Fig. 29 is a diagram showing the lengths of the parts under consideration and their positions with respect to each other and to the axes of coordinates.

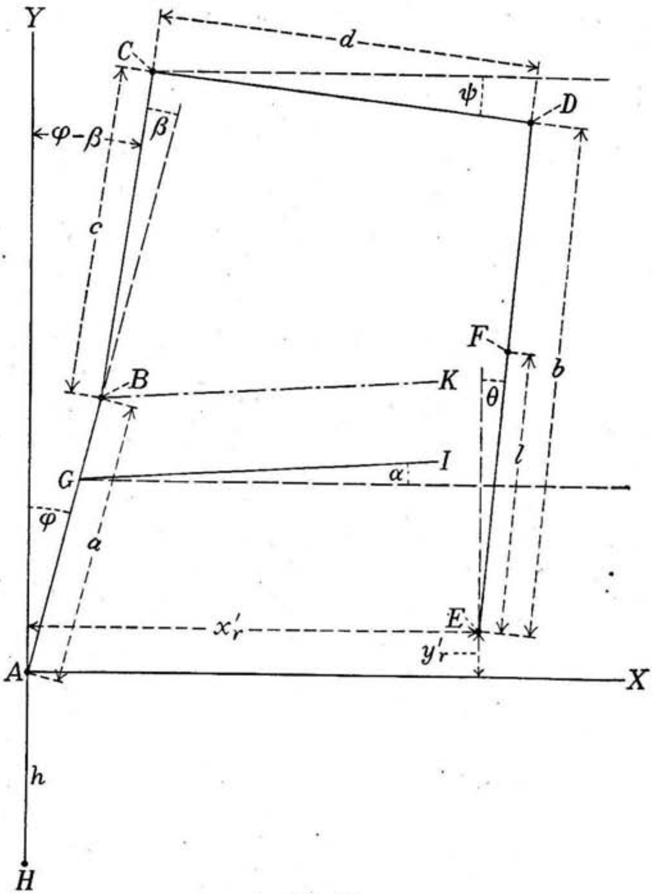

Fig. 29.

The origin of coordinates is assumed at A, the projection of the axis of the gun-lever pin when the gun is in battery. The forces are considered as co-planar and the axis of X will be taken as horizontal and the axis of Y as vertical. ABC is the median line of the gun lever. a is the length of the part between the axis of the gun-lever pin and the axis of the gun-lever axle, and ϕ is the angle which this part makes with the vertical; c is the length of the part between the axes of the gun-lever axle and the gun trunnions, and $\phi - \beta$ is the angle which this part makes with the vertical. β is a constant angle. CD represents the axis of the gun, the points C and D representing the axes of the gun trunnions and elevating-band trunnions, respectively; d is the distance between these points; and ψ is the angle between the axis of the gun and the horizontal. At the instant of firing ψ is equal to the angle of elevation. ED represents the elevating arm pivoted to a fixed part (the elevating slide of the rear transom) of the carriage at E and to the trunnions of the elevating band at D; and θ is the angle which the elevating arm makes with the vertical. b is the length of the elevating arm and l the length of the part from the pivot E to the center of gravity F of the arm. x'_r and y'_r are the coordinates of E. The gun-lever axle is carried in bearings at B in the top carriage, which during recoil slides along the roller path GI inclined at an angle α with the horizontal. The path of the axis of the gun-lever axle during recoil is indicated by the dot and dash line BK. H is the center of mass of the counterweight directly under the origin of coordinates, and h is its distance below that point.

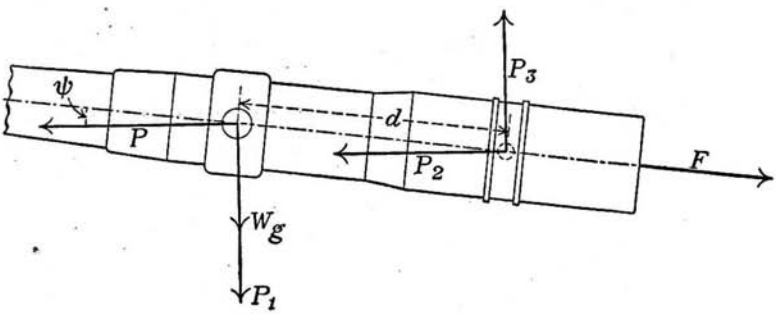

Fig. 30.

44. Forces on the Gun. — Fig. 30 represents the gun and the forces acting on it at the instant when the maximum powder pressure occurs. The velocity corresponding to the point of inflection in Fig. 27 is 10 f. s. which is therefore the velocity of free recoil at the instant of maximum powder pressure. From Table 2 the corresponding distance traveled by the gun in free recoil is .01151 ft. = .1381 ins. The actual distance recoiled by the gun will be somewhat less than this, so that for all practical purposes it will be sufficiently accurate to consider the coordinates of the various points of the system and the dimensions of the angles at the instant when the maximum powder pressure occurs as having the same values as they had when the gun was about to be fired.

At the time of maximum powder pressure the gases will exert a force F on the gun in the direction of its axis. The weight of the gun W_g is a force acting upon it through its center of mass which is assumed to be in the axis of the gun trunnions. The gun levers and the elevating arm restrain the free movement of the gun, retarding it in the direction of the axis of X and giving it an acceleration in the direction of the axis of Y. The forces exerted on the gun by these parts must be normal to the surfaces in contact and must, therefore, intersect the axes of the gun trunnions and elevating-band trunnions, respectively, but as their direction is not known the force which the gun levers exert on the gun is resolved into horizontal and vertical components P and P_1, respectively, and the force exerted on the gun by the elevating arm is resolved into a horizontal component P_2 and a vertical component P_3.

Since the forces may be considered as co-planar the algebraic sum of their horizontal components must equal the product of the mass of the gun by its horizontal acceleration and similarly for the vertical components. The algebraic sum of the moments of the forces with respect to the center of mass must also equal the moment of inertia of the gun, taken with respect to the axis through its center of mass perpendicular to the plane of the forces, multiplied by the angular acceleration about that axis. We may then write the following equations:

$$F \cos \psi - P - P_2 = M_g d^2x_g/dt^2 \qquad (1)$$

$$-F \sin \psi - W_g - P_1 + P_3 = M_g d^2y_g/dt^2 \qquad (2)$$

$$P_2 d \sin \psi - P_3 d \cos \psi = \Sigma m r_g^2 \times d^2\psi/dt^2 \qquad (3)$$

In the above equations the subscript g indicates that the symbols under which it is placed refer to the gun. The subscript a will be similarly used for the gun levers and the subscripts c, e, and w for the top carriage, elevating arm, and counterweight, respectively.

45. Forces on the Gun Levers. — Fig. 31 represents the gun levers and the forces acting thereon.

The top circle represents the bearings in the gun levers for the gun trunnions to which the upper ends of the levers are attached,

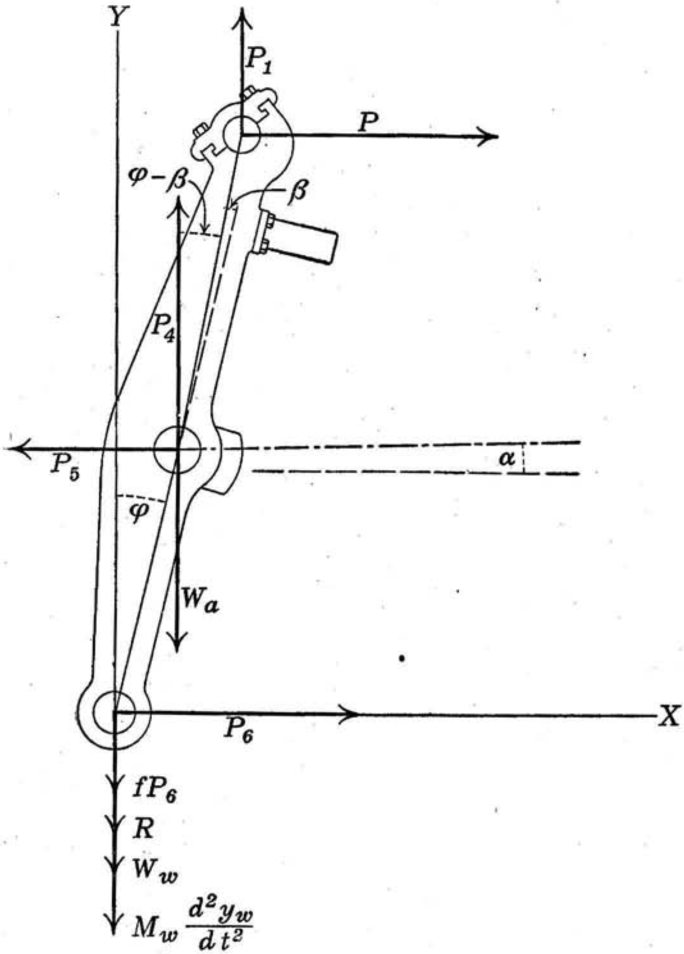

Fig. 31.

the middle circle the trunnions of the gun-lever axle carried in bearings in the top carriage, and the bottom circle the bearings for the gun-lever pins to which the lower ends of the levers are attached and to which are also fastened the counterweight and the hydraulic recoil cylinder. The gun-lever pins, the lower ends of the gun levers, the counterweight, and the recoil cylinder are constrained by the construction of the carriage to move only in a vertical direction.

The forces P and P_1 which the gun levers exert on the gun are exerted in the opposite directions as shown in Fig. 31 by the gun on the gun levers. The top carriage resists the translation of the gun levers and the force exerted by it upon them must be normal to the surfaces in contact and must, therefore, intersect the axis of the gun-lever axle trunnions. As the direction of this force is not known it will be resolved into horizontal and vertical components P_5 and P_4 as shown in the figure. The weight of the gun levers W_a acts at the center of mass as shown. The lower ends of the levers tend to move to the front around the gun-lever axle under the action of the forces brought upon them by the discharge of the gun, but are prevented from doing so by the construction of the carriage which brings a force P_6 to bear upon them as shown. The force P_6 also produces a force of friction fP_6 between the vertical guides of the carriage and the cross-head, which is attached to the gun-lever pins and forms a part of the counterweight. This friction is due to the upward movement of the lower part of the levers, counterweight, etc. f is the coefficient of sliding friction. The force R, which is the constant resistance to recoil of the hydraulic cylinder, acts vertically on the lower ends of the gun levers through the cross-head and gun-lever pins as does the weight W_w of the counterweight. The counterweight must also be given an acceleration in the direction of the axis of Y during recoil and the vertical force on the lower ends of the gun levers necessary for this purpose is measured by the product of the mass of the counterweight by its acceleration. This force is represented by

$$M_w d^2y_w/dt^2.$$

Placing the algebraic sum of the components of the various forces acting on the gun levers in the direction of each of the two axes equal, respectively, to the product of the mass of the gun levers and its acceleration in each of those directions, we have

$$P - P_5 + P_6 = M_a d^2x_a/dt^2 \tag{4}$$

$$P_1 + P_4 - W_a - fP_6 - R - W_w - M_w d^2y_w/dt^2 = M_a d^2y_a/dt^2 \tag{5}$$

Placing the algebraic sum of the moments of the forces about the center of mass of the gun levers equal to the moment of inertia of the gun levers, taken with respect to an axis through their center of mass perpendicular to the plane of the forces, multiplied by the angular acceleration about that axis; and representing the coordinates of the centers of mass of the gun, gun levers, and gun-lever pins by x_g, y_g; x_a, y_a; and y_p, respectively, $(x_p = 0)$ we have, since the center of mass of the gun levers is in the axis of their trunnions,

$$P(y_g - y_a) - P_1(x_g - x_a) - P_6(y_a - y_p) - [fP_6 + R + W_w + M_w d^2y_w/dt^2]x_a = \Sigma m r_a^2 d^2\phi/dt^2 \quad (6)$$

46. Forces on the Top Carriage. — Fig. 32 represents the top carriage and the forces acting thereon.

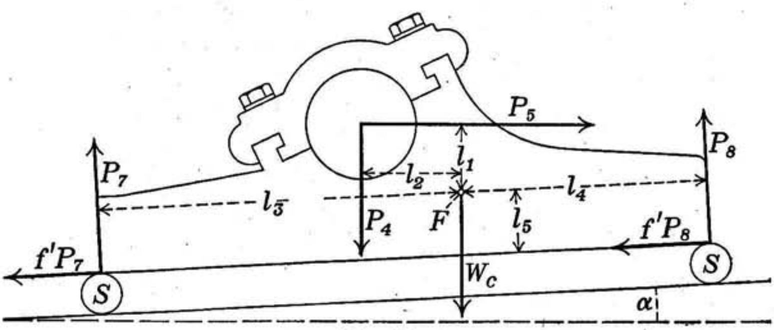

Fig. 32.

