will occupy exactly the same position as did the first before the cylinder was rotated and, consequently, the end of the helical tooth on the cylinder will now engage with it. Another complete revolution of the cylinder will again rotate the gear through an arc measured on its pitch circumference equal to its circular pitch, will restore the helical tooth on the cylinder to its original position, and will bring a third tooth space of the gear in position to be engaged by the tooth of the cylinder, and so on; so that continuous rotation of the cylinder will cause continuous rotation of the gear.

In this combination the gear with helical teeth is called a *worm-wheel* and the cylindrical rack with a helical tooth is called a *worm*. A worm and worm-wheel form a particular case of screw gearing, so called because of the screw-like action of the teeth of the gears.

In this discussion reference has been had to a length of the helical tooth on the worm sufficient to make one complete turn only about its axis. This length is all that is necessary to secure

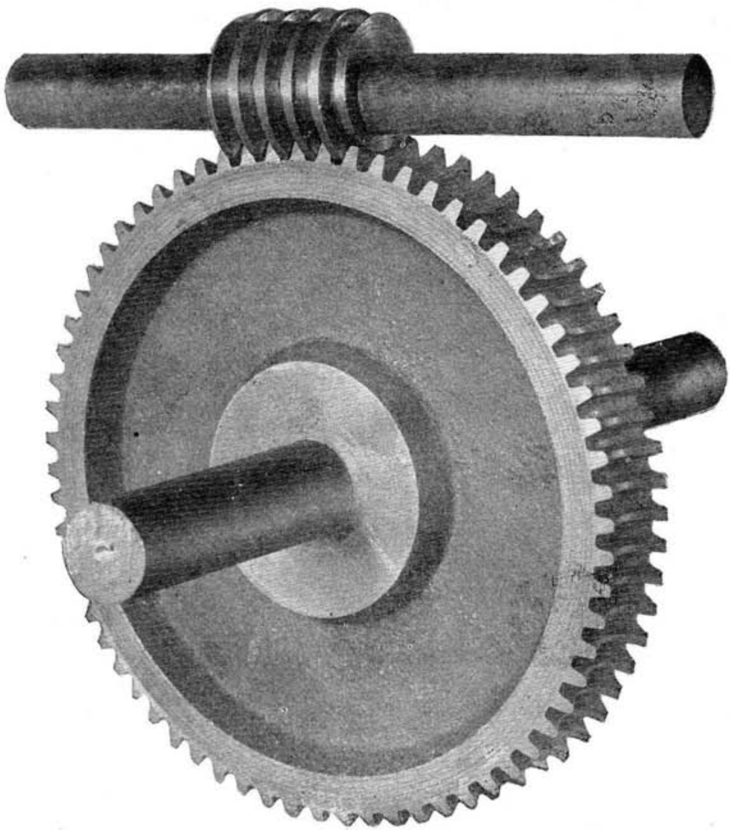
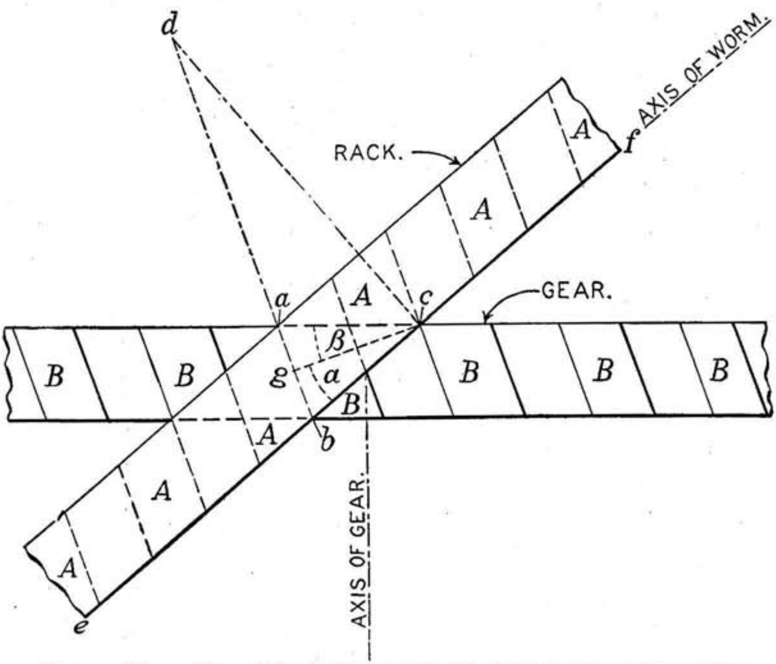


Fig. 82. — Worm and Worm-Wheel.

continuous rotation of the worm-wheel, but in practice several turns are used as this distributes the pressure over two or more teeth on the wheel.

Fig. 82 shows a worm and worm-wheel in mesh.

Referring again to Fig. 81, it is evident that the rack with inclined teeth need not be perpendicular to the axis of the spur gear provided the helical teeth on the gear make the same angle with its axis as do the inclined teeth on the rack. Such a rack and spur gear are shown in plan in Fig. 83, the teeth of the rack being marked *A* and those of the gear being marked *B*.



NOTE. — The outlines of the teeth shown in this figure correspond to sections of the teeth at the pitch surfaces.

Fig. 83.

When this combination is used as a rack and gear the rack must be held in guideways to compel it to move only in the direction of its longest dimension, and the gear must be prevented from moving in a direction parallel to the shaft by collars on the shaft or other suitable means. If the guideways be removed, however, and the rack given a motion of translation at right angles to the direction of its longest dimension it will still cause a limited rotation of the gear; and, by giving the rack the proper width and wrapping it around a cylinder whose axis is parallel to the longest dimension of the rack, it will become a worm as before which, when rotated continuously about its axis, will cause continuous rotation of the gear, now called a worm-wheel. It is evident from this that it is not necessary that the axes of a worm and worm-wheel be at right angles to each other, although such is generally the case.

In Fig. 83 the circular pitch of the gear is equal to the distance *ac*, which is the distance between corresponding points of adjacent teeth of the rack measured in a direction at right angles to the axis of the gear. Now in order to rotate the gear through an arc measured on its pitch circumference equal in length to *ac* by a movement of the rack at right angles to its longest dimension over a distance equal to its width, it is evident that the width of the rack must be made equal to the distance *cd*, the point *d* being the intersection of the edge of the tooth passing through *a* by a line drawn through *c* at right angles to the longest dimension of the rack. It is also apparent that, when the width of the rack is made equal to *cd* and the rack is rolled into a cylinder whose axis is *ef*, the various teeth of the rack will form so many complete turns of a continuous helical tooth on the cylinder, and that one complete revolution of the cylinder or worm will rotate the gear, now called a worm-wheel, through an arc measured on its pitch circumference equal to its circular pitch, which is equal to *ac*. The pitch of the helix on the worm will then be equal to *bc*, and the circular pitch of the wheel, equal to *ac*, is equal to the oblique projection on a plane at right angles to the axis of the wheel of the pitch of the helix of the worm, the projecting lines being parallel to the teeth of the wheel.

Let *cg* be a line, in a plane parallel to the axes of the worm and wheel, drawn perpendicular to the teeth through their point of contact *c*, and let α be the angle which this line makes with the axis *ef* of the worm, and β be the angle which it makes with a plane at right angles to the axis of the wheel; then if α is greater than β a point on the pitch circumference of the wheel will, during one complete revolution of the worm, move through an arc less in length than the pitch of the helix on the worm, and vice versa if α is less than β. If α is equal to β a point on the pitch circumference of the wheel will, during one complete revolution of the worm, move through an arc equal in length to the pitch of the helix on the worm, the same as it would if the axes of the worm and wheel were perpendicular to each other.

Since the axis of the worm is oblique to the axis of the wheel, the point of contact of the teeth during one revolution of the worm travels in a diagonal line across the rim of the wheel instead of remaining in its central plane as it does when the axes of the worm and wheel are perpendicular to each other. On this account the rim of the wheel must not be too narrow or the teeth will not remain in contact during one complete revolution of the worm.

Spiral Gears. — If instead of wrapping the rack shown in Fig. 83 around a cylinder whose axis is parallel to the longest dimension of the rack, it be wrapped around one whose axis is perpendicular to that dimension, the continuous rotation of this cylinder will also cause continuous rotation of the gear, and the combination so formed is called a pair of spiral gears. Fig. 84 shows a pair of spiral gears in mesh.