The horizontal and vertical forces P_5 and P_4 which the top carriage exerts on the gun levers are exerted in the opposite directions as shown in Fig. 32 by the gun levers on the top carriage. The top carriage rests on rollers S, and for the purpose of the determination of the forces it will be assumed that it bears on the front and rear rollers only. These rollers exert forces P_7 and P_8 on the top carriage in a direction normal to the surface of its roller path, that is, upward but making an angle α with the vertical. These forces when the top carriage moves to the rear give rise to the forces of friction $f'P_7$ and $f'P_8$ parallel to the surface of the roller path and passing through the points of contact of the top carriage with the rollers. f' is the coefficient of rolling friction. The weight W_c of the top carriage acts on it through its center of mass F as shown. The lever arms of the forces P_5, P_4, P_7, P_8, $f'P_7$, and $f'P_8$ with respect to the center of mass are l_1, l_2, l_3, l_4, and l_5, respectively, the lever arms of $f'P_7$ and $f'P_8$ being equal.

Placing the algebraic sum of the components of the forces in the direction of each of the two axes equal, respectively, to the product of the mass of the top carriage and its acceleration in

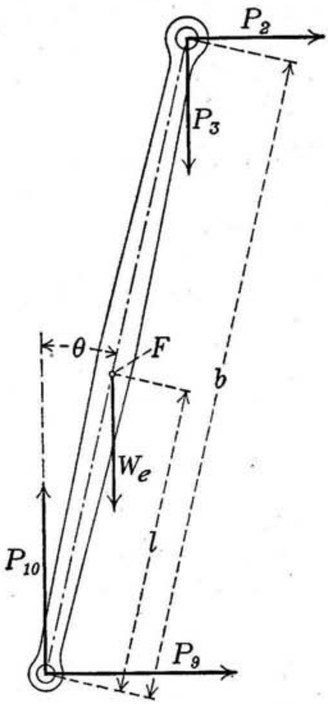

Fig. 33.

each of those directions, and the algebraic sum of the moments of the forces about the center of mass equal to zero, since there is no rotation of the top carriage, we have the following equations:

$$P_5 - P_7 \sin\alpha - P_8 \sin\alpha - f'P_7 \cos\alpha - f'P_8 \cos\alpha = M_c d^2x_c/dt^2 \quad (7)$$

$$P_7 \cos\alpha + P_8 \cos\alpha - P_4 - f'P_7 \sin\alpha - f'P_8 \sin\alpha - W_c = M_c d^2y_c/dt^2 \quad (8)$$

$$P_5 l_1 - P_4 l_2 + P_7 l_3 - P_8 l_4 + f'P_7 l_5 + f'P_8 l_5 = 0 \quad (9)$$

47. Forces on the Elevating Arm.—Fig. 33 represents the elevating arm and the forces acting thereon.

The horizontal and vertical forces P_2 and P_3, which the elevating arm exerts on the gun, are exerted in the opposite directions as shown in the figure by the gun on the elevating arm. A pin through the lower end of the elevating arm rotates in bearings in the elevating slide which, when the gun is being elevated or depressed, slides in inclined guideways in the rear transom of the carriage. At other times the elevating slide is stationary. Motion of translation of the pin forming the lower part of the elevating arm under the action of the weight of the arm and the forces P_2 and P_3 is prevented by a force exerted on the pin by the elevating slide. This force is normal to the surfaces in contact and, therefore, intersects the axis of the pin, but as its direction is not known it must be resolved into unknown horizontal and vertical components P_9 and P_{10} as shown. The weight of the elevating arm acts through its center of mass F.

Placing the algebraic sum of the components of these forces in the direction of each of the two axes equal, respectively, to the product of the mass of the elevating arm and its acceleration in each of those directions; and the algebraic sum of the moments of the forces about the center of mass equal to the moment of inertia of the elevating arm, taken with respect to the axis through its center of mass perpendicular to the plane of the forces, multiplied by the angular acceleration about that axis, we have

$$P_2 + P_9 = M_e d^2x_e/dt^2 \qquad (10)$$

$$P_{10} - P_3 - W_e = M_e d^2y_e/dt^2 \qquad (11)$$

$$P_2(b - l)\cos\theta + P_3(b - l)\sin\theta - P_9 l\cos\theta + P_{10} l\sin\theta = \Sigma m r_e^2 d^2\theta/dt^2 \qquad (12)$$

48. Reduction of the Number of Unknown Quantities in Equations (1) to (12).—In these equations F, the force of the powder gases, and all fixed dimensions are known. For any assumed position of the gun in recoil the angles ϕ, $\phi - \beta$, ψ, and θ as well as the coordinates of the centers of mass and other important points, are known from the construction of the carriage; and further it is sufficiently accurate for all practical purposes to assume that their values when the maximum powder pressure occurs are the same as they were immediately before the gun was fired. The angles α and β are constant. The weights, masses, and moments of inertia of the various parts are also known from the construction of the carriage. There are, therefore, in the twelve equations twenty-four unknown quantities of which twelve, including the constant resistance of the recoil cylinder, are forces and twelve are accelerations, linear and angular.

There are, however, certain definite relations, true at all times, between the centers of mass and other important points of the various parts of the carriage. These relations may be expressed in terms of the trigonometrical functions of the variable angles ϕ, $\phi - \beta$, ψ, and θ; and from them by differentiation may be obtained similar relations between the various velocities, linear and angular; and by differentiating again, similar relations between the accelerations, linear and angular. Having obtained these relations between the accelerations, all of them may be expressed in terms of $d^2\phi/dt^2$, and the trigonometrical functions of the variable angles which are known for any assumed position of the gun in recoil, thus reducing the unknown quantities in equations (1) to (12), inclusive, to one angular acceleration and twelve forces. A further reduction of the number of unknown quantities to twelve, thereby permitting the solution of the equations and the determination of the values of the various forces, may be made by obtaining independently the value of R, the constant resistance of the recoil cylinder.

49. Method of Determining *R*.—After the powder gases have ceased to act on the gun, the work of the resistance R over the remainder of its path during the time the system is coming to rest plus the work done on the system against the force of gravity during this time is equal to the sum of the kinetic energies of translation and rotation possessed by the system at the beginning of the interval of time considered. Letting x'' and y'' with the proper subscripts represent the coordinates of the centers of mass of the various parts of the system in the recoiled position, and x and y with the proper subscripts the coordinates at any time during recoil, we may write the following equation:

$$\left.\begin{aligned}
&R\,(y''_w - y_w) + W_w\,(y''_w - y_w) + W_a\,(y''_a - y_a) + W_c(y''_c - y_c) \\
&- W_g\,(y_g - y''_g) - W_e(y_e - y''_e) = (M_g/2)\,(dx_g/dt)^2 \\
&+ (M_g/2)\,(dy_g/dt)^2 + (\Sigma m r_g^2/2)(d\psi/dt)^2 + (M_a/2)\,(dx_a/dt)^2 + \\
&(M_a/2)\,(dy_a/dt)^2 + (\Sigma m r_a^2/2)\,(d\phi/dt)^2 + (M_c/2)\,(dx_c/dt)^2 \\
&+ (M_c/2)\,(dy_c/dt)^2 + (M_e/2)\,(dx_e/dt)^2 + (M_e/2)\,(dy_e/dt)^2 \\
&+ (\Sigma m r_e^2/2)\,(d\theta/dt)^2 + (M_w/2)\,(dy_w/dt)^2
\end{aligned}\right\} \quad (13)$$

In stating equation (13) the resistances of friction have been omitted as they depend largely on the pressures between the parts which are unknown. As a result the value of R deduced from equation (13), while sufficiently exact, will be somewhat larger than required to bring the system to rest at the desired point.

The variable coordinates, y_w, y_a, etc., in equation (13) may be expressed in terms of the trigonometrical functions of the variable angles, and the differentials may be expressed in terms of these functions and $d\phi/dt$. The variable angles may also be expressed in terms of ϕ. This having been done it is then only necessary, in order to solve equation (13) for R, to select some value for ϕ which occurs after the powder gases have ceased to act on the gun, and to determine the corresponding value of $d\phi/dt$. The data given by the curve of free recoil of the gun will enable the required values of ϕ and $d\phi/dt$ to be chosen with sufficient accuracy, and the value of ϕ which we shall select is that corresponding to the instant the powder gases cease to act on the gun.

50. Value of ϕ and of the Velocity of Restrained Recoil of the Gun at the Instant the Powder Gases Cease to Act on the Gun. — From Table 2 it is seen that the distance traveled by the gun in free recoil up to the time when the projectile leaves the bore is .18486 ft. and the corresponding velocity of free recoil is 24.648 f. s. The value of x_g when the gun is in battery is $x'_g = 1.9628$ ft. and, assuming

$$x_g - x'_g = .18486$$

whence

$$x_g = .1849 + 1.9628 = 2.1477,$$

the corresponding value of ϕ in free recoil from equation (14) below is approximately 14° 9′.4. Also the distance traveled by the gun in free recoil up to the time τ when the powder gases cease to act upon it is 1.6786 ft., the corresponding maximum velocity of free recoil is 32.64 f. s. and, assuming

$$x_g = 1.6786 + x'_g,$$

the corresponding value of ϕ in free recoil is approximately 23° 37′.8.

A number of foreign ordnance engineers of prominence assume, in solving problems of a nature similar to this, that the maximum velocity of restrained recoil is instantly acquired and equal to the maximum velocity of free recoil; but the experience of the Ordnance Department, U. S. Army, has shown that a closer approximation than this, and one sufficiently exact for all practical purposes, is to assume that due to the restraint of the recoil brake the velocity of the gun in restrained recoil at the instant the powder gases cease to act on it is, for carriages of this type, about eight-tenths of the maximum velocity of free recoil, and that the value of the angle ϕ at the same instant is equal to its value when the gun is in battery plus about eight-tenths of its increase in value in free recoil up to the time τ.* The value of ϕ when the gun is in battery being 13°, its value when the powder gases cease to act will be assumed as 21°. The assumed values of ϕ and v_r lie between those obtained above for free recoil at the instant the projectile leaves the bore and at the instant the powder gases cease to act on the gun, respectively.