Spiral gears are also a particular form of screw gearing and do not differ in principle from a worm and worm-wheel as may be inferred from the manner in which they are developed from a rack and spur gear, or as may perhaps be more readily seen from the following discussion. Suppose the cylinder formed by wrapping the rack of Fig. 83 around an axis perpendicular to its longest dimension be increased in length until one of its helical teeth makes a complete turn about it, and that all teeth except this one be removed from the cylinder. Then, except for the fact that the pitch of the helix on the cylinder is relatively so great in comparison with the diameter and width of the gear that the tooth of the gear would probably rotate out of contact with that of the cylinder before the latter could make one complete revolution, one such revolution of the cylinder would cause a point on the pitch circumference of the gear to move through an arc equal in length to the oblique projection on a plane at right angles to the axis of the gear of the pitch of the helix on the cylinder, the projecting lines being parallel to the teeth of the gear; and, furthermore, by increasing the diameter of the gear, keeping the inclina-

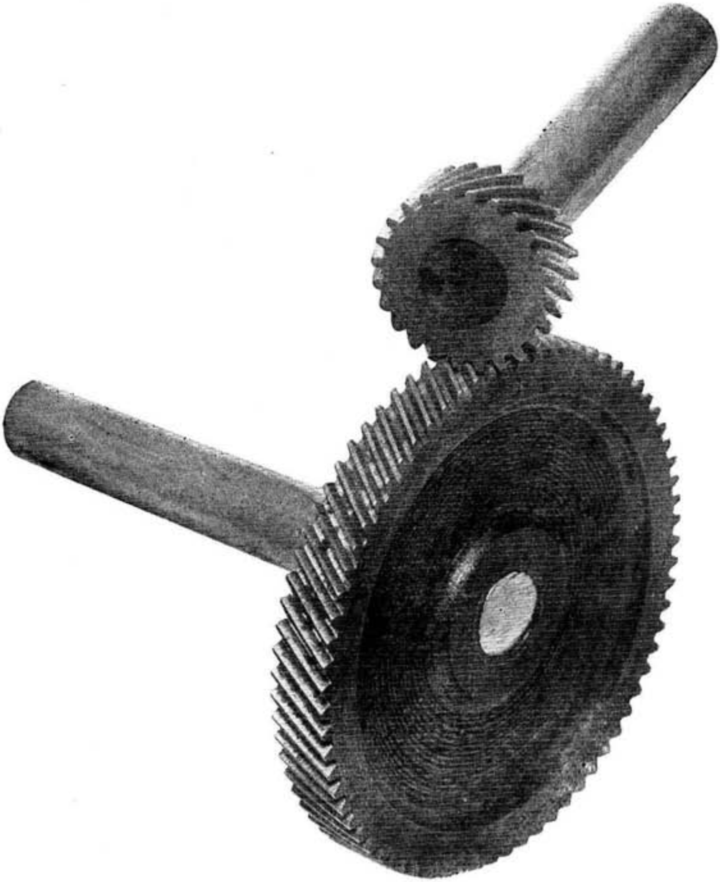


Fig. 84. — Spiral Gears.

tion of its helical teeth the same, the curvature of its pitch circle can be reduced to such an extent that, when the width of the gear is sufficiently increased, its tooth will remain in contact with that of the cylinder during one complete revolution of the latter.

It is thus seen that by increasing the length of one of the spiral gears developed from the rack and spur gear of Fig. 83 and increasing the diameter and the width of the other, keeping the inclination of the helical teeth the same in both cases, a combination has been produced in which, if the first gear be rotated through a complete revolution, the other will be rotated through an arc measured on its pitch circumference equal in length to the oblique projection on a plane at right angles to its axis of the pitch of the helix of the first; and, if the circular pitch of the teeth on the second gear be made equal to this arc, continuous rotation of the first gear will cause continuous rotation of the second. This combination has now become a worm and worm-wheel differing in no essential from the worm and wheel formed by wrapping the rack of Fig. 83 around a cylinder whose axis is parallel to the longest dimension of the rack; and this transformation has been effected without changing the angle between the axes of the gears or those between the teeth and the axes, but only by increasing the length of one gear and the diameter, width, and circular pitch of the other, which changes evidently do not affect the principle upon which the gears operate.

97. Distinction between Worm-Gearing and Spiral Gearing. — If the helical teeth formed by wrapping the rack around a cylinder make so great an angle with its axis that they make one or more complete turns around it in the length of the cylinder, the resulting gear is called a worm and the gear-wheel with which it meshes is called a worm-wheel. By examination of Figs. 81 and 83 it is seen that the helical teeth formed by wrapping the rack around a cylinder whose axis is parallel to the longest dimension of the rack make several turns around that axis in the length of the cylinder, and this is why the resulting gear was called a worm and that with which it meshes, a worm wheel.

If the helical teeth formed by wrapping the rack around a cylinder make so small an angle with its axis that they do not make a complete turn about it in the length of the cylinder, the resulting gear and the one with which it meshes are called spiral gears. By examination of Fig. 83 it is seen that when the rack is wrapped around a cylinder whose axis is perpendicular to the longest dimension of the rack, the helical teeth so formed will make but a small portion of a turn around the axis of the cylinder. For this reason the resulting gear and that with which it meshes were called spiral gears.

98. Velocity Ratio of Worm-Gears.—It has already been shown that by choosing a suitable width for the rack shown in Fig. 81 and then rolling it into a cylinder whose axis is parallel to the longest dimension of the rack, the pitch of the helix on the cylinder may be made equal to the circular pitch of the gear. Similarly, by choosing a suitable width for the rack shown in Fig. 83 and rolling it into a cylinder whose axis is parallel to the longest dimension of the rack, the pitch of the helix on that cylinder may be made of such a length that its oblique projection on a plane at right angles to the axis of the gear, by lines parallel to the teeth of the gear, will be equal to the circular pitch of the gear. Each separate tooth of either rack will then form one complete turn of a helix on the cylinder and, furthermore, the various turns formed by the teeth of the rack will become so many parts of one continuous helical tooth or thread on the cylinder, so that the resulting worm is called a *single-threaded worm.*

One revolution of a single-threaded worm will rotate the worm-wheel through an arc measured on its pitch circumference equal in length to its circular pitch, and to cause the wheel to make a complete revolution the worm must make as many revolutions as there are teeth on the wheel. Since the single thread on the worm is in reality a single tooth, the angular velocities of the worm and worm-wheel are to each other inversely as the numbers of their teeth, as is the case with all toothed gears.

If the width of the rack be increased to twice its former amount, the inclination of the teeth remaining the same as before, every alternate tooth of the rack, when it is rolled into a cylinder whose axis is parallel to that of the first cylinder, will form part of a continuous helical tooth or thread around the cylinder; and there will be two such threads, the worm in this case being called a *double-threaded worm.* Since the pitch of the helix on the worm is double what it was before, either one of the two threads will engage only with every alternate tooth of the gear, the remaining teeth of the gear engaging with the other thread on the worm. One revolution of a double-threaded worm will rotate the worm-wheel through an arc measured on its pitch circumference equal in length to twice its circular pitch, and to cause the wheel to make a complete revolution the worm must make half as many revolutions as there are teeth on the wheel.

If there are three separate threads on the worm it is called a *triple-threaded worm*, and so on. When any multiple-threaded worm meshes with a worm-wheel the angular velocities of the worm and wheel are inversely as the numbers of the teeth or threads on the worm and the worm-wheel. The ratio of the angular velocities of a worm and worm-wheel in mesh is independent of their radii. The teeth on a worm are referred to as threads.

Velocity Ratio of Spiral Gears. — The angular velocities of a pair of spiral gears in mesh are inversely as the numbers of the teeth on the gears, and the ratio of the angular velocities is independent of the radii of the gears except in the one case where the axes of the gears are perpendicular to each other and the helix angle between the teeth of each gear and a line perpendicular to its axis is forty-five degrees. In this case the numbers of the teeth on the gears are directly proportional to, and the angular velocities inversely proportional to, the pitch radii of the gears.

Tooth Curves of Screw Gears. — In view of the development of screw gearing from a rack and spur gear it is evident that the tooth surfaces may belong to either the involute or the cycloidal system. The involute system is generally preferred for worms because the involute thread for racks and worms has straight sides, and is on this account more readily cut in the lathe than the cycloidal thread.

99. Shafts Connected by Screw Gearing. — Drivers. — Pitch of Screw Gearing. — Since the axes of spiral gears and worm and worm-wheels in mesh are in different planes and not parallel to each other, such gearing is used to transmit rotary motion between shafts that are neither parallel nor intersecting. The worm and worm-wheel are used particularly when it is desired to obtain a large velocity ratio between the shafts.

Either one of a pair of spiral gears in mesh may be the driver.

In practice, however, that spiral gear is taken as the driver which has the smaller helix angle between its teeth and a line perpendicular to its axis, since greater efficiency of the gears results from doing so.