51. Determination of the Values of the Coordinates of the Centers of Mass in Terms of the Trigonometrical Functions of the Angles ϕ, $\phi - \beta$, ψ, θ, and the Constant Angle α. — Letting x' and y' with the proper subscripts represent the coordinates of the centers of mass of the various parts when the gun is in battery, and x'_r and y'_r the coordinates of the axis of the pin connecting the lower end of the elevating arm to the elevating slide, we may write the following equations from the relations shown in Fig. 29:

$$x_g = a \sin \phi + c \sin (\phi - \beta) \tag{14}$$

$$x_a = a \sin \phi \tag{15}$$

$$x_c = a \sin \phi + x'_c - x'_a \tag{16}$$

$$x_e = x'_r + l \sin \theta \tag{17}$$

$$x_e = x_g + d \cos \psi - (b - l) \sin \theta = a \sin \phi + c \sin (\phi - \beta) + d \cos \psi - (b - l) \sin \theta \tag{18}$$

$$y_g = y'_a + (x_a - x'_a) \tan \alpha + c \cos (\phi - \beta) = y'_a + a \sin \phi \tan \alpha - x'_a \tan \alpha + c \cos (\phi - \beta) \tag{19}$$

* A more rigorous method of determining the velocity of the gun in restrained recoil and the value of ϕ at the instant the powder gases cease to act on the gun is used by the gun-carriage division of the Ordnance Department. It is too extensive for discussion here, but the results obtained by it have shown that the approximate method outlined above is sufficiently accurate for all practical purposes.

$$y_a = y'_a + (x_a - x'_a) \tan \alpha = y'_a + a \sin \phi \tan \alpha - x'_a \tan \alpha \tag{20}$$

$$y_c = y'_a + a \sin \phi \tan \alpha - x'_a \tan \alpha - (y'_a - y'_c) \tag{21}$$

$$y_p = y'_a + a \sin \phi \tan \alpha - x'_a \tan \alpha - a \cos \phi \tag{21½}$$

$$y_w = y'_a + a \sin \phi \tan \alpha - x'_a \tan \alpha - a \cos \phi - h \tag{22}$$

$$y_e = y'_r + l \cos \theta \tag{23}$$

$$y_e = y_g - d \sin \psi - (b - l) \cos \theta = y'_a + a \sin \phi \tan \alpha - x'_a \tan \alpha + c \cos (\phi - \beta) - d \sin \psi - (b - l) \cos \theta \tag{24}$$

52. Determination of the Values of ψ and θ when the Value of ϕ is Known or Assumed. — Equating the values of x_e from equations (17) and (18) we have

$$b \sin \theta = a \sin \phi + c \sin (\phi - \beta) + d \cos \psi - x'_r$$

and representing $a \sin \phi + c \sin (\phi - \beta) - x'_r$ by Z,

$$b \sin \theta = d \cos \psi + Z \tag{25}$$

Equating the values of y_e from equations (23) and (24) we have

$$b \cos \theta = y'_a + a \sin \phi \tan \alpha - x'_a \tan \alpha + c \cos (\phi - \beta) - d \sin \psi - y'_r$$

and representing

$y'_a + a \sin \phi \tan \alpha - x'_a \tan \alpha + c \cos (\phi - \beta) - y'_r$ by Y,

$$b \cos \theta = -d \sin \psi + Y \tag{26}$$

Squaring equations (25) and (26) and adding we have

$$b^2 = d^2 + 2\, dZ \cos \psi - 2\, dY \sin \psi + Z^2 + Y^2$$

which may be put in the form

$$Z \cos \psi - Y \sin \psi = (b^2 - d^2 - Z^2 - Y^2)/2\, d$$

Dividing by $\sqrt{Z^2 + Y^2}$

$$\frac{Z}{\sqrt{Z^2 + Y^2}} \cos \psi - \frac{Y}{\sqrt{Z^2 + Y^2}} \sin \psi = \frac{b^2 - d^2 - Z^2 - Y^2}{2\, d \sqrt{Z^2 + Y^2}} \tag{26½}$$

Let $$A = \tan^{-1} Z/Y$$

whence $$\sin A = \frac{Z}{\sqrt{Z^2 + Y^2}}$$

and $$\cos A = \frac{Y}{\sqrt{Z^2 + Y^2}}$$

Replacing in equation ($26\frac{1}{2}$)

$$\frac{Z}{\sqrt{Z^2 + Y^2}} \text{ by } \sin A \text{ and } \frac{Y}{\sqrt{Z^2 + Y^2}} \text{ by } \cos A$$

we have

$$\sin A \cos \psi - \cos A \sin \psi = \sin (A - \psi) = \frac{b^2 - d^2 - Z^2 - Y^2}{2\, d \sqrt{Z^2 + Y^2}} \quad (27)$$

in which $A = \tan^{-1} Z / Y$.

For any value of ϕ the corresponding value of ψ can be obtained from equation (27), and having ϕ and ψ the corresponding value of θ can be obtained from equation (26).

53. Determination of the Velocities in Terms of the Trigonometrical Functions of the Angles, and of $d\phi / dt$. — Differentiating both sides of equations (14) to (24), inclusive, excepting equation ($21\frac{1}{2}$), and dividing by dt we have

$$dx_g / dt = a \cos \phi \, (d\phi / dt) + c \cos (\phi - \beta) \, (d\phi / dt) \quad (28)$$

$$dx_a / dt = a \cos \phi \, (d\phi / dt) \quad (29)$$

$$dx_c / dt = a \cos \phi \, (d\phi / dt) \quad (30)$$

$$dx_e / dt = l \cos \theta \, (d\theta / dt) \quad (31)$$

$$dx_e / dt = a \cos \phi \, (d\phi / dt) + c \cos (\phi - \beta) \, (d\phi / dt) - d \sin \psi \, (d\psi / dt) - (b - l) \cos \theta \, (d\theta / dt) \quad (32)$$

$$dy_g / dt = a \tan \alpha \cos \phi \, (d\phi / dt) - c \sin (\phi - \beta) \, (d\phi / dt) \quad (33)$$

$$dy_a / dt = a \tan \alpha \cos \phi \, (d\phi / dt) \quad (34)$$

$$dy_c / dt = a \tan \alpha \cos \phi \, (d\phi / dt) \quad (35)$$

$$dy_w / dt = a \tan \alpha \cos \phi \, (d\phi / dt) + a \sin \phi \, (d\phi / dt) \quad (36)$$

$$dy_e / dt = -l \sin \theta \, (d\theta / dt) \quad (37)$$

$$dy_e / dt = a \tan \alpha \cos \phi \, (d\phi / dt) - c \sin (\phi - \beta) \, (d\phi / dt) - d \cos \psi \, (d\psi / dt) + (b - l) \sin \theta \, (d\theta / dt) \quad (38)$$

Equating the values of dx_e / dt given by equations (31) and (32) we have

$$b \cos \theta \, (d\theta / dt) = a \cos \phi \, (d\phi / dt) + c \cos (\phi - \beta) \, (d\phi / dt) - d \sin \psi \, (d\psi / dt) \quad (39)$$

and equating the values of dy_e / dt given by equations (37) and (38) we have

$$-b \sin \theta \, (d\theta / dt) = a \tan \alpha \cos \phi \, (d\phi / dt) - c \sin (\phi - \beta) \, (d\phi / dt) - d \cos \psi \, (d\psi / dt) \quad (40)$$

Multiplying equation (39) by $\sin\theta$ and equation (40) by $\cos\theta$ and adding we have

$$\begin{aligned}&\sin\theta\, a\cos\phi\,(d\phi/dt) + \sin\theta\, c\cos(\phi-\beta)\,(d\phi/dt)\\ &\quad+\cos\theta\, a\tan\alpha\cos\phi\,(d\phi/dt) - \cos\theta\, c\sin(\phi-\beta)\,(d\phi/dt)\\ &\quad= \sin\theta\, d\sin\psi\,(d\psi/dt) + \cos\theta\, d\cos\psi\,(d\psi/dt)\end{aligned}$$

or

$$\frac{d\psi}{dt} = \left\{\begin{matrix}\dfrac{\sin\theta\,\{a\cos\phi + c\cos(\phi-\beta)\}}{d\,(\sin\theta\sin\psi + \cos\theta\cos\psi)} + \\ \dfrac{\cos\theta\,\{a\tan\alpha\cos\phi - c\sin(\phi-\beta)\}}{d\,(\sin\theta\sin\psi + \cos\theta\cos\psi)}\end{matrix}\right\}\frac{d\phi}{dt} \qquad (41)$$

Dividing equation (39) by $b\cos\theta$ we have

$$\frac{d\theta}{dt} = \frac{a\cos\phi + c\cos(\phi-\beta)}{b\cos\theta}\,\frac{d\phi}{dt} - \frac{d\sin\psi}{b\cos\theta}\,\frac{d\psi}{dt}$$

or replacing $d\psi/dt$ by its value in terms of $d\phi/dt$ from equation (41) and representing the coefficient of $d\phi/dt$ in that equation by X

$$\frac{d\theta}{dt} = \left\{\frac{a\cos\phi + c\cos(\phi-\beta)}{b\cos\theta} - \frac{d\sin\psi}{b\cos\theta}X\right\}\frac{d\phi}{dt} \qquad (42)$$

For any value of $d\phi/dt$ the corresponding values of $d\psi/dt$ and $d\theta/dt$ can be obtained from equations (41) and (42), respectively, when the angle ϕ is known. The values of all the linear velocities can then be obtained from equations (28) to (37), inclusive.

54. Determination of the Accelerations in Terms of the Trigonometrical Functions of the Angles, and of $d^2\phi/dt^2$. — Differentiating both sides of equations (28) to (38), inclusive, and dividing by dt we obtain

$$\begin{aligned}d^2x/dt^2 = \{a\cos\phi + c\cos(\phi-\beta)\}\,(d^2\phi/dt^2) - \{&a\sin\phi\\ &+ c\sin(\phi-\beta)\}\,(d\phi/dt)^2\end{aligned} \qquad (43)$$

$$d^2x_a/dt^2 = a\cos\phi\,(d^2\phi/dt^2) - a\sin\phi\,(d\phi/dt)^2 \qquad (44)$$

$$d^2x_c/dt^2 = a\cos\phi\,(d^2\phi/dt^2) - a\sin\phi\,(d\phi/dt)^2 \qquad (45)$$

$$d^2x_e/dt^2 = l\cos\theta\,(d^2\theta/dt^2) - l\sin\theta\,(d\theta/dt)^2 \qquad (46)$$

$$\left.\begin{aligned} d^2x_e/dt^2 = &\{a\cos\phi + c\cos(\phi-\beta)\}(d^2\phi/dt^2) \\ &- d\sin\psi\,(d^2\psi/dt^2) - (b-l)\cos\theta\,(d^2\theta/dt)^2 \\ &-\{a\sin\phi + c\sin(\phi-\beta)\}(d\phi/dt)^2 \\ &- d\cos\psi\,(d\psi/dt)^2 + (b-l)\sin\theta\,(d\theta/dt)^2 \end{aligned}\right\} \quad (47)$$

$$\begin{aligned} d^2y_g/dt^2 = &\{a\tan\alpha\cos\phi - c\sin(\phi-\beta)\}(d^2\phi/dt^2) \\ &- \{a\tan\alpha\sin\phi + c\cos(\phi-\beta)\}(d\phi/dt)^2 \end{aligned} \quad (48)$$

$$d^2y_a/dt^2 = a\tan\alpha\cos\phi\,(d^2\phi/dt^2) - a\tan\alpha\sin\phi(d\phi/dt)^2 \quad (49)$$

$$d^2y_c/dt^2 = a\tan\alpha\cos\phi\,(d^2\phi/dt^2) - a\tan\alpha\sin\phi\,(d\phi/dt)^2 \quad (50)$$

$$\begin{aligned} d^2y_w/dt^2 = &\{a\tan\alpha\cos\phi + a\sin\phi\}(d^2\phi/dt^2) - \{a\tan\alpha\sin\phi \\ &- a\cos\phi\}(d\phi/dt)^2 \end{aligned} \quad (51)$$

$$d^2y_e/dt^2 = -l\sin\theta\,(d^2\theta/dt^2) - l\cos\theta\,(d\theta/dt)^2 \quad (52)$$

$$\left.\begin{aligned} d^2y_e/dt^2 = &\{a\tan\alpha\cos\phi - c\sin(\phi-\beta)\}(d^2\phi/dt^2) \\ &- d\cos\psi\,(d^2\psi/dt^2) + (b-l)\sin\theta\,(d^2\theta/dt^2) \\ &- \{a\tan\alpha\sin\phi + c\cos(\phi-\beta)\}(d\phi/dt)^2 \\ &+ d\sin\psi\,(d\psi/dt)^2 + (b-l)\cos\theta\,(d\theta/dt)^2 \end{aligned}\right\} \quad (53)$$

Equating the values of d^2x_e/dt^2 from equations (46) and (47) we have

$$\left.\begin{aligned} &b\cos\theta\,(d^2\theta/dt^2) - b\sin\theta\,(d\theta/dt)^2 \\ &= \{a\cos\phi + c\cos(\phi-\beta)\}(d^2\phi/dt^2) - d\sin\psi\,(d^2\psi/dt^2) \\ &- \{a\sin\phi + c\sin(\phi-\beta)\}(d\phi/dt)^2 - d\cos\psi\,(d\psi/dt)^2 \end{aligned}\right\} \quad (54)$$

and equating the values of d^2y_e/dt^2 from equations (52) and (53)

$$\left.\begin{aligned} &-b\sin\theta\,(d^2\theta/dt^2) - b\cos\theta\,(d\theta/dt)^2 = \\ &\{a\tan\alpha\cos\phi - c\sin(\phi-\beta)\}(d^2\phi/dt^2) - d\cos\psi(d^2\psi/dt^2) \\ &- \{a\tan\alpha\sin\phi + c\cos(\phi-\beta)\}(d\phi/dt)^2 + d\sin\psi\,(d\psi/dt)^2 \end{aligned}\right\} (55)$$

Multiplying equation (54) by $\sin\theta$ and equation (55) by $\cos\theta$ and adding we have

$$\left.\begin{aligned} -b(d\theta/dt)^2 = &[\sin\theta\{a\cos\phi + c\cos(\phi-\beta)\} \\ &+ \cos\theta\{a\tan\alpha\cos\phi - c\sin(\phi-\beta)\}](d^2\phi/dt^2) \\ &- [d\sin\theta\sin\psi + d\cos\theta\cos\psi](d^2\psi/dt^2) \\ &- [\sin\theta\{a\sin\phi + c\sin(\phi-\beta)\} + \cos\theta\{a\tan\alpha\sin\phi \\ &+ c\cos(\phi-\beta)\}](d\phi/dt)^2 - [d\sin\theta\cos\psi \\ &- d\cos\theta\sin\psi](d\psi/dt)^2 \end{aligned}\right\}$$

or

$$\frac{d^2\psi}{dt^2} = \left\{ \begin{aligned} & \frac{\sin\theta\,\{a\cos\phi + c\cos(\phi-\beta)\}}{d\sin\theta\sin\psi + d\cos\theta\cos\psi}\,\frac{d^2\phi}{dt^2} \\ & + \frac{\cos\theta\,\{a\tan\alpha\cos\phi - c\sin(\phi-\beta)\}}{d\sin\theta\sin\psi + d\cos\theta\cos\psi}\,\frac{d^2\phi}{dt^2} \\ & - \frac{\sin\theta\,\{a\sin\phi + c\sin(\phi-\beta)\}}{d\sin\theta\sin\psi + d\cos\theta\cos\psi}\left(\frac{d\phi}{dt}\right)^2 \\ & - \frac{\cos\theta\,\{a\tan\alpha\sin\phi + c\cos(\phi-\beta)\}}{d\sin\theta\sin\psi + d\cos\theta\cos\psi}\left(\frac{d\phi}{dt}\right)^2 \\ & - \frac{\sin\theta\cos\psi - \cos\theta\sin\psi}{\sin\theta\sin\psi + \cos\theta\cos\psi}\left(\frac{d\psi}{dt}\right)^2 \\ & + \frac{b}{d\sin\theta\sin\psi + d\cos\theta\cos\psi}\left(\frac{d\theta}{dt}\right)^2 \end{aligned} \right\} \qquad (56)^*$$

Dividing equation (54) by $b\cos\theta$ and solving for $d^2\theta/dt^2$ we have

$$\frac{d^2\theta}{dt^2} = \left\{ \begin{aligned} & \frac{a\cos\phi + c\cos(\phi-\beta)}{b\cos\theta}\,\frac{d^2\phi}{dt^2} \\ & - \frac{d\sin\psi}{b\cos\theta}\,\frac{d^2\psi}{dt^2} \\ & - \frac{a\sin\phi + c\sin(\phi-\beta)}{b\cos\theta}\left(\frac{d\phi}{dt}\right)^2 \\ & - \frac{d\cos\psi}{b\cos\theta}\left(\frac{d\psi}{dt}\right)^2 + \tan\theta\left(\frac{d\theta}{dt}\right)^2 \end{aligned} \right\}$$

or representing the coefficients of

$$d^2\phi/dt^2, \quad (d\phi/dt)^2, \quad (d\psi/dt)^2, \text{ and } (d\theta/dt)^2$$

in equation (56) by X, X', X'', and X''', respectively, we have

$$\frac{d^2\theta}{dt^2} = \left\{ \begin{aligned} & \frac{a\cos\phi + c\cos(\phi-\beta) - d\sin\psi\,X}{b\cos\theta}\,\frac{d^2\phi}{dt^2} \\ & - \frac{a\sin\phi + c\sin(\phi-\beta) - d\sin\psi X'}{b\cos\theta}\left(\frac{d\phi}{dt}\right)^2 \\ & - \frac{d\,(\cos\psi - \sin\psi X'')}{b\cos\theta}\left(\frac{d\psi}{dt}\right)^2 \\ & - \frac{d\sin\psi X''' - b\sin\theta}{b\cos\theta}\left(\frac{d\theta}{dt}\right)^2 \end{aligned} \right\} \qquad (57)$$

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* When this equation is used to determine the value of $d^2\psi/dt^2$ the numerical work can be simplified somewhat by noting that $\sin\theta\sin\psi + \cos\theta\cos\psi = \cos(\theta-\psi)$ and $\sin\theta\cos\psi - \cos\theta\sin\psi = \sin(\theta-\psi)$.

For any value of $d^2\phi/dt^2$ the corresponding values of $d^2\psi/dt^2$ and $d^2\theta/dt^2$ can be obtained from equations (56) and (57), respectively, when the angle ϕ and the angular velocities are known. The values of the linear accelerations can then be obtained from equations (43) to (53), inclusive.

55. Determination of the Value of *R* from Equation (13) (Gun fired at 0° Elevation). — The following data are known from the construction of the gun and carriage:

TABLE 3.

$\alpha = 1° 20'$	$y'_r = 0.5563$ ft.	$W_g = 12764$ lbs.
$\phi' = 13°$	$y'_g = 9.2942$ "	$M_g = 396.88$
$\phi'' = 88°$	$y''_g = 4.6612$ "	$W_a = 6040.8$ lbs.
$\beta = 2°$	$y'_a = 4.2224$ "	$M_a = 187.84$
$(\phi' - \beta) = 11°$	$y''_a = 4.3008$ "	$W_c = 2528$ lbs.
$(\phi'' - \beta) = 86°$	$y'_c = 3.7571$ "	$M_c = 78.61$
$\theta' = 3° 58'$	$y''_c = 3.8355$ "	$W_e = 707$ lbs.
ψ' = angle of elevation	$y'_e = 4.6548$ "	$M_e = 21.984$
$\psi'' = 5°$	$y''_e = 2.2571$ "	$W_w = 19797$ lbs.
$x'_g = 1.9628$ ft.	$y'_p = 0.0$ "	$M_w = 615.58$
$x''_g = 9.4857$ "	$y'_w = -3.0$ "	$\Sigma mr^2_g = 14967$
$x'_a = 0.975$ "	$y''_w = 1.1492$ "	$\Sigma mr^2_a = 1250.4$
$x''_a = 4.3317$ "	$a = 4.3333$ "	$\Sigma mr^2_e = 155.08$
$x'_c = 1.6401$ "	$b = 8.75$ "	$l_1 = 0.4653$ ft.
$x''_c = 4.9968$ "	$c = 5.1667$ "	$l_2 = 0.6651$ "
$x'_e = 7.2145$ "	$d = 5.5$ "	$l_3 = 2.4875$ "
$x''_e = 10.579$ "	$l = 4.1042$ "	$l_4 = 1.6792$ "
$x'_r = 7.0002$ "	$h = 3.0$ "	$l_5 = 0.4255$ "

In the computation of *R* it is customary to assume that the gun is fired at zero degrees elevation, whence $\psi = 0$, and this assumption will be made here. No difficulty arises, however, in computing *R* for any other given elevation of the gun when fired, the method to be pursued in that case being exactly similar to that which follows.

For the reasons given in article 50, page 78, the value of ϕ at the instant the powder gases cease to act on the gun will be taken as 21° and the corresponding velocity of the gun in restrained recoil as

$$\sqrt{(dx_g/dt)^2 + (dy_g/dt)^2} = .8\,V_f = 26.112 \text{ f. s.},$$

V_f being the maximum velocity of free recoil equal to 32.64 f. s.

With $\phi = 21°$ the values of ψ and θ, computed from equations (27) and (26), respectively, are

$$\psi = -7'.3 \qquad \theta = 11° 26'.3$$

With $\phi = 21°$ and $\theta = 11° 26'.3$ the values of y_w, y_a, y_c, y_g, and y_e, computed from equations (22), (20), (21), (19), and (23), respectively, are

$$\begin{aligned} y_w &= -2.8078 \text{ ft.} \qquad & y_g &= 9.1226 \text{ ft.} \\ y_a &= 4.2374 \text{ ft.} & y_e &= 4.5790 \text{ ft.} \\ y_c &= 3.7721 \text{ ft.} \end{aligned}$$

With $\sqrt{(dx_g/dt)^2 + (dy_g/dt)^2} = 26.112$ f. s. and $\phi = 21°$, the value of $d\phi/dt$, computed from equations (28) and (33), is

$$d\phi/dt = 2.8788 \text{ radians per sec.}$$

With $d\phi/dt = 2.8788$ radians per sec., $\phi = 21°$, $\psi = -7'.3$, and $\theta = 11° 26'.3$, the values of $d\psi/dt$, $d\theta/dt$, dx_g/dt, dy_g/dt, dx_a/dt, dy_a/dt, dx_c/dt, dy_c/dt, dx_e/dt, dy_e/dt, and dy_w/dt, computed from equations (41), (42), (28), (33), (29), (34), (30), (35), (31), (37), and (36), respectively, are as follows:

$$\begin{aligned} d\psi/dt &= .1147 \text{ radians per sec.} \qquad & dx_c/dt &= 11.645 \text{ f. s.} \\ d\theta/dt &= 2.9978 \text{ radians per sec.} & dy_c/dt &= 0.2708 \text{ f. s.} \\ dx_g/dt &= 25.709 \text{ f. s.} & dx_e/dt &= 12.059 \text{ f. s.} \\ dy_g/dt &= -4.5715 \text{ f. s.} & dy_e/dt &= -2.4401 \text{ f. s.} \\ dx_a/dt &= 11.645 \text{ f. s.} & dy_w/dt &= 4.741 \text{ f. s.} \\ dy_a/dt &= 0.2708 \text{ f. s.} \end{aligned}$$

Substituting in equation (13) the values of the coordinates and the velocities just determined, together with the values of the coordinates when the gun is from battery and the values of the masses and moments of inertia from Table 3, and solving for R, we obtain

$$R = 37312 \text{ lbs.}$$

56. Determination of the Values of the Forces from Equations (1) to (12), Inclusive. Gun Fired at 0° Elevation. — The first step in the solution of equations (1) to (12), inclusive, is to determine the values of all the accelerations in terms of $d^2\phi/dt^2$. To do this it is again necessary to obtain the values of $d\phi/dt$,

$d\psi/dt$, and $d\theta/dt$ this time, however, corresponding to the position of the parts at the instant of maximum pressure of the powder gases. As stated in article 44, page 71, it will be sufficient to consider that the values of the coordinates and the angles are at this instant the same as just before the gun was fired. The value of $\sqrt{(dx_g/dt)^2+(dy_g/dt)^2}$ will again be taken as eight-tenths of the velocity of free recoil which at this time, as previously determined, is 10 f. s.

With $\sqrt{(dx_g/dt)^2 + (dy_g/dt)^2} = .8 \times 10 = 8$ f. s.

the values of $d\phi/dt$, $d\psi/dt$, and $d\theta/dt$ computed from equations (28), (33), (41), and (42), respectively, are

$$d\phi/dt = .8569 \text{ radians per sec.}$$
$$d\psi/dt = -.04256 \text{ radians per sec.}$$
$$d\theta/dt = .9123 \text{ radians per sec.}$$

With these values of $d\phi/dt$, $d\psi/dt$, and $d\theta/dt$ and the known values of the angles, the values of all the accelerations in terms of $d^2\phi/dt^2$ may be obtained as follows:

From equation (43) $d^2x_g/dt^2 = 9.2937\ (d^2\phi/dt^2) - 1.44$ (43′)
From equation (44) $d^2x_a/dt^2 = 4.2219\ (d^2\phi/dt^2) - .716$ (44′)
From equation (45) $d^2x_c/dt^2 = 4.2219\ (d^2\phi/dt^2) - .716$ (45′)
From equation (48) $d^2y_g/dt^2 = -.887585(d^2\phi/dt^2) - 3.741$ (48′)
From equation (49) $d^2y_a/dt^2 = .098265\ (d^2\phi/dt^2) - .0167$ (49′)
From equation (50) $d^2y_c/dt^2 = .098265\ (d^2\phi/dt^2) - .0167$ (50′)
From equation (51) $d^2y_w/dt^2 = 1.07299\ (d^2\phi/dt^2) + 3.083$ (51′)
From equation (56) $d^2\psi/dt^2 = -.04421\ (d^2\phi/dt^2) + .62915$ (56′)
From equation (57) $d^2\theta/dt^2 = 1.0647\ (d^2\phi/dt^2) - .1083$ (57′)
From equation (46) $d^2x_e/dt^2 = 4.3592\ (d^2\phi/dt^2) - .2363$ (46′)
From equation (52) $d^2y_e/dt^2 = -.30228\ (d^2\phi/dt^2) - 3.408$ (52′)

By substituting these expressions for the accelerations and the value of $R = 37312$ lbs. in equations (1) to (12), inclusive, the number of unknown quantities therein will be reduced to twelve and the equations can be readily solved.