In the case of a worm and worm-wheel the worm is the driver. When the angle between the threads of the worm and a line perpendicular to its axis is less than the angle of friction, which is generally the case, the worm-wheel cannot drive the worm. If, however, this angle is greater than the angle of friction the worm-wheel can drive the worm.

The *circular pitch* of a spiral gear is the distance between corresponding points of adjacent teeth measured on the cylindrical pitch surface at right angles to the axis of the gear. The distance measured on the pitch surface at right angles to the teeth is the *normal pitch*, and that measured in a direction parallel to the axis of the gear is the *axial pitch*. For two spiral gears to run together the normal pitches must be the same in both, and, if the axes of the gears are perpendicular to each other, the circular pitch of the one must also be equal to the axial pitch of the other.

The term circular pitch is not used in connection with a worm, but it is in connection with a worm-wheel and has the same significance with regard to the latter as for a spiral gear. The *axial pitch* of a worm is the distance measured on its pitch surface in a direction parallel to its axis between corresponding points of adjacent threads of a multiple-threaded worm or of adjacent parts of the same thread of a single-threaded worm. In the case of a single-threaded worm the axial pitch is the same as the pitch of the helix formed by its thread. The distance between corresponding points of adjacent threads of a multiple-threaded worm or of adjacent parts of the same thread of a single-threaded worm measured on its pitch surface at right angles to the threads is the *normal pitch*. The axial and normal pitches of a worm-wheel are the same as for a spiral gear. For a worm and worm-wheel to run together the normal pitches must be the same in both and, when their axes are perpendicular to each other, as is generally the case, the axial pitch of the worm must be equal to the circular pitch of the worm-wheel.

The *lead* of a worm is the distance any separate thread advances in the direction of the axis of the worm while making one complete turn around it. It is the same as the pitch of the helix formed by the thread.

100. Wheel Train. — Velocity Ratio of First and Last Shafts Connected by a Wheel Train. — A series of gears interposed between two shafts is called a wheel or gear train. In Fig. 85

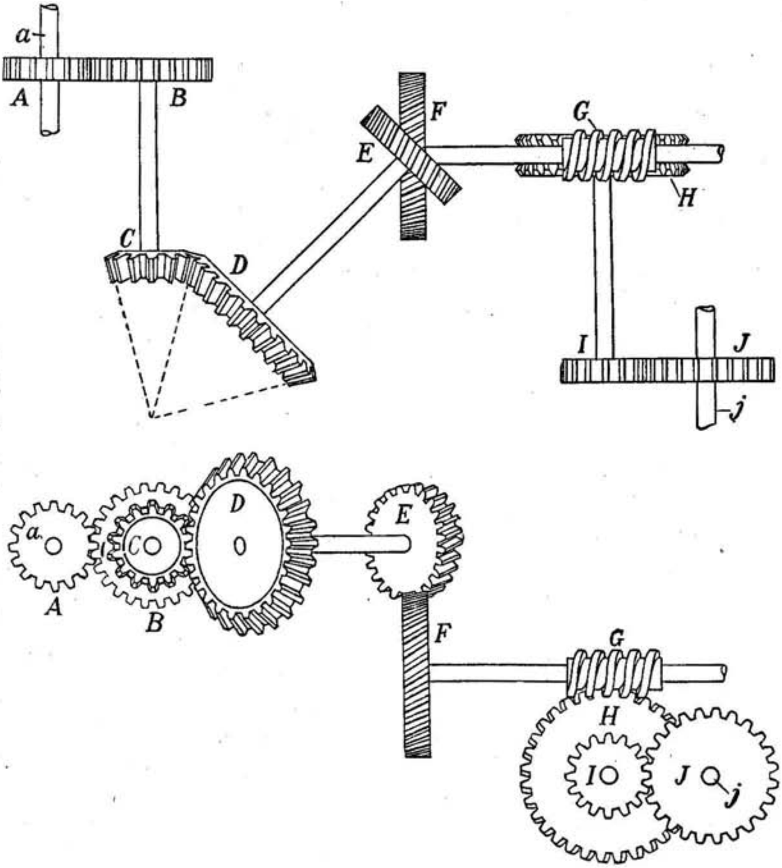


Fig. 85.

is shown a wheel train connecting the driving shaft *a* with the driven shaft *j*.

A spur gear *A* on the shaft *a* meshes with another *B*. On the same shaft with *B* is a bevel gear *C* meshing with another bevel gear *D* carried by a third shaft, on the other end of which is a spiral gear E. The gear E meshes with a second spiral gear F and on the same shaft with F is a single-threaded worm G driving a worm-wheel H. On the other end of the shaft which carries H is a spur gear I that drives the spur gear J on the shaft j.

Let it be required to find the ratio of the angular velocities of the shafts a and j or, what is the same thing, the ratio of the numbers of revolutions per minute of the shafts. Let N_a represent the number of revolutions per minute of the shaft a and of the gear A, N_b the number of revolutions per minute of the gear B, N_c the number of revolutions per minute of the gear C, etc.; and let T_a represent the number of teeth of A, T_b the number of teeth of B, etc. Then

$$\frac{N_a}{N_b} = \frac{T_b}{T_a}; \quad \frac{N_c}{N_d} = \frac{T_d}{T_c}; \quad \frac{N_e}{N_f} = \frac{T_f}{T_e}; \quad \frac{N_g}{N_h} = \frac{T_h}{T_g}; \quad \frac{N_i}{N_j} = \frac{T_j}{T_i}$$

Multiplying together the first terms of the various equalities and placing the product equal to the product of all the last terms of the equalities, we have

$$\frac{N_a \times N_c \times N_e \times N_g \times N_i}{N_b \times N_d \times N_f \times N_h \times N_j} = \frac{T_b \times T_d \times T_f \times T_h \times T_j}{T_a \times T_c \times T_e \times T_g \times T_i} \qquad (1)$$

But since B and C are on the same shaft $N_b = N_c$, and, similarly, $N_d = N_e$, $N_f = N_g$, and $N_h = N_i$. Making these substitutions in equation (1) we obtain

$$\frac{N_a}{N_j} = \frac{T_b \times T_d \times T_f \times T_h \times T_j}{T_a \times T_c \times T_e \times T_g \times T_i} \qquad (2)$$

Referring to Fig. 85 it will be seen that the gears B, D, F, H, and J are all driven gears or followers and the gears A, C, E, G, and I are all drivers, and we may, therefore, write that *when two shafts are connected by a wheel train the ratio of the angular velocities of the driving and driven shafts or of the numbers of revolutions per minute of these shafts is equal to the product of the numbers of the teeth of all the followers divided by the product of the numbers of the teeth of all the drivers.*

101. Idlers. — If a wheel train connecting two shafts consists of three or more gears meshing directly the one into the other as shown in Fig. 86, the velocity ratio of the driving and driven shafts is the same as if the gears on these shafts meshed directly into each other without the interposition of the intermediate gears, for the gear *B* is a follower with respect to *A* and a driver with respect to *C* and the number of its teeth will appear both in the numerator and the denominator of the fraction representing

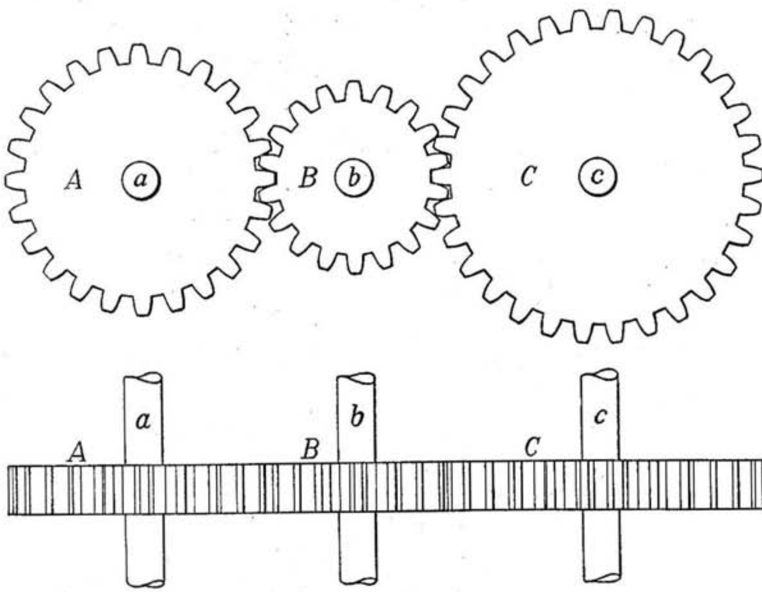


Fig. 86.

the ratio of the angular velocities of the shafts *a* and *c*, and will, consequently, cancel out of the fraction. If another gear be interposed between *B* and *C* it also will act both as a follower and a driver and can, consequently, have no effect on the velocity ratio of *a* and *c*.