Rewriting the equations with these substitutions and replacing the symbols of the constants therein by their values from

Table 3 we have, since the value of ψ in the case under consideration is 0 and the value of F corresponding to a powder pressure of 36000 lbs. per sq. in. is

$$36000 \times \pi\,(6)^2/4 = 1017860 \text{ lbs.},$$

$$1017860 - P - P_2 = 396.88\,[9.2937\,(d^2\phi/dt^2) - 1.44] \qquad (1')$$

$$-12764 - P_1 + P_3 = 396.88\,[-.887585\,(d^2\phi/dt^2) - 3.741] \qquad (2')$$

$$-5.5\,P_3 = 14967\,[-.04421\,(d^2\phi/dt^2) + .62915] \qquad (3')$$

$$P - P_5 + P_6 = 187.84\,[4.2219\,(d^2\phi/dt^2) - .716] \qquad (4')$$

$$\left.\begin{aligned} &P_1 + P_4 - 6040.8 - .15\,P_6 - 37312 - 19797 \\ &\quad - 615.58\,[1.07299\,(d^2\phi/dt^2) + 3.083] \\ &\quad = 187.84\,[.098265\,(d^2\phi/dt^2) - .0167] \end{aligned}\right\} \qquad (5')$$

$$\left.\begin{aligned} &5.0718\,P - .9878\,P_1 - 4.2224\,P_6 - .975\,[.15\,P_6 \\ &+ 37312 + 19797 + 615.58\,\{1.07299\,(d^2\phi/dt^2) \\ &+ 3.083\}] = 1250.4\,(d^2\phi/dt^2) \end{aligned}\right\} \qquad (6')$$

$$\left.\begin{aligned} &P_5 = .02327\,P_7 - .02327\,P_8 - .005 \times .99972\,P_7 \\ &\quad - .005 \times .99972\,P_8 = 78.61\,[4.2219\,(d^2\phi/dt^2) - .716] \end{aligned}\right\} \qquad (7')$$

$$\left.\begin{aligned} &.99972\,P_7 + .99972\,P_8 - P_4 - .005 \times .0237\,P_7 \\ &- .005 \times .0237\,P_8 - 2528 \\ &= 78.61\,[.098265\,(d^2\phi/dt^2) - .0167] \end{aligned}\right\} \qquad (8')$$

$$\begin{aligned} .4653\,P_5 - .6651\,P_4 + 2.4875\,P_7 - 1.6792\,P_8 + .4255 \\ \times .005\,P_7 + .4255 \times .005\,P_8 = 0 \end{aligned} \qquad (9')$$

$$P_2 + P_9 = 21.984\,[4.3592\,(d^2\phi/dt^2) - .2363 \qquad (10')$$

$$P_{10} - P_3 - 707 = 21.984\,[-.30228\,(d^2\phi/dt^2) - 3.408] \qquad (11')$$

$$\begin{aligned} 4.6347\,P_2 + .3214\,P_3 - 4.0944\,P_9 + .2839\,P_{10} \\ = 155.08\,[1.0647\,(d^2\phi/dt^2) - .1083] \end{aligned} \qquad (12')$$

SOLUTION OF EQUATIONS (1′) TO (12′), INCLUSIVE.

From equation (3′) $P_3 = 120.3\,(d^2\phi/dt^2) - 1712$ (3″)

Substituting this value of P_3 in equation (2′)

$$P_1 = 472.57\,(d^2\phi/dt^2) - 12991.3 \qquad (2'')$$

Substituting the value of P_3 in equation (11′)

$$P_{10} = 113.66\,(d^2\phi/dt^2) - 1079.92 \qquad (11'')$$

Substituting in equation (12′) the values of P_3 from equation (3″), P_{10} from equation (11″), and P_9 from equation (10′),

$$P_2 = 55.739\,(d^2\phi/dt^2) + 93.80 \qquad (12'')$$

From equation (1′) $P = 1018337.7 - 3744.24\,(d^2\phi/dt^2)$ (1″)

From equation (6′) $P_6 = 1171851 - 4888.27\,(d^2\phi/dt^2)$ (6″)

From equation (4′) $P_5 = 2190323.19 - 9425.41\,(d^2\phi/dt^2)$ (4″)

From equation (5′) $P_4 = 253813.38 - 26.828\,(d^2\phi/dt^2)$ (5″)

In equation (7′) the coefficient of P_7 and P_8 when the two terms containing P_7 and the two containing P_8 are consolidated is .02827, and in equation (8′) when these terms are consolidated the coefficient is .9996. Therefore, by multiplying equation (7′) by $\frac{.9996}{.02827}$ and adding the result to equation (8′), P_7 and P_8 are eliminated and

$$35.359\,P_5 - P_4 - 2528 = 35.359 \times 78.61\,[4.2219\,(d^2\phi/dt^2) - .716] + 78\ 61\,[.098265\,(d^2\phi/dt^2) - .0167] \qquad (7'')$$

Substituting for P_5 and P_4 in equation (7″) their values from equations (4″) and (5″)

$$344493\,(d^2\phi/dt^2) = 77193650 \qquad (7''')$$

Whence $d^2\phi/dt^2 = 224.08$ radians per sec. per sec. (7^{IV})

With the value of $d^2\phi/dt^2 = 224.08$ we obtain

From equation (1″) $P = 179338$ lbs. (1‴)

From equation (2″) $P_1 = 92902$ lbs. (2‴)

From equation (12″) $P_2 = 12584$ lbs. (12‴)

From equation (3″) $P_3 = 25245$ lbs. (3‴)

From equation (5″) $P_4 = 135748$ lbs. (5‴)

From equation (4″) $P_5 = 78223$ lbs. (4‴)

From equation (6″) $P_6 = 76476$ lbs. (6‴)

$fP_6 = .15\,P_6 = 11471$ lbs. (6^{IV})

From equation (51′) and the value of $M_w = 615.58$ from Table 3,

$$M_w\,(d^2y_w/dt^2) = 149907 \text{ lbs.} \qquad (51'')$$

The total vertical force acting on the lower ends of the gun levers through the gun-lever pins is

$$fP_6 + R + W_w + M_w d^2y_w/dt^2 = 11471 + 37312 + 19797 + 149907 = 218487 \text{ lbs.} \qquad (51''')$$

To obtain the values of P_7 and P_8 replace P_4 and P_5 in equation (9′) by their values from equations (5‴) and (4‴), respectively, and solve for P_8 in terms of P_7. Then substitute this value of P_8 and the numerical values of P_4 and $d^2\phi/dt^2$ in equation (8′) and solve for P_7. The value of P_8 then follows from that of P_7.

From equation (9′) $\quad P_8 = 1.4845\,P_7 - 32127 \qquad$ (9″)

From equation (8′) $\quad P_7 = 69304$ lbs. $\qquad$ (8″)

From equation (9″) $\quad P_8 = 70755$ lbs. $\qquad$ (9‴)

From equation (10′) after substituting therein the values of P_2 and $d^2\phi/dt^2$

$$P_9 = 8885 \text{ lbs.} \qquad (10'')$$

From equation (11″) after substituting therein the value of $d^2\phi/dt^2$

$$P_{10} = 24386 \text{ lbs.} \qquad (11''')$$

57. Computation of the Values of the Forces when the Gun is Fired at Any Angle of Elevation. — When Fired at Extreme Angles of Elevation and Depression. — The determination of the forces acting on the carriage has been made under the supposition that the gun was fired at zero degrees elevation. The method used is, however, equally applicable to a case with any other angle of elevation. In the latter case the force of the powder gases will have a vertical as well as a horizontal component, but the value of each component will be known and no additional unknown quantities will be introduced into the equations. x'_r and y'_r, the coordinates of the axis of the pin at the lower end of the elevating arm, will vary with the elevation given to the gun, but their values corresponding to any given elevation can be readily obtained from the drawings of the carriage.

In order that the maximum forces on each part of the gun carriage may be determined for use in so proportioning the various parts that the stresses in them will not be dangerously great, the forces should be computed not only for an elevation of the gun of zero degrees but also for the extreme angles of elevation and depression for which the carriage is designed. For the same reason it is often necessary, particularly in the case of barbette and mobile artillery carriages, to compute the forces corresponding to different positions of the gun in recoil, generally its positions in and from battery.

58. Maximum Values of the Forces on a Disappearing Carriage during Recoil. — Velocity and Acceleration Curves of the Recoiling Parts. — In the case of a disappearing carriage of the service type the peculiar constraint of the moving parts during recoil is such that the maximum accelerations of some of the parts occur somewhat later than the instant of maximum powder pressure. However, by plotting the velocity curves of the various parts from the relations established in equations (14) to (42), inclusive, it is seen that the maximum accelerations of all parts occur at times sufficiently near that of the maximum powder pressure to warrant the assumption that the values of the forces computed for that instant vary but little from their maximum values.

In plotting the velocity curves referred to, the velocity of recoil of the top carriage is first measured by a Sebert velocimeter and may then be plotted both as a function of time and of space. For any measured velocity the corresponding coordinates x_c and y_c of the center of mass of the top carriage are known and from them may be obtained by either of equations (16) or (21) the corresponding value of ϕ. The corresponding values of the coordinates of the centers of mass of the other moving parts, and the values of the angles ψ and θ, can then be obtained from equations (14) to (27), inclusive. The corresponding velocities, linear and angular, of the other moving parts can now be determined from the measured velocity of the top carriage and equations (28) to (42), inclusive. As the velocity of the top carriage is measured both as a function of time and of space, the velocity curves of the other moving parts may also be plotted as functions either of time or of space. The accelerations are obtained by measuring the angles which the tangents to the velocity curves as a function of time make with the axis of time. The tangents of these angles give the required accelerations, which may then be plotted either as functions of time or of space.

Having plotted an acceleration curve of one moving part, the acceleration curves of the other moving parts may, if desired, be plotted from the data given by the plotted acceleration curve and its corresponding velocity curves as a function of time and of space, and the relations established in equations (14) to (57), inclusive.

If it is desired to determine the acceleration curves of the various moving parts before the carriage is constructed, it may be done in the manner to be described later in article 66.

59. Computation of the Values of the Forces at Any Instant While the Projectile is in the Bore. — If we wish to compute the values of the forces on the parts of the carriage at any time while the projectile is in the bore other than the time of maximum powder pressure, the method to be followed is similar to that described for the instant of maximum powder pressure. For example, suppose it is desired to compute the values of the forces at the instant the projectile leaves the bore. The value of R, the constant resistance of the recoil cylinder, will be equal to 37312 lbs. as before. The value of the force F at this instant can be obtained from the curve of pressures while the projectile is in the bore plotted by the methods of interior ballistics. The velocity of the gun in restrained recoil can be determined very closely as before from a consideration of the data given by Table 2, and the value of ϕ may be obtained from those data and equation (14). The values of the angles ψ and θ and of the coordinates of the centers of mass at the instant the projectile leaves the bore can then be obtained from equations (14) to (27), inclusive. The value of $d\phi/dt$ can be found from the velocity of the gun in restrained recoil selected after consideration of Table 2, and equations (28) and (33). The values of $d\psi/dt$ and $d\theta/dt$ then follow from equations (41) and (42), respectively, and the values of the accelerations in terms of $d^2\phi/dt^2$ from equations (43) to (57), inclusive.