By examination of Fig. 86 it will be seen that gears *A* and *B* turn in opposite directions as do gears *B* and *C* and, consequently, *A* and *C* and the shafts to which they are fastened turn in the same direction. If another gear be interposed between *A* and *C* the latter will turn in the opposite direction from *A*; and, in general, if the number of gears meshing directly the one into the other as shown in Fig. 86 is odd, the first and last gears will turn in the same direction, but if the number is even the first and last gears will turn in opposite directions. All the gears in such a train except the first and last are called *idlers*, and their function is principally to change the direction of rotation of the driven shaft since they cannot change its angular velocity. In an exceptional case when shafts are so far apart that two gears connecting them would be inconveniently large, idlers might be used to bridge over the distance between them.

102. Relation between the Power and the Resistance in a Wheel Train. — Referring to Fig. 85, the shaft a is driven by some source of energy which applies a force P called the power at the end of a lever arm p with respect to the axis of the shaft. The source of energy may be human as when the shaft a is turned by the hand at the extremity of the lever p, or the shaft may be driven by still another gear-wheel not shown in the figure, or by a belt. If driven by another gear-wheel the power will be the force exerted between the teeth of the wheels and its lever arm may be taken as the pitch radius of the driven wheel on the shaft a. If the shaft is driven by a belt the power will be the tension in the belt, or, more strictly, the difference in tension between the upper and lower parts of the belt considered as horizontal, and its lever arm will be the radius of the pulley increased by one-half the thickness of the belt. The energy received from the source is transmitted through the wheel train and, neglecting friction, is all delivered to the shaft j where it is made to perform useful work in overcoming a resistance R having a lever arm r with respect to the axis of the shaft j.

Let it be required to find the power P that must be applied with a lever arm p to the shaft a to overcome through the wheel train shown in Fig. 85 a resistance R having a lever arm r with respect to the shaft j, (a) when friction is neglected, and (b) when the friction of the gearing is considered.

Friction Neglected. — Neglecting friction is equivalent to assuming that all the energy supplied to the shaft a is transmitted to the shaft j and, therefore, the work of the power P in one minute must be equal to the work of the resistance R in one minute. Since the number of revolutions per minute of a is N_a the path of the power in one minute is $N_a \times 2\,\pi p$ and the work performed by it in one minute is $P \times N_a \times 2\,\pi p$; and, similarly, the work performed by the resistance in one minute is $R \times N_j \times 2\pi r$, whence

$$P \times N_a \times 2\pi p = R \times N_j \times 2\pi r$$

or

$$P = R \times \frac{r}{p} \times \frac{N_j}{N_a} \tag{3}$$

and replacing the ratio $\frac{N_j}{N_a}$ in equation (3) by its value from equation (2)

$$P = R \times \frac{r}{p} \times \frac{T_a \times T_c \times T_e \times T_g \times T_i}{T_b \times T_d \times T_f \times T_h \times T_j} \tag{4}$$

and we may write that *the power required to overcome a resistance through any given wheel train, when friction is neglected, is equal to the resistance multiplied by the ratio of the lever arms of the resistance and the power with respect to the axes of the shafts of the last and first wheels, respectively, of the train, multiplied by the product of the numbers of the teeth of all the drivers in the train divided by the product of the numbers of the teeth of all the followers.*

Friction of Gearing Considered. — When energy is transmitted from one shaft to another through a wheel train, part of it is lost in performing wasteful work due to the friction between the teeth of the gears and that between the shafts and their bearings. To determine the latter it is necessary to know the normal pressures between all the shafts and their bearings; the radii and angular velocities of all the shafts; and the coefficients of friction, which vary with the normal pressures, the speeds of the rubbing surfaces, the materials in contact, the nature of the lubricant, if any, and the temperature. While the energy lost by friction between the shafts and their bearings can be taken into account with reasonable accuracy in any particular case, it will not be considered in this discussion as it is ordinarily considerably less than that lost through the friction of the gearing.

By analytical methods and by experiment the percentage of energy transmitted to energy received has been determined for a pair of each of the different classes of gears. These percentages are called efficiencies and they are always fractions less than unity. Represent the efficiencies of

spur gears by E_s,
bevel gears by E_b,
spiral gears by E_{sp}, and
worm-gears by E_w.

Now since the velocity ratio of the shafts *a* and *j* is constant and not affected by friction, the loss of energy must appear entirely in the reduction of the value of the resistance *R* and, therefore, the value of *P* that will produce a given value of *R* must be the theoretical value given by equation (4) divided by the product of the efficiencies of all the pairs of gears in the wheel train, whence

$$P = R \times \frac{r}{p} \times \frac{T_a \times T_c \times T_e \times T_g \times T_i}{T_b \times T_d \times T_f \times T_h \times T_j} \times \frac{1}{E_s \times E_b \times E_{sp} \times E_w \times E_s} \quad (5)$$

It should be noted that this equation is true only when *R* is the resistance and *P* the driving force. If *P* becomes the resistance and *R* the driving force, the term involving the efficiencies must be inverted since in this case it is *P* instead of *R* that must be multiplied by this term.

103. Efficiencies of Various Classes of Gears. — The efficiency of a pair of spur gears varies from about .90 at very slow speeds to about .985 when the linear speed of a point on the pitch circumference is equal to or greater than 200 feet per minute.

When the linear speed of a toothed gear is referred to it is to be understood as meaning the speed of a point on its pitch circumference.

The efficiency of a pair of spiral or worm-gears varies with the helix angle which the thread makes with a line perpendicular to the axis of the gear having the smaller helix angle, and also with the linear speed of a point on the pitch circumference. At very slow speeds the efficiency varies from about .30 for a helix angle of five degrees to about .75 for a helix angle of forty-five degrees. At speeds equal to or greater than 200 feet per minute the efficiency varies from about .75 for a helix angle of five degrees to about .90 for a helix angle of forty-five degrees. The low efficiency of spiral and worm-gearing is largely due to the thrusts of the helical threads in directions parallel to the axes of the shafts, which develop great pressures against the end bearings that are required to prevent longitudinal movement of the shafts. It is evident from this why the efficiency increases with the helix angle of the threads. By interposing ball or roller bearings between the shafts and their end bearings the efficiency of spiral and worm-gearing is much increased.

The efficiency of a pair of bevel gears varies from about .85 at very slow speeds to about .95 at speeds equal to or greater than 200 feet per minute.

By reference to equation (5) it is seen how great is the increase required in the value of the power over its theoretical value to produce a given value of the resistance when the number of gears in the train is large or when one or more of the pairs consist of screw gears. Screw gears are valuable, however, when it is desired to transmit motion between nonparallel shafts in different planes, and worm-gears are particularly useful when a large velocity ratio between shafts is required.

104. Example. — The elevating mechanism of the 5-inch barbette carriage, model of 1903, is shown diagrammatically in Fig. 87.

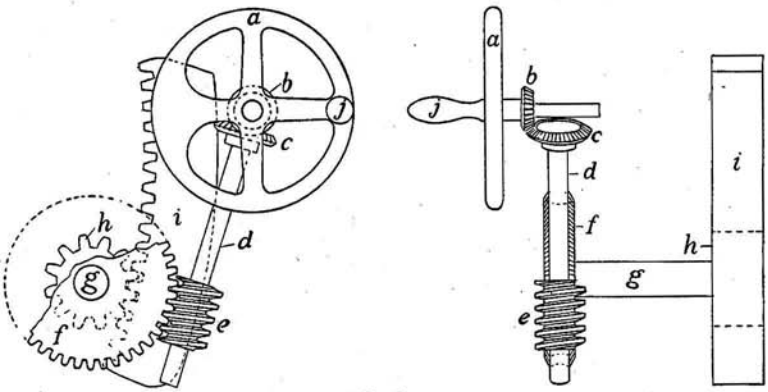


Fig. 87.