By substituting these data together with the values of the constants from Table 3 in equations (1) to (12), inclusive, they may be readily solved as before giving the value of $\frac{d^2\phi}{dt^2}$ and the values of the forces P to P_{10} at the instant the projectile leaves the bore.

60. Computation of the Values of the Forces at any Instant after the Powder Gases have Ceased to Act on the Gun. — After the powder gases have ceased to act on the gun the angular velocity $d\phi/dt$ corresponding to any assumed value of ϕ can be obtained from equation (13), or more readily from equation (13″) as explained later in article 64. The values of ψ, θ, $d\psi/dt$, $d\theta/dt$, of the coordinates of the centers of mass, and of the accelerations in terms of $d^2\phi/dt^2$, can then be determined in the manner already explained. The value of R is 37312 lbs. as before.

Substituting these data and the values of the constants from Table 3 in equations (1) to (12), inclusive, and making $F = 0$ since the powder gases have ceased to act on the gun, the equations may be solved giving the values of $d^2\phi/dt^2$ and the forces P to P_{10} corresponding to the assumed value of ϕ. A negative value for $d^2\phi/dt^2$ obtained in the solution of equations (1) to (12) indicates that the rotation of the gun levers is being retarded.

61. Centripetal Accelerations. — It will be noted that each expression for the value of the different accelerations in terms of the angular acceleration $d^2\phi/dt^2$, equations (43) to (57), inclusive, contains at least one term in which the square of an angular velocity occurs. This is due to the rotary motion of the parts and the consequent centripetal accelerations along the radii equal to $V^2/\rho = \rho\omega^2$ in which ρ is the radial distance of the center of mass of the body from the center about which it is rotating, V is the linear velocity of the center of mass, and ω the angular velocity of the rotating body. The components of the centripetal accelerations in the directions of the axes of X and Y must enter in the expressions for the various linear accelerations in those directions, and due to the relations between the variable angles the expressions for the angular accelerations $d^2\theta/dt^2$ and $d^2\psi/dt^2$ will also contain terms involving the squares of the angular velocities.

As a general rule, however, when forces whose duration is very short, like those of the powder gases in a gun, are considered, the accelerations produced by them will be relatively very great as compared with the velocities because of the very short period of time during which the forces act. Referring to the expressions for the various accelerations given on pages 87 and 88, the numerical terms represent the values of the terms into which the squares of the angular velocities entered; and as the angular acceleration $d^2\phi/dt^2$ has been determined to be 224.08 radians per sec. per sec., it follows that the terms containing the squares of the angular velocities are of relatively little importance as regards their effect on the computed values of the forces. On this account and because of the labor involved in considering them, it is customary to neglect such terms in computing the forces brought upon a gun carriage when the gun is fired. Had this been done in the case under consideration, it would not have been necessary to determine any of the angular velocities of the parts at the instant of maximum powder pressure, and the last terms of the acceleration equations on pages 87 and 88 would not appear.

62. Effect of the Movement of the Parts on the Intensities of the Forces. — It is well to consider the effect of permitting movement of the parts of the gun carriage when the gun is fired. As a result of such movement the total force brought upon the upper ends of the gun levers is but 201970 lbs., this being the square root of the sum of the squares of its horizontal and vertical components P and P_1. The total force on the upper end of the elevating arm is 28208 lbs., its horizontal and vertical components being P_2 and P_3, respectively. Were the gun levers rigidly fixed in position the whole force of the powder gases, 1017860 lbs., would be brought against their upper ends; but when the parts of the carriage are permitted to move, as in this case, by far the larger part of this force is absorbed in giving acceleration to the gun without causing any stress in the carriage parts.

If it were not for the translation of the center of mass of the gun levers to the rear on the top carriage during recoil of the gun, the gun and counterweight would have about the same acceleration. Comparing the mass of the counterweight, 615.58, with that of the gun, 396.88, it will be seen that if the top carriage were not permitted to move to the rear, more than half of the force exerted by the powder gases would have to be transmitted through the gun levers to the counterweight in order to give it about the acceleration of the gun. The design of the carriage in this case would of course have to be changed for the counterweight would have to move in the arc of a circle instead of in a vertical line as at present. The construction of the carriage, which permits the cen[...]anslate to the rear on the top carriage, materially reduces the acceleration of the counterweight and permits the use of much lighter gun levers than would otherwise be possible. The acceleration of the gun in the horizontal direction at the instant of maximum powder pressure obtained from equation (43′) by making $d^2\phi/dt^2 =$ 224.08 radians per sec. per sec. is 2078.42 ft. per sec. per sec., and its acceleration in the vertical direction obtained from equation (48′) is 202.37 ft. per sec. per sec. The acceleration of the counterweight occurs only in the vertical direction and its value at the instant of maximum powder pressure obtained from equation (51′) is only 243.21 ft. per sec. per sec., or about one-ninth of the horizontal acceleration of the gun.

DETERMINATION OF THE PROFILE OF THE THROTTLING GROOVES IN THE RECOIL CYLINDER.

63. Formula for the Area of Orifice. — Experimental Modification to Allow for the Contraction of the Liquid Vein. — The area of orifice is given by equation (20), page 287, Lissak's Ordnance and Gunnery, which is as follows:

$$a^2 = \gamma A^3 v_r^2 / 2\, gP \tag{58}$$

in which a is the area of orifice in square feet, A is the effective area of the piston in square feet, v_r is the velocity of restrained recoil, P is the total pressure on the piston, g is the acceleration due to gravity, and γ is the weight of a cubic foot of the liquid in the recoil cylinder.

This expression, however, does not take into account the contraction of the liquid vein. From actual measurements of velocities of recoil and the corresponding pressures in the recoil cylinders, made at the Sandy Hook Proving Ground with a Sebert velocimeter and a special form of the ordinary indicator used for indicating the steam pressure corresponding to different positions of the piston in the cylinder of a steam-engine, it has been determined that the value of the area of orifice required to give the desired pressure on the piston is obtained by substituting for the velocity of flow of the liquid through the orifice

$$v_l = v_r A / a \tag{59}$$

given by equation (16), page 287, Lissak's Ordnance and Gunnery, a larger value

$$v_{lc} = bv_l + c = (bv_rA/a) + c \qquad (60)$$

in which b and c are experimental constants that vary with the diameter of the recoil cylinder, the velocity of recoil, and the shape of the orifice.

Following the lines of the discussion on page 287, Lissak's Ordnance and Gunnery, the pressure required to produce a velocity of flow v_{lc} through an orifice is the pressure due to a column of liquid whose height is given by the equation

$$v^2_{lc} = 2\,gh \qquad (61)$$

Substituting for v_{lc} its value from equation (60) and solving for h, we have

$$h = [(bv_rA/a) + c]^2/2\,g \qquad (62)$$

The weight of a cubic foot of the liquid being γ, the weight of a column whose area of cross-section is equal to that of the piston is $A\gamma h$. $A\gamma h$ is, therefore, P, the pressure on the piston, and multiplying both sides of equation (62) by $A\gamma$ we obtain

$$P = A\gamma h = [(bv_rA/a) + c]^2A\gamma/2\,g \qquad (63)$$

Solving equation (63) for a we have

$$a = bv_rA/[\sqrt{2\,gP/A\gamma} - c] \qquad (64)$$

which is the form of equation used in place of equation (58) when it is desired to take into account the contraction of the liquid vein. The values of b and c for the 6-inch disappearing carriage, model of 1905 MI, have been taken as 1.3 and 122, respectively. Making these substitutions in equation (64) it becomes

$$a = 1.3\,v_rA/[\sqrt{2\,gP/A\gamma} - 122] \qquad (65)$$

To express the area of orifice and the area of the piston in square inches divide a and A by 144 and $\sqrt{A}$ by $\sqrt{144}$ obtaining

$$a_{\square''} = 1.3\,v_rA_{\square''}/[\sqrt{288\,gP/A_{\square''}\gamma} - 122] \qquad (66)$$

The density of oil used in the cylinder of the carriage being .85, the weight of a cubic foot of the liquid is

$$\gamma = .85 \times 62.5 = 53.125 \text{ lbs.}$$

The total resistance of the recoil cylinder is 37312 lbs. of which about 1100 lbs. is the friction in the stuffing-box against the piston-rod. The total pressure of the liquid against the piston is, therefore,

$$P = 37312 - 1100 = 36212 \text{ lbs.}$$

The diameter of the piston is 7.23 ins. and that of the piston-rod is 3.25 ins., so that the effective area of the piston is

$$A\square'' = \pi\{(7.23)^2 - (3.25)^2\}/4 = 32.76 \text{ sq. ins.}$$

Substituting these values of γ, P, and A in equation (66) and taking $g = 32.2$, we have

$$a\square'' = \frac{1.3 \times 32.76\, v_r}{\left(\frac{288 \times 32.2 \times 36212}{32.76 \times 53.125}\right)^{\frac{1}{2}} - 122} = .1342\, v_r \qquad (67)$$

The piston-rod of this carriage is stationary, and since the recoil cylinder is attached to the cross-head to which the counterweight is also attached, the cylinder has the same movement as the counterweight, and we may write

$$a\square'' = .1342\, dy_w/dt \qquad (68)$$

64. Profile of the Throttling Grooves. — The areas of the throttling grooves are calculated to correspond to the setting of the throttling valve-stem in the 4th notch which gives an opening through the valve of .17 sq. in. In addition the piston has a clearance in the cylinder of .02 in. on the diameter which gives a constant area for the escape of the liquid around the piston equal to

$$\pi\,[(7.25/2)^2 - (7.23/2)^2] = .2275 \text{ sq. in.}$$

Both of these areas must be subtracted from $a\square''$ to get the area of the throttling grooves. There are two such grooves and they have a uniform width of 1.25 ins., so that their depth d at any point corresponding to a velocity dy_w/dt of the counterweight will be

$$d = \frac{a\square'' - .17 - .2275}{2 \times 1.25} = \frac{.1342\,(dy_w/dt) - .3975}{2.50}$$

or

$$d = .05369\,(dy_w/dt) - .159 \qquad (69)$$

The profile of each throttling groove will be a curve whose ordinates are the values of d given by equation (69) for various values of dy_w/dt, and whose abscissas are the values of y_w corresponding to the same values of dy_w/dt.

The value of y_w for any assumed value of ϕ is given by equation (22). The value of $d\phi/dt$ for any assumed value of ϕ equal to or greater than 21°, the value of ϕ corresponding to the instant when the powder gases cease to act on the gun, can be obtained from equation (13), and with this value of $d\phi/dt$ the corresponding value of dy_w/dt can be obtained from equation (36). Substituting in equation (13) the values of the linear and angular velocities in terms of $d\phi/dt$ from equations (28) to (42), inclusive, solving for $d\phi/dt$, and representing, for the sake of simplicity of expression, the coefficients of $d\phi/dt$ in equations (41) and (42) by X and U, respectively, we obtain

$$\frac{d\phi}{dt} = \frac{\begin{array}{l}[R\,(y''_w - y_w) + W_w\,(y''_w - y_w) + W_a\,(y''_a - y_a) \\ \quad + W_c\,(y''_c - y_c) - W_g\,(y_g - y''_g) - W_e(y_e - y''_e)]^{\frac{1}{2}}\end{array}}{\left[\begin{array}{l}\frac{1}{2}M_g\,[a\cos\phi + c\cos(\phi-\beta)]^2 + \frac{1}{2}M\ \ [a\tan\alpha\cos\phi \\ \quad - c\sin(\phi-\beta)]^2 + \frac{1}{2}\Sigma mr^2_g X^2 + \frac{1}{2}M_a[a\cos\phi]^2 \\ \quad + \frac{1}{2}M_a\,[a\tan\alpha\cos\phi]^2 + \frac{1}{2}\Sigma mr^2_a + \frac{1}{2}M_c\,[a\cos\phi]^2 \\ \quad + \frac{1}{2}M_c\,[a\tan\alpha\cos\phi]^2 + \frac{1}{2}M_e\,[l\cos\theta\,U]^2 \\ \quad + \frac{1}{2}M_e\,[l\sin\theta U]^2 + \frac{1}{2}\Sigma mr^2_e U^2 \\ \quad + \frac{1}{2}M_w\,[a\tan\alpha\cos\phi + a\sin\phi]^2\end{array}\right]^{\frac{1}{2}}} \qquad (13')$$

in which

$$X = \frac{\sin\theta\,[a\cos\phi + c\cos(\phi-\beta)] + \cos\theta\,[a\tan\alpha\cos\phi - c\sin(\phi-\beta)]}{d\,[\sin\theta\sin\psi + \cos\theta\cos\psi]}$$

and

$$U = \frac{a\cos\phi + c\cos(\phi-\beta) - d\sin\psi X}{b\cos\theta}$$

Substituting in equation (13′) the known values of the resistance R, of the masses, moments of inertia, and the ordinates of the centers of mass when the parts are in the recoiled position, it becomes

$$\frac{d\phi}{dt} = \frac{\begin{array}{l}[333719 - 57109\,y_w - 6041\,y_a - 2528\,y_c - 12764\,y_g \\ \quad -707\,y_e]^{\frac{1}{2}}\end{array}}{\left[\begin{array}{l}198.44\,[a\cos\phi + c\cos(\phi-\beta)]^2 + 198.44\,[a\tan\alpha\cos\phi \\ \quad - c\sin(\phi-\beta)]^2 + 7483.5\,X^2 + 93.92\,[a\cos\phi]^2 \\ \quad + 93.92\,[a\tan\alpha\cos\phi]^2 + 625.2 + 39.305\,[a\cos\phi]^2 \\ \quad + 39.305\,[a\tan\alpha\cos\phi]^2 + 10.992\,[l\cos\theta U]^2 \\ \quad + 10.992\,[l\sin\theta U]^2 + 77.54\,U^2 \\ \quad + 307.79\,[a\tan\alpha\cos\phi + a\sin\phi]^2\end{array}\right]^{\frac{1}{2}}} \qquad (13'')$$

The first step in the solution of equation (13″) to determine the value of $d\phi/dt$ corresponding to any assumed value of ϕ, is to calculate ψ and θ from equations (27) and (26), respectively, and from them and the assumed value of ϕ, the corresponding values of X and U. With ϕ, ψ, θ, X, and U known, the value of the denominator of equation (13″) may be determined. The values of y_w, y_a, y_c, y_g, and y_e may then be obtained from equations (22), (20), (21), (19), and (23), respectively, and the value of the numerator of equation (13″) determined. The value of $d\phi/dt$ follows. In this manner the values of $d\phi/dt$ for $\phi = 30°$, 40°, 50°, 60°, 70°, 80°, and 85° have been calculated. These values, together with the value of $d\phi/dt$ for 21° already determined, and the corresponding values of ψ, θ, y_w,* dy_w/dt, and d, equation (69), are shown in the following table, in which are also shown for comparison with dy_w/dt the corresponding velocities v_c of the top carriage determined by obtaining dx_a/dt from equation (29) and dividing it by cos α.

TABLE 4.

ϕ	ψ	θ	y_w inches.	$d\phi/dt$ radians per sec.	dy_w/dt (v_w) ft. per sec.	d inches.	v_c ft. per sec.
21°	−0° 7′.3	11° 26′.3	2.31	2.8788	4.741	0.096	11.645
30°	+0° 43′.8	20° 41′.5	5.97	2.8295	6.378	0.183	10.621
40°	+2° 37′.5	30° 38′.8	11.34	2.7213	7.790	0.259	9.036
50°	+5° 4′.2	40° 6′.7	17.90	2.5915	8.770	0.312	7.220
60°	+7° 25′.5	48° 50′.7	25.44	2.3927	9.100	0.330	5.126
70°	+8° 50′.5	56° 30′.3	33.75	2.0592	8.456	0.295	3.053
80°	+8° 6′.7	62° 34′	42.56	1.4214	6.091	0.168	1.070
85°	+6° 34′.4	64° 40′	47.07	0.7326	3.1692	0.111	0.2767
88°	+5°	65° 31′.4	49.79	0	0	0	0

As the profile of the throttling groove is a smooth curve the number of points given in Table 4 will ordinarily be sufficient to enable it to be plotted from the abscissa $y_w = 2.31$ ins. to the

* For convenience in discussing the profile of the throttling groove, y_w will hereafter be referred to the center of mass of the counterweight when the gun is in battery instead of to the center of coordinates heretofore assumed. It will also be expressed in inches. Its values under these conditions are obtained by adding the value of $h = 3$ ft. to the values of y_w obtained from equation (22) and multiplying the results by twelve to reduce feet to inches.

end, although after plotting it from the table it may be found desirable to calculate one or two more points on the curve to determine with certainty the location of its maximum ordinate and the point at which it ends. On account of the subtractive term in equation (69) the throttling groove does not commence at the point where y_w is zero and does not extend to the point where y_w is 49.79 ins. From equation (69), d will be equal to zero when dy_w/dt is equal to $.159/.05369 = 2.961$ f. s. Until dy_w/dt attains this value the groove will not commence and it will cease as soon as dy_w/dt decreases to this value after having reached its maximum. The point where the groove ends becomes apparent at once when its profile is plotted to a large scale with a sufficient number of calculated points. The point where it commences is determined as described in the next paragraph.

Equations (13), (13′), and (13″) are not applicable to values of ϕ less than 21° for then the powder gases are still acting on the gun. $d\phi/dt$ can not, therefore, be obtained from equation (13″) for values of y_w less than 2.31 ins. Owing to the rapidly changing force exerted by the powder gases on the gun and to the impracticability of deducing a simple equation expressing the value of the force as a function of time or space, it is not practicable to obtain by analytical methods reliable values for dy_w/dt while the powder gases are acting, although these values may be obtained approximately from the curve of free recoil of the gun as a function of space. In the case of those barbette and mobile artillery carriages in which the resistance to recoil is constant and applied in the direction of the axis of the gun, the curve showing the velocity of restrained recoil may be accurately determined by graphical methods as described in Par. 166, pages 283 and 284, Lissak's Ordnance and Gunnery. The general form of the profile of the throttling grooves for such carriages is well known and it is of the same character beyond the point where the powder gases cease to act as the profile of the throttling groove of the disappearing carriage. It is a reasonable assumption, therefore, that the part of the profile of the throttling groove which corresponds to the position of the parts while the powder gases are acting is also similar in the two cases. For values of y_w less than 2.31 ins., therefore, the profile of the throttling groove of the 6-inch disappearing carriage, model of 1905 MI, is obtained by prolonging the curve already plotted back to the origin and giving to it the ordinary form.

Each new type or model of carriage is tested at the Sandy Hook Proving Ground, and any necessary changes in its design and in the areas of the throttling orifices are determined there. These changes are then applied to all carriages of that model before they are issued to the service.

Fig. 34 shows the complete profile of the throttling groove for this carriage as it would be furnished to the shops for cutting the grooves in the recoil cylinder.

It will be observed on examining this figure that above the word "PISTON" the curve is straight for a distance of 2.25 ins., its ordinate there being the maximum ordinate. This arrangement is necessary because the piston is 2.25 ins. long and the smallest opening between it and the wall at the bottom of the groove, or the controlling orifice, is at the upper edge of the piston until the maximum ordinate of the groove is reached during recoil. On account of the length of the cylindrical piston the maximum orifice could not be obtained if the straight part of the profile of the groove did not exist, for before the upper edge of the piston reached the maximum ordinate the lower edge would have passed it and the opening at the lower edge would have become the controlling orifice. The straight part of the profile permits the orifice at the upper edge of the piston to be the controlling orifice until it reaches the maximum ordinate, after which the orifice at the lower edge of the piston becomes the controlling orifice until recoil ends.

65. Velocities of the Counterweight and the Top Carriage Compared. — Advantages of the Recoil Cylinder in the Counterweight and of the Counter-Recoil Buffer Acting Against the Top Carriage. — The velocities of the top carriage shown in the last column of Table 4 have been calculated for comparison with those of the counterweight, and to illustrate by an example taken from a service carriage the statements made in Par. 200, page 346, Lissak's Ordnance and Gunnery, as to the reasons for changing the position of the recoil cylinders of the disappearing carriage from the top carriage to the counterweight while retaining the action of the counter-recoil buffer on the top carriage. By examination of the table it will be seen that the velocity of the top

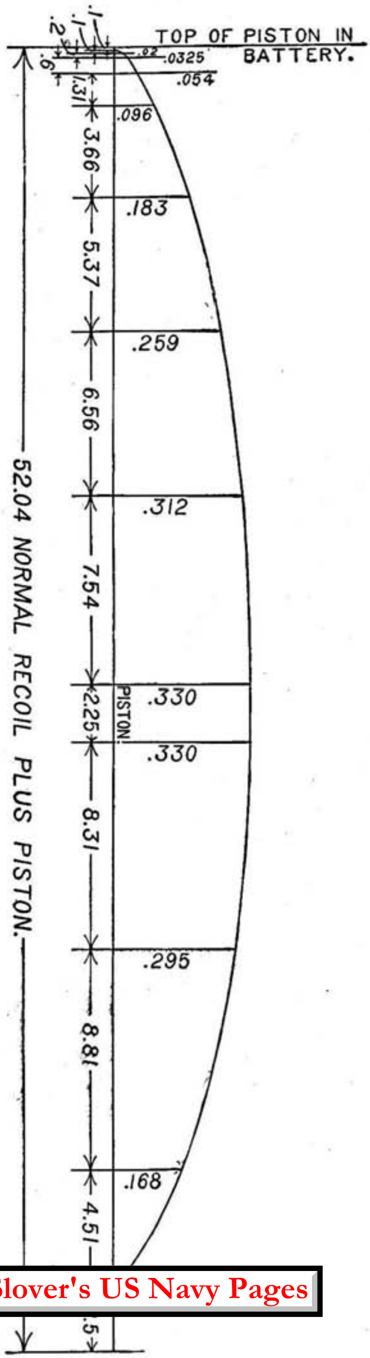
TOP OF PISTON IN BATTERY.
.02
.0325
.054
.096
.183
.259
.312
.330
.330
.295
.168
3.66
5.37
6.56
7.54
2.25
8.31
8.81
4.51
1.31
PISTON
52.04 NORMAL RECOIL PLUS PISTON.

Fig. 34.

carriage toward the end of recoil is much less than that of the counterweight, so that a recoil cylinder in the counterweight gives better control of the final movement of the gun in recoil than does one placed in the top carriage. As the loading angle and the height of the breech of the gun are greatly affected by a comparatively small change in the total amount of recoil, it is important that this variation be as small as possible.

The relations between the velocities of the counterweight and top carriage at any point are the same in counter-recoil as in recoil, so that when the gun is almost in battery during counter-recoil the velocity of the top carriage is much greater than that of the counterweight and a counter-recoil buffer acting against the top carriage gives better results than would one acting on the counterweight.

66. Velocity and Acceleration Curves of the Recoiling Parts.— Article 58 gives a description of the method of plotting the velocity and acceleration curves of the recoiling parts of a disappearing carriage based upon the measurement of the velocities of the top carriage by the Sebert velocimeter. If these curves are desired before the carriage is manufactured they can be obtained from the velocities of restrained recoil used in the calculations of the profile of the throttling grooves. For example, Table 4 gives the velocity of recoil of the counterweight as a function of space corresponding to a number of positions of the counterweight in recoil after the powder gases have ceased to act on the gun. The velocity of the counterweight corresponding to a number of its positions while the powder gases are acting may also be obtained by substituting in equation (69) the values of d scaled from the early part of the profile of the throttling groove. By plotting the reciprocals of the velocities of the counterweight as a function of space and integrating under the curve up to any ordinate, the area will represent the time corresponding to the space over which the counterweight has recoiled, these operations being the same as those for obtaining the curve of the velocity of the projectile in the bore as a function of time. Having the times corresponding to the various distances recoiled by the counterweight, its velocity of recoil may be plotted as a function of time as well as of space. The velocity of recoil of each of the other moving parts and the corresponding accelerations of all the moving parts may now be plotted as a function either of time or space in a manner similar to that described in article 58.