A hand-wheel *a* with crank-handle *j* is mounted on a short horizontal shaft carried in bearings in the elevating bracket. The elevating bracket is bolted to the platform bracket and the latter is bolted to the pivot yoke. On the shaft to which the hand-wheel *a* is attached is a bevel gear *b* engaging with another *c* on an inclined shaft *d* also carried in bearings in the elevating bracket. On the other end of this shaft is a worm *e* engaging with a worm-wheel *f*, on the same short horizontal shaft *g* with which is the spur gear or pinion *h* that meshes with the circular rack *i*. The shaft *g* is carried in bearings in the elevating bracket and the rack *i* is attached to the cradle carrying the gun. The circular rack i forms a part of a spur wheel with internal teeth whose axis is the axis of the trunnions of the cradle. The trunnion may, therefore, be considered as the shaft to which i is attached. The following data are known from the construction of the carriage, viz.:

Lever arm of crank-handle with respect to axis of first shaft [= 5 ins.
Number of teeth in bevel gear b = 18.
Number of teeth in bevel gear c = 30.
Worm is single-threaded (number of teeth = 1).
Number of teeth in worm-wheel = 40.
Number of teeth in spur pinion = 12.
Number of teeth in spur gear of which rack is a part = 175.
Radius of trunnions = 2.505 ins.
Coefficient of friction at trunnions = .1.
Weight of parts resting on trunnions (gun and cradle) = 14900 lbs.
Efficiency of a pair of bevel gears = .85.
Efficiency of a pair of worm-gears = .35.
Efficiency of pinion and rack (spur gears) = .90.

Neglecting the friction between the shafts and their bearings, let it be required to determine the force P that must be exerted on the crank-handle j to start the gun in elevation, assuming that the center of mass of the parts resting on the trunnions is in the axis of the trunnions so that the weight has no moment with respect to that axis.

Solution. — The resistance to be overcome is the force of friction at the cylindrical surfaces of the trunnions, which is equal to 14900 × .1 = 1490 lbs. This resistance has a lever arm of 2.505 ins. with respect to the axis of the trunnions, and the lever arm of the power with respect to the axis of the first shaft is 5 ins. Therefore, noting from Fig. 87 that the gears b, e, and h are drivers and the gears c, f, and i are followers, we have, in accordance with the principles discussed in deducing equation (5),

$$P = 1490 \times \frac{2.505}{5} \times \frac{18 \times 1 \times 12}{30 \times 40 \times 175} \times \frac{1}{.85 \times .35 \times .90} = 2.87 \text{ lbs.} \quad (6)$$

2.87 lbs. is, therefore, the force on the crank-handle required to start the gun in elevation or, assuming that the coefficient of friction at the trunnions is constant, it is the force required to *keep* the gun moving in elevation with a uniform angular velocity.

Force on the Crank-Handle Required to Produce in a given Time a given Angular Velocity of the Rotating Parts. — To attain a given angular velocity starting from rest it is necessary, however, to give the rotating parts an angular acceleration until the given angular velocity is reached. To determine the increase in the force required on the crank-handle to give the gun a desired angular velocity in elevation in a given time is not a difficult problem providing we know the moment of inertia of the rotating parts about the axis of the trunnions. Having decided on the angular velocity desired and the time in which it is to be attained, the angular acceleration is found by dividing the angular velocity by the time. To produce an angular acceleration $d^2\phi/dt^2$ about a fixed axis requires a moment in foot pounds about that axis equal to $\Sigma mr^2 \times d^2\phi/dt^2$, in which Σmr^2 is the moment of inertia in mass units and feet of the rotating parts about the axis. While the force required to produce this angular acceleration is actually applied at the pitch circumference of the rack fastened to the cradle, it is immaterial where it be taken as applied provided its moment with respect to the axis of the trunnions is $\Sigma mr^2 \times d^2\phi/dt^2$. For convenience assume the force to be applied at the surface of the trunnions as is the friction due to the weight of the parts. Then, the radius of the trunnions being 2.505 ins., the force is

$$\frac{\Sigma mr^2 \times d^2\phi/dt^2}{2.505/12} \text{ lbs.}$$

If now the value of the resistance, 1490 lbs., in equation (6) be increased by the value of the force just determined, the resulting value of P will be the force on the crank-handle required to overcome the resistance of the friction at the trunnions and to give the gun and other rotating parts the desired angular velocity in elevation in a given time. Computations such as these are important in designing the gearing of a gun carriage, for it is necessary that the gun can be elevated or traversed through a given number of degrees in a reasonably short time without too much effort being required at the crank-handles.

105. Pressure between the Teeth of any Pair of Gears in a Wheel Train. — Although it has been shown that the action line of the pressure between the teeth when the involute system is used makes an angle of 15°, and when the cycloidal system is used an angle varying from 30° to 0°, with the common tangent to the pitch circles, no serious error will be committed by assuming it to be coincident with this tangent. Referring to Fig. 85, the pressure between the teeth of any two gears as C and D can be determined if either the power P or the resistance R and the corresponding lever arm are known. If the resistance R and its lever arm are known the pressure between the teeth of C and D may be regarded as a power with respect to the rest of the train and determined as was the power P except that the part of the train between A and D must not be considered. The gears C and D being bevel gears, their pitch surfaces are conical and the lever arm of the pressure between the teeth with respect to the axis of either should be taken equal to the pitch radius at the middle of the length of the teeth. (In practice this length is called the face of the teeth.) If the power P and its lever arm are known the pressure between the teeth of C and D can be regarded as a resistance and its value determined by considering only that portion of the wheel train between A and D.

Frequently only the horse-power delivered to the first or last shaft of a wheel train and the speed of the shaft are known. Horse-power is a measure of the rate of doing work and one horse-power is equal to 33000 ft. lbs. per min. Let the number of horse-power delivered to the shaft a, Fig. 85, be represented by H and the number of revolutions per minute of the shaft by N_a; then if the point of application of the force required to produce this number of horse-power be at a distance of p inches from the axis of the shaft, the path it will follow in one minute is $2\pi p N_a/12$ ft., and the intensity of the force required to produce H horse-power when acting with a lever arm of p inches is

$$P = \frac{H \times 33000 \times 12}{2\pi p N_a}$$

The moment of this force (the power) with respect to the axis of the shaft is, in in. lbs.,

$$Pp = \frac{H \times 33000 \times 12}{2\pi N_a}. \qquad (7)$$

Similarly, if the horse-power delivered to the shaft j be represented by H and the number of revolutions per minute of the shaft by N_j, the moment of the resistance in in. lbs. is

$$Rr = \frac{H \times 33000 \times 12}{2\,\pi N_j} \tag{8}$$

Having obtained either the moment of the power from equation (7) or the moment of the resistance from equation (8), the moment of the other can be obtained from equation (5), or more readily from equation (9) below, which is equation (5) put in a more convenient form for this purpose.

$$Pp = Rr \times \frac{T_a T_c T_e T_g T_i}{T_b T_d T_f T_h T_j} \times \frac{1}{E_s E_b E_{sp} E_w E_s} \tag{9}$$

If the moment of the power is known the moment of the pressure between the teeth of any pair of gears in the train can be obtained by regarding it as the moment of the resistance and considering only the part of the train between the point of application of the power and that of the pressure between the teeth. If the moment of the resistance is known the moment of the pressure between the teeth of any pair of gears in the train can be obtained by regarding it as the moment of the power and considering only the part of the train between the point of application of the resistance and that of the pressure between the teeth. The pressure between the teeth of the gears can then be obtained by dividing the moment of the pressure by the pitch radius of the driving gear in case the pressure is regarded as a resistance, and by the pitch radius of the driven gear in case the pressure is regarded as a power.

106. Stresses in the Shafts. — The pressure between the teeth of a pair of gear-wheels causes a torsional moment on each shaft equal to the pressure multiplied by the pitch radius of the gear on that shaft.* This pressure also causes a bending moment and a shearing stress in each shaft. The bending moment on each shaft will depend on the pressure and the distance between the supports (called the bearings) of the shaft. When the torsional and bending moments and the diameters of the shafts are known,

* Assuming that the action line of the pressure between the teeth is tangent to the pitch circles of both gears.

the stresses can be computed as explained in Chapter IV. The torsional and bending stresses should be combined by equations (24) and (25), Chapter IV.

107. Stresses in the Gear-Teeth. — The teeth of gears may be considered as cantilevers subjected to a bending force equal to the pressure between the teeth. In practice the height of a tooth above the pitch circumference is made equal to the circular pitch divided by π. The depth below the pitch circumference is equal to the height above it increased by a clearance equal to .05 times the circular pitch, the clearance being allowed to permit the rounding of the corners at the bottoms of the tooth spaces and to prevent the bottoming of the teeth in the tooth spaces. The thickness of the tooth at the pitch circumference is equal to one-half the circular pitch. The thickness at the root varies with the circular pitch and also with the size of the gear. Special formulas have been devised for determining the stresses in gear-teeth based upon their variation of form and, consequently, of the variation in position of the weakest section, which generally occurs a little above the root of the tooth. These formulas are given in mechanical engineering hand-books and in works on machine design. One of the best is Lewis's formula which is as follows:

$$S = \frac{W}{pf\left(.124 - \frac{.684}{n}\right)} \tag{10}$$

in which S is the maximum bending stress; W is the pressure between the teeth in pounds; p is the circular pitch in inches; f is the face of the teeth in inches, that is, the length of the teeth measured in a direction parallel to the axis of spur gears, and along the element of contact in bevel gears; and n is the number of teeth in the gear. This formula will answer for spur gears, spiral gears, and worm-wheels. The tooth of the worm is always stronger than that of the worm-wheel with which it meshes.

For bevel gears Lewis's formula is as follows:

$$S = \frac{W}{pf\left(.124 - \frac{.684}{n}\right)} \times \frac{D}{d} \tag{11}$$

in which S, W, f, and n have the same meanings as before, p is the circular pitch in inches at the large end of the bevel-gear tooth, D is the pitch diameter at the large end of the tooth, and d is the pitch diameter at the small end. To make allowance for shocks when the speed of the gearing is increased the factors of safety should increase with the speed.

In the absence of a special formula the maximum stress in gear-teeth may be approximately obtained with the error on the safe side by considering the whole pressure uniformly distributed along the top edge of the tooth and at right angles to the plane passing through the center of the tooth and the axis of the wheel, by taking the length of the cantilever as the total height of the tooth including the clearance, and by considering the dangerous section as located at the root and as having a thickness equal to that of the teeth at the pitch circumference, that is, equal to one-half the circular pitch. The height of bevel-gear teeth should be taken equal to half the sum of the heights at the large and small ends, and the thickness as half the sum of the thicknesses at the large and small ends.

108. Stresses in the Arms of Gear-Wheels. — The teeth of large gears, or gear-wheels, are formed on a rim connected to a nave or hub by several arms. The stresses in the arms may be determined by considering them as cantilevers subjected to a bending moment equal to the pressure between the teeth multiplied by the length of the arm divided by the number of arms. In order to err somewhat on the safe side the length of the arm is taken equal to the pitch radius of the gear.

Proportions for the Rims of Gear-Wheels. — Arbitrary proportions for the rims of gear-wheels based on experience have been adopted. These proportions are given in mechanical engineering hand-books and standard works on machine design. A standard thickness for rims of gear-wheels whose circular pitch is greater than one and one-half inches is one-half the circular pitch. For gear-wheels having pitches less than one and one-half inches the thickness of the rim is taken as

$$.4\,p + 1/8 \text{ in.} \tag{12}$$

in which p is the circular pitch in inches. A rib is added to the under side of the rim. The width of the rib is the same as that of the arms of the gear-wheel and its height is equal to the thickness of the rim.

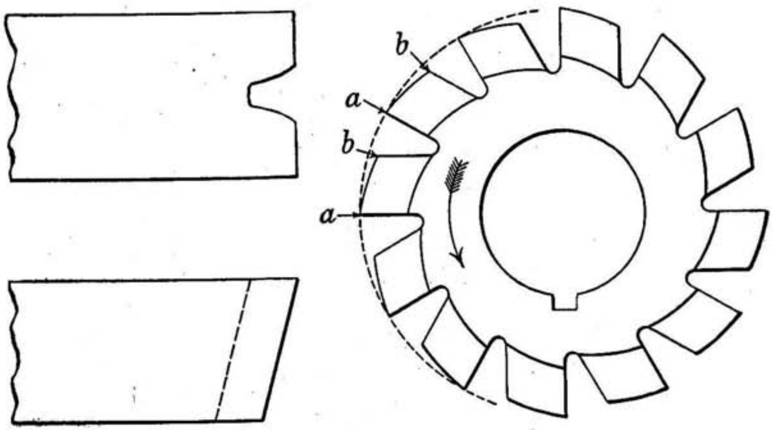
Proportions for the Hubs of Gear-Wheels. — Arbitrary proportions for the hubs of gear-wheels adopted as a result of experience are also given in mechanical engineering hand-books and standard works on machine design. If the gear-wheel has to transmit the full power of the shaft the thickness of the hub is made equal to the radius of the shaft. If the gear-wheel does not have to transmit the full power of the shaft the thickness of the hub is given by the following formula:

$$t = \sqrt[3]{fpR}/3 \qquad (13)$$

in which t is the thickness of the hub in inches, f is the face of the teeth in inches, p is the circular pitch in inches, and R is the pitch radius in inches. The length of the hub varies from f to $1.4f$.

109. Gear Cutting. — The teeth of spur gears, spiral gears, and worm-wheels are cut in a universal milling machine such as that which the cadets have operated in their practical work. When a large amount of gear cutting has to be done special milling machines are used which are peculiarly adapted to the work and are not capable of performing the miscellaneous character of work done by the regular milling machine. The particular feature of the special machines which renders them most useful is that they are automatic in their action. After being started they continue to work without further attention until all the teeth are cut. After one tooth space has been cut the gear-wheel is moved past the cutter and rotated through the proper angle so as to be ready for the next cut, all by the automatic operation of the machinery. As the principles of the automatic machines are the same as those of the milling machine with which the cadets are familiar, the latter will be referred to in the descriptions that follow.

Rotary Cutters. — The profiles of the teeth of the milling cutters used are exact duplicates of those of the tooth spaces they are required to produce. The manufacture of milling cutters is a specialty not much practised by the ordinary machine shop for, by reason of the special appliances used and the quantity made at one time, the manufacturers of the cutters can furnish a better article for less money than it would cost to make it in an ordinary shop. A brief description of the general method of making cutters for gear-teeth is as follows: A piece of tool steel is turned in a lathe to the proper diameter and width and then notches for the teeth are cut in the milling machine. The piece is then taken back to the lathe and the outline of the teeth cut by a lathe tool, shown in Fig. 88, the cutting edges of which have the outline of a tooth space and have been shaped by filing to a template prepared from a drawing of the tooth curves. As the top surfaces of the teeth of a milling cutter are not concentric with its axis but are of a spiral character to give proper clearance and keenness to the cutting edges, as shown by the lines *ab* in Fig. 88 in which the points *a* are at the cutting edges, the lathe



Lathe Tool. Rotary Cutter.

Fig. 88.

is provided with an attachment called a backing off or relieving attachment which forces the lathe tool gradually in as the work is revolving past it from *a* to *b* and then draws it out quickly to be in position to cut the edge *a* at the proper height on the next tooth. After the outlines of the teeth have been cut in the lathe the cutter is hardened and tempered.

110. Cutting the Teeth of Spur Gears. — The wheel or gear before the teeth are cut is called a gear-blank. This is turned to the required shape and size in the lathe. It is then mounted on centers between the index head and tail stock of the milling machine. The rotary cutter is placed on the arbor with its center in the vertical plane containing the axis of the gear-blank, and the table raised to give the proper depth of cut. The machine is then started, the feed mechanism thrown in, and the table fed past the cutter. It is then drawn back by hand, the blank turned through the proper angle by the index mechanism, and another cut taken and so on.

Cutting the Teeth of Spiral Gears. — The method of cutting the teeth of spiral gears is essentially the same in principle as for spur gears, the only difference being that the teeth are helical. The table of the milling machine is, therefore, set at the proper angle with the axis of the spindle, and the index head is connected by the requisite gearing to the feed screw of the table in order to give the gear-blank such a combined motion of translation and rotation as will secure the proper helix angle, these operations being similar to those performed by the cadets in cutting the spiral teeth of a reamer.

111. Cutting the Teeth of a Worm and Worm-Wheel. — The worm is turned and threaded in the lathe, the thread being cut in practically the same way as the thread of an ordinary bolt except that a roughing tool is first used, being followed by the finishing tool. The roughing tool has a square nose slightly narrower than the tooth space at the bottom of the thread. When the diameter at the bottom of the thread has been reduced to the prescribed size with the roughing tool, the finishing tool is put in the tool post and the thread finished with it. The finishing tool is very carefully made and filed to a template whose profile is the same as that of the tooth space of the worm.

In order that the teeth of a worm and worm-wheel whose axes are perpendicular to each other shall be in contact over a considerable arc measured on the pitch surface of the worm in a direction parallel to its thread, the face of the wheel is made concave to fit the curvature of the worm and the teeth are cut in a milling machine using a rotary cutter called a hob. The worm-wheel blank is turned in a lathe. If the number of worm-wheels under manufacture justifies the expense, particularly if the wheels are large, an attachment is made or purchased for turning the concave face of the wheel. This attachment is placed on the tool carriage of the lathe and permits the cutting tool to be fed around a fixed center on the carriage, thus turning the concave face to the required radius. The longitudinal feed of the lathe is not used with this attachment. The cross feed is used only to obtain the proper depth of cut. If only a few worm-wheels are to be made, particularly if the wheels are small, an attachment such as described would not be used. Instead the concave face of the wheel is first roughed out with a round-nose tool to approximately the correct outline as shown by a template, using the power longitudinal feed and operating the cross feed by hand. The cut is started at the right edge of the face of the wheel and the tool is fed to the left by power while being fed in by hand in such manner as to make it follow approximately the arc of a circle. When the tool reaches the center of the face it is withdrawn from the work and a new cut is started at the left edge of the face, the tool now being fed to the right by power while being fed in by hand as before. When the second cut reaches the first at the center of the face the tool is again withdrawn from the work, and the operations just described are repeated as often as may be necessary until the face of the wheel is rough turned to approximately the correct size and shape. The face is then finished to the template using a broad-nose finishing tool whose cutting edge is rounded to the same radius as the template. This tool is ordinarily narrower than the face of the wheel and is only used to dress off the high spots indicated by the template, being shifted from one spot to another of the face of the wheel for this purpose. When the finishing tool is used all feeding is done by hand.

The hob, see Fig. 89, is made of tool steel and is almost the exact counterpart of the worm, the only differences being that its diameter is slightly greater to provide for the clearances at the bottoms of the tooth spaces of the wheel, that it is fluted in a direction parallel to its axis so as to form cutting teeth, and that the top surfaces of the teeth like those of the milling cutter referred to in article 109 are of a spiral form to give proper clearance and keenness to the cutting edges. It is turned and threaded in the lathe and fluted in the milling machine. The top surfaces of the teeth are then filed to give them the proper shape and the hob is finally hardened and tempered.

To cut the teeth of the worm-wheel it is mounted on an arbor between the index centers of the milling machine and left free to rotate thereon. It is then brought directly under the hob which is fastened to an arbor in the spindle of the machine, the axes of the hob and worm-wheel being perpendicular to each other. The machine is started and the table raised until the hob teeth sink a certain distance into the face of the wheel. The rotation of the hob while cutting will, because of its cutting threads, cause the wheel to rotate in the same way as if it were in mesh with its worm. After the wheel has made one complete revolution the hob is sunk deeper into it and another cut taken on the teeth all around the circumference, these operations being repeated until the tooth spaces in the wheel are cut to the proper depth. Very frequently, and especially if the wheel is large, it is "gashed" by an ordinary milling cutter before being hobbed as when this is done there is no danger of the wheel not rotating properly with the hob, which sometimes occurs when the wheel is not gashed. A gash is cut over every tooth space, the table of the machine being turned so that the gashes are parallel to the helices of the finished teeth at the central plane of the wheel. The wheel does not rotate when the gashes are being made but it is turned through the proper angle between gashes by the index mechanism.

In some machines the worm-wheel while being hobbed is not left free to rotate on the centers but is required by gearing to rotate at exactly the correct speed with reference to that of the hob. When worm-wheels are hobbed in these machines the preliminary gashing is not needed.

Fig. 89 shows the operation of hobbing a worm-wheel.

When the axes of a worm and worm-wheel are not to be perpendicular to each other, the wheel is not hobbed but instead its teeth are cut with an ordinary rotary cutter in the same way as the teeth of a spiral gear.

112. Cutting the Teeth of Bevel Gears. — Because of the conical pitch surfaces of bevel gears a section of the tooth at one end is larger than at the other, and the tooth outline varies in size from the large end to the small end; consequently perfectly formed bevel-gear teeth cannot be cut by a rotary cutter. Nevertheless such teeth were in the past cut in the milling machine

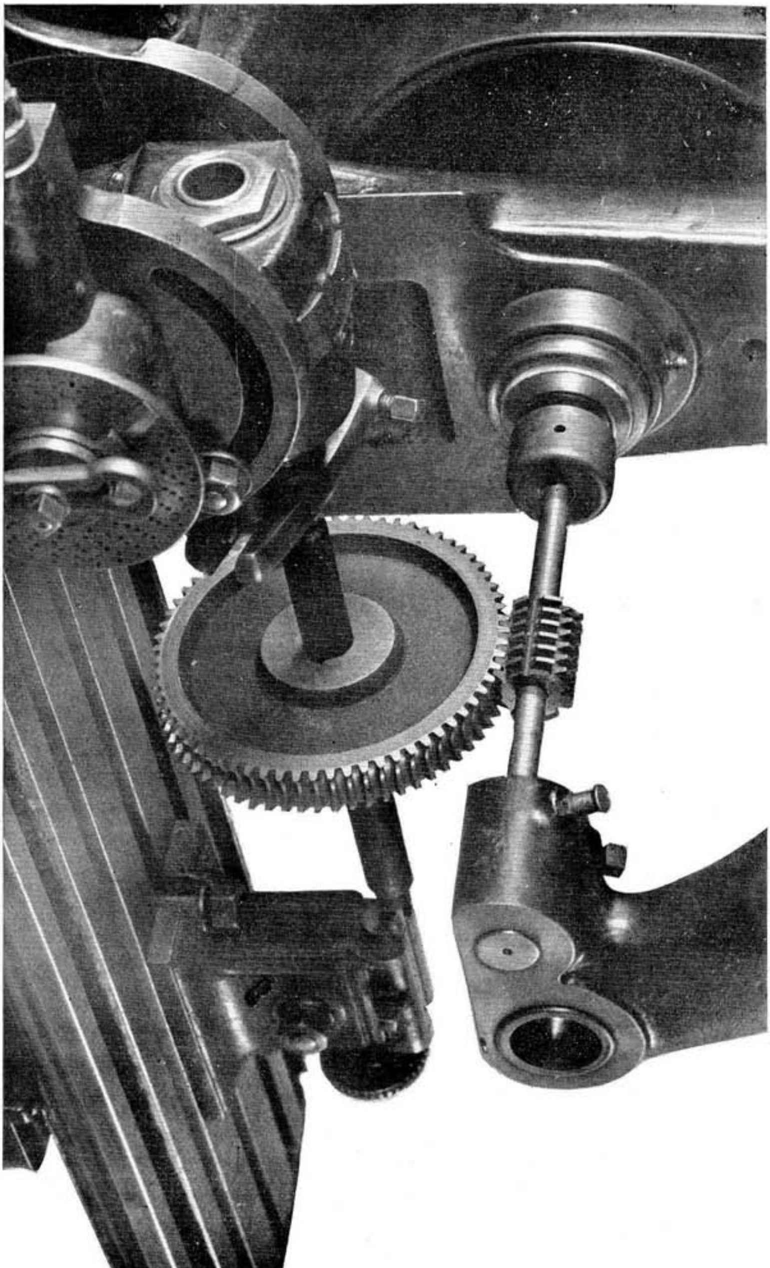


Fig. 89. — Hobbing a Worm-Wheel.

with a rotary cutter and this practice is followed to a considerable extent at the present time.

After the gear-blank has been turned in the lathe, if the teeth are to be cut in a milling machine it is mounted on an arbor in the spindle of the index head, which is rotated around a horizontal axis until the middle element of the surface that will form the bottom of the tooth space is in a horizontal plane. A rotary cutter whose width is that of the tooth space at its small end and the profile of whose cutting edges is the same as that of the tooth space at its large end is so mounted on an arbor in the spindle of the machine that its center and the axis of the gear-blank are

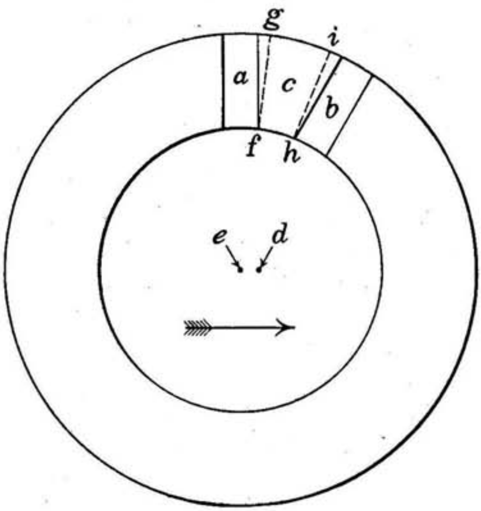
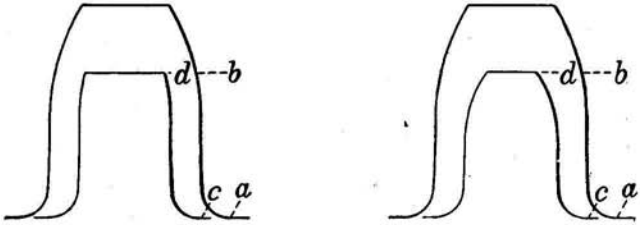


Fig. 90.

contained in the same vertical plane, and two spaces, *a* and *b*, are cut as shown in Fig. 90, the index mechanism being used to obtain the proper spacing.

This will leave the tooth *c* too wide at the large end. The table of the milling machine is then moved by the cross feed in the direction of the arrow a certain distance *ed*, which moves the center of the gear-blank to the right of the center of the cutter. The gear-blank is then revolved until the cutter will just enter the cut *a* at the small end of what will be the finished tooth space. The center of the blank being now to the right of the center of the cutter, the path of the latter when the blank is fed up to it will not be parallel to the original cut *a*, but will make an angle with it; and the cutter will cut the left side of the tooth *c* as shown by the dotted line *fg*, the amount of metal removed varying from practically nothing at the small end of the tooth to an amount at the large end dependent on the distance *ed* which the center of the gear-blank has been moved to the right of the center of the cutter. The center of the blank is next moved as far to the left of the center of the cutter as it was to the right and, after the blank is revolved until the cutter will enter *b* at the small end of which will be the finished tooth space, a cut is taken on the right side of *c* as shown by the dotted line *hi*. The thickness of *c* at the pitch circumference at the large end is then measured; if too thick the operations just described are repeated, the center of the blank being set a little further to the right and to the left of the center of the cutter than it was before. When the correct thickness of the tooth at the pitch circumference at the large end is obtained, cuts are taken all around the blank with its center set at the proper distance to the right of the center of the cutter, and then with the center of the blank set at the same distance to the left of the center of the cutter. The index mechanism is used to rotate the blank through an angle corresponding to the circular pitch.



Before filing. After filing.

Fig. 91.

This method will produce a proper outline at the large end of the tooth but toward the small end it will not be curved enough, as will be seen from Fig. 91, which shows the large end of a bevel-gear tooth placed over the small end of the same tooth, it being understood that the part of the cutter that forms the portion *ab* of the large end must also form the whole of the side *cd* of the small end of the tooth. The parts of the teeth near their small ends must, therefore, be rounded with a file.

113. Planing Bevel-Gear Teeth. — The proper way to cut bevel-gear teeth is to plane them, using for this purpose a specially designed shaper. The teeth of all bevel gears for gun carriages are required to be planed.

Since the tooth surfaces of bevel gears are generated by cones rolling on conical pitch surfaces whose apexes are at the point of intersection of the shafts, or by planes rolling on base cones whose apexes are at the same point, it is evident that the elements

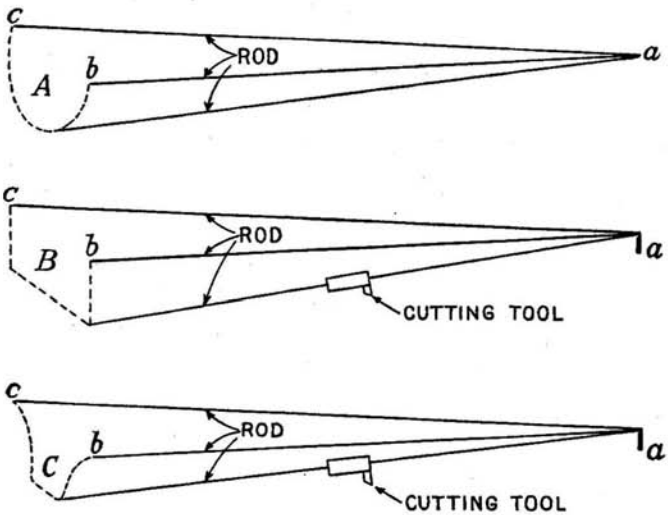


Fig. 92.

of the tooth surfaces are straight lines *converging to the point of intersection of the shafts*. If, therefore, the point of a planing tool can be made to follow these elements, the surfaces cut by it will be correct tooth surfaces.

If one end of a rod be pivoted at *a*, Fig. 92, by a universal joint and the other end be moved around the inside of a semi-circle *A*, it is apparent that it will by its motion generate the half surface of a cone, and that any section of this half cone parallel to the plane of the semi-circle *A* will also be a semi-circle whose size will vary with the distance of the section from *a*. If the end of the rod instead of being moved continuously around the semi-circle be moved around it step by step, the successive positions of the rod being quite close to each other, a slide carrying a cutting tool can be placed on the rod and made to move forward toward *a* and back again at every successive position of the rod, which in this case would be bent downward for a short distance at its end before being pivoted at *a* in order to enable the point of the cutting tool to travel directly toward *a* and back. If a metal bar not too long is placed between *A* and *a* and the first position of the free end of the rod carrying the slide and cutting tool is at *b*, it is apparent that, during the successive movements of the free end of the rod around the semi-circle from *b* to *c*, the cutting tool on the slide, by moving forward and backward each time the rod takes a new position, will cut in the bar a hollow half cone whose apex is at *a* and whose sections parallel to the plane of *A* will all be semi-circles.* If instead of moving the free end of the rod around the inside of the semi-circle it be moved around the inside of the rectangle *B* from *b* to *c*, the tool can be made to cut a pyramidal trough whose apex is at *a* in a metal bar placed between *B* and *a*. Any section of this trough parallel to the plane of *B* will give three sides of a rectangle similar to *B*, the size of the rectangle varying with the distance of the section from *a*. And by substituting for the rectangle a template having the outline of a correctly shaped tooth space, the tool can be made to cut a correctly shaped tooth space in a bevel-gear blank so placed between *C* and *a* that the apex of its conical pitch surface is at *a*. All elements of the tooth surfaces so cut will converge toward *a* and the difference in the size of the tooth at its large and small ends will depend upon the length of its face and the angle at the apex of the pitch cone. The teeth of bevel gears of different size can be cut by varying the distance of the gear-blank from the point *a*.

The construction of a bevel-gear shaper is somewhat complicated but the principle upon which it operates is not difficult to understand after the discussion in the preceding paragraph. The ram of the shaper moves in guideways in a saddle pivoted

* To permit the end of the rod to be fed around the inside of the semi-circle, rectangle, or template while the tool is cutting the bar (or gear blank), the latter would ordinarily have to be first roughed out by central cuts.

at a point in front by a universal joint. The rear part of the saddle carries a roller which the mechanism of the machine keeps hard pressed against a template whose outline is that of a correctly shaped tooth space on a large scale. The ram carries the cutting tool and is moved backward and forward by the ordinary shaper mechanism. After each stroke of the shaper the feed mechanism moves the roller at the rear end of the saddle downward while at the same time it is kept hard pressed by springs or otherwise against the tooth outline of the template. In this way the rear part of the saddle, which corresponds to the rod in Fig. 92, follows the outline of a correctly shaped tooth space while its front end is pivoted at a fixed point. The ram of the shaper corresponds to the slide on the rod of Fig. 92. The bevel-gear blank on which the teeth are to be cut is placed in front of the ram with the apex of its pitch cone at the point to which the front end of the saddle is pivoted. The position of the gear-blank in front of the ram can be varied to suit the size of gear being cut. After one tooth space has been cut the blank is automatically rotated through the proper angle to place it in position for another cut by a mechanism similar to the index mechanism of a milling machine. In this machine the tool would be fed downward by the downward movement of the rear end of the saddle along the tooth outline of the template, and one side of a tooth would be cut at a time.

CHAPTER VI.

COUNTER-RECOIL SPRINGS.

114. Helical Springs. — The counter-recoil springs used in gun carriages for returning the gun into battery after recoil has ended are made of steel bars coiled into helices and hence are called helical springs. The cross-section of the bars from which the springs are made is generally either circular or rectangular. The steel from which the bars are forged must be of the best quality, and since an exceptionally high elastic limit is required the carbon content of the steel is high and the springs are hardened and tempered before use; and, as it is necessary to retain as much as possible of the increased elastic limit caused by the hardening process, the tempering temperature is low.

Fig. 93 shows two counter-recoil springs, one coiled from a bar of circular cross-section and the other coiled from a bar of rectangular cross-section.

In order that each end of a counter-recoil spring shall bear evenly against the piston of the spring rod or other part against which it acts, the ends of the end coils are closed down against the adjoining coils and ground flat so that the end surfaces of the springs are truly at right angles to its axis. When this is the case the force acting on the spring is uniformly distributed over its end surface and the effect is the same as if the force were concentrated at the axis of the spring and acted in the direction of that axis.

115. Torsional Stresses and Strains in a Straight Bar. — As the stresses and strains produced in the material of a helical spring by a force acting on it in the direction of its axis are almost entirely those of torsion, the effect of a twisting force on a straight bar will be considered before studying the stresses and strains produced in a helical spring by a force acting to compress or elongate it.