In this way we can obtain, before the carriage is built, the total time of recoil, and the position, velocity, and acceleration of each of the recoiling parts of a disappearing carriage corresponding to any interval of time measured from the instant when recoil began.

67. Method of Designing a Gun Carriage. — The forces on the various parts of a gun carriage having been determined, it is then necessary to compute the stresses caused by them in the parts and to so proportion the parts that the stresses will be kept within permissible limits. The method of doing this will be explained in Chapter IV. From what has preceded it is apparent that the masses, and therefore the weights, and the distribution of the mass of each part, represented by its moment of inertia, affect to a considerable degree the forces brought upon the parts when the gun is fired; while on the other hand, the intensities of the forces govern in most cases the sizes and distribution of the mass of the parts. It is evident, therefore, that the design of a gun carriage, as of any other machine where the strength of the parts must be considered, is a matter of trial in which the actual amount of work involved is largely a matter of the skill of the designer and his experience with the type of gun carriage or machine being designed. The general outline of the carriage is first laid down on the drawing and the dimensions of the parts determined approximately by experience and judgment. The weights of the parts are then calculated from the drawing by obtaining their volumes and multiplying them by the weights of a unit volume of the various materials used. The centers of mass of the parts and their moments of inertia are calculated in accordance with the principles given in works on mechanics. (The method of calculating centers of mass and moments of inertia of parts from a drawing will be illustrated later in Chapter IV.)

The forces on the parts are next calculated and then the stresses caused by them. It will generally happen that the stresses in some of the parts are too great for safety and that in other parts they are smaller than they need be. Those parts in which the stresses are too great must be increased in size or changed in shape in such manner as to decrease the stresses to a reasonable figure. If it is important that the weight of the carriage be kept as small as possible, as is the case with all mobile artillery carriages, those parts in which the stresses are smaller than they need be must be decreased in size and weight until the stresses are raised to the maximum permissible limits. If weight is not important and the unnecessarily large size of the parts is not objectionable otherwise, the parts may not be changed.

If the changes in the parts are so considerable as to require a redetermination of the weights, centers of mass, moments of inertia, and finally of the forces on the parts, this must be done; and the stresses on the parts due to the changed forces must again be determined. These processes are repeated until the sizes, weights, and shapes of the parts are considered satisfactory.

CHAPTER IV.

STRESSES IN PARTS OF GUN CARRIAGES.

68. Strength of Materials. — The determination of the stresses in parts of gun carriages requires a knowledge of strength of materials, but as this subject is studied by the cadets of the first class in the course of civil and military engineering the principles involved will not be deduced here. The most important facts and formulas will, however, be stated and briefly explained.

69. Stresses of Tension and Compression. — Let AB, Fig. 35, be a rod fixed at the end A. If a force T acts upon it in the direction of its axis to elongate it as shown, the force will produce

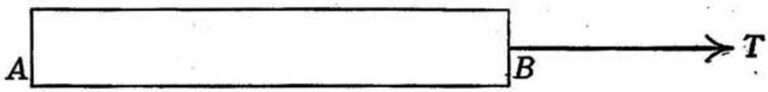

Fig. 35.

a stress of tension in the rod which will be distributed uniformly over each cross-section between B and A, the intensity of the stress per unit of area being equal to the total force divided by the area of cross-section considered.

If the direction of the force be reversed it will compress the rod producing in it a stress of compression. This stress also will be distributed uniformly over each cross-section between B and A and its intensity per unit of area will equal the total force divided by the area of cross-section considered.

If the force T be not applied at the end of the rod but at some section nearer A it will cause a stress in the sections between it and A but none in those between it and the free end B. If a number of forces act instead of one force T each will produce a stress in every section affected by it and the algrebaic sum of these stresses in any section will be the total stress in the section.

In general, if a rod is in equilibrium under the action of a number of forces, all those that are parallel to its axis will cause either tension or compression in the rod. If any section of the rod be considered the tension or compression in it will be due to all of the forces acting on one side (either side) of that section; and if on either side there are no forces acting there will be no stress in the section. All forces acting toward a section produce compression therein and all those acting away from it produce tension therein.

70. Shearing Stress. — If two flat plates are laid one upon the other and fastened together by rivets any force applied to the plates to slide one along the other will be a simple shearing force as applied to the rivets. This force will be resisted by a shearing stress in the cross-sections of the rivets lying in the plane of contact of the plates. The shearing stress will be distributed uniformly over the cross-sections of the rivets and its intensity per unit of area will be equal to the total shearing force divided by the total area of cross-section.

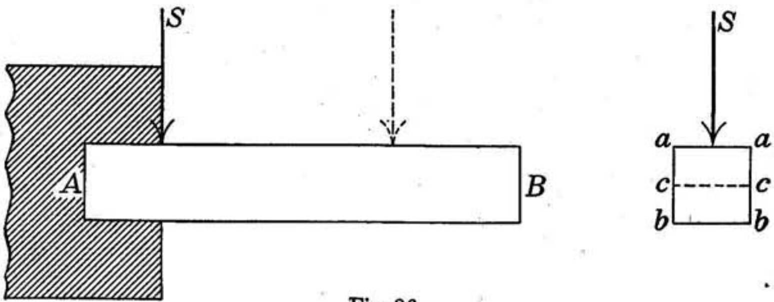

Fig. 36.

If a solid bar be considered instead of the plates the same force would tend to slide one part past the other in the same manner as if they were separate parts fastened together by rivets as before, and the stress developed between them would be one of simple shear.

Let *AB*, Fig. 36, be a block imbedded in a wall and *S* a force applied to the block through the center of the section next the wall as shown. The force will tend to slide that portion of the block projecting from the wall past the portion imbedded therein and will produce in the section next the wall a shearing stress uniformly distributed over it, whose intensity per unit of area will equal the total force divided by the area of the section.

If the force S be not applied next the wall but at a distance from it as shown by the dotted force line it will produce a shearing stress in every section between it and the wall but none in the sections between it and the free end of the block. It will also produce a bending stress in the sections between it and the wall which will be discussed later. In this and every other case of a shearing stress caused by a force that also produces a bending stress, the shearing stress is not distributed uniformly over the section but varies from a maximum at the axis of the section (called the neutral axis) which is perpendicular to the force to zero at the points or lines of the section most distant from the neutral axis. Thus in Fig. 36 *aabb* is the section next the wall. The shearing stress therein due to the force next the wall is uniformly distributed, but that due to the force shown by the dotted line varies from a maximum at the line *cc* to zero at the lines *aa* and *bb*. It is ordinarily assumed that a shearing stress, however caused, is uniformly distributed over the section since when caused by a force that also produces a bending stress it is generally much smaller than the latter.

If a number of shearing forces act on a piece, each will produce a shearing stress in every section affected by it and the algebraic sum of these stresses in any section will be the total shearing stress in the section.

In general, if a piece is in equilibrium under the action of a number of forces, all those that are perpendicular to its axis will cause shear in the piece, and the total shearing stress in any section will be due to all the forces perpendicular to the axis of the piece acting on one side (either side) of that section. If on either side there are no forces acting there will be no stress in the section.

TORSIONAL STRESS.

71. Rod Fixed at One End. — Let AB, Fig. 37, be a rod fixed at the end A* and acted on at the free end by a force F perpendicular to its axis as shown. If this force intersected the axis of the rod it would produce in the rod only shearing and bending

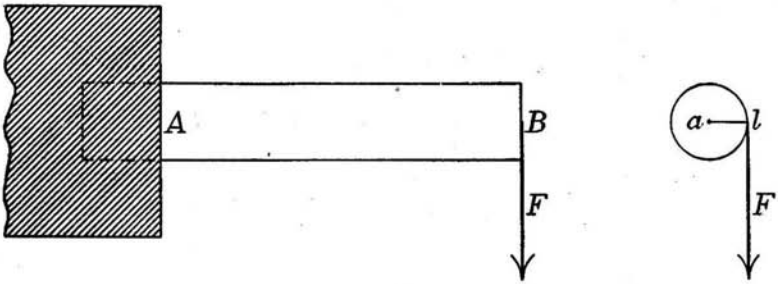

Fig. 37.

stresses as already explained, but having a lever arm with respect to the axis, it twists the rod producing in it a torsional stress also. Shearing stresses have already been considered and bending stresses will be taken up later, so that the twisting or torsional stress in the rod will alone be discussed at present.

The torsional moment of the force at any section between it and the fixed end of the rod is

$$M_t = F \times al \tag{1}$$

in which M_t is the torsional moment, F is the intensity of the force, and al is its lever arm in inches with respect to the axis of the rod. The torsional stress will be proportional to the torsional

* In order that a rod or beam may be fixed at one or both ends a portion of its length at one or both ends must be clamped in some manner that will prevent all movement of the portions clamped. The rod may be clamped by having its end or ends embedded in a wall or by any other suitable means. When the length of a rod that is fixed at one or both ends is referred to, it is to be understood as meaning the length of that portion which is not clamped; and when the fixed end of a rod fixed at one end only is referred to, it is to be understood as referring to the section of the rod at the end of the clamp next the unclamped portion of the rod. Similarly, when the ends of a rod that is fixed at both ends are referred to, it is to be understood as meaning the sections at the end of both clamps next the unclamped portion of the rod.

moment but its intensity on any elementary area of a cross-section will vary directly with the distance of that area from the axis. If a number of twisting forces act on the rod each will produce a torsional moment at every section affected by it and the algebraic sum of these moments at any section will be the total torsional moment at the section. If there is no twisting force at the free end of the rod there will be no torsional moment at any section between the free end and that section whose plane contains the twisting force nearest to the free end.

Rod Fixed at Both Ends.* — In Fig. 37 the rod is fixed at one end only; if fixed at both ends as shown in Fig. 38 the force F will produce a torsional moment on each side of the section in whose plane it is contained but the moment will not be equal to

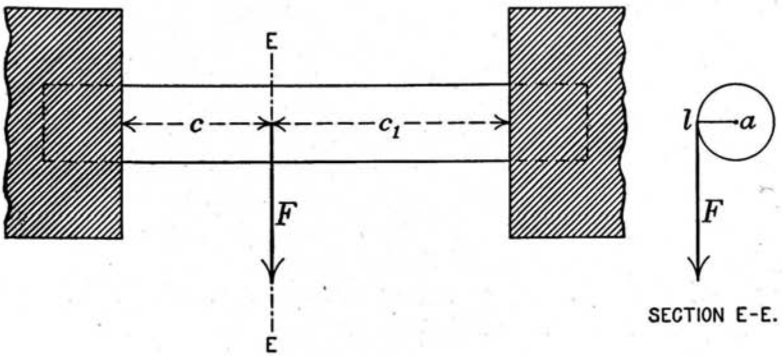

Fig. 38.

$F \times al$ and it will be different on the two sides of the section. If c and c_1 represent the distances of this section from the left and right ends, respectively, of the rod, the torsional moment at any section within the portion c will be

$$M_t = F \times al \times \frac{c_1}{c + c_1} \qquad (2)†$$

and that at any section within the portion c_1 will be

$$M_t = F \times al \times \frac{c}{c + c_1} \qquad (3)†$$

* See footnote, page 109.

† From Reuleaux's Constructor.

the larger of the two moments pertaining to the section whose plane contains the force.

General Case of Torsional Moments. — When a rod is in equilibrium under the forces acting on it and all the twisting forces and couples either to the left or to the right of a section are known, the torsional moment at that section is the algebraic sum of the individual moments with respect to the axis of the rod of each of the twisting forces and couples on either side of the section. The reactions between the rod and its supports must be included among the twisting forces and couples on either side of the section, and when this can be done it is unnecessary to consider whether the rod is fixed at one or both ends. If on either side of the section there are no twisting forces or couples there is no torsional moment at the section.

Intensity of Torsional Stress. — Let I_p be the polar moment of inertia of a cross-section of the rod, M_t the torsional moment at the section, and S'''_t the torsional stress per unit of area at a unit's distance from the axis; then, as shown on pages 38 and 39, Fiebeger's Civil Engineering,

$$S'''_t = M_t / I_p$$

and the stress per unit of area at any distance r from the axis will be

$$rS'''_t = M_t r / I_p \qquad (4)$$

The maximum stress will evidently occur at that point of the section most distant from the axis.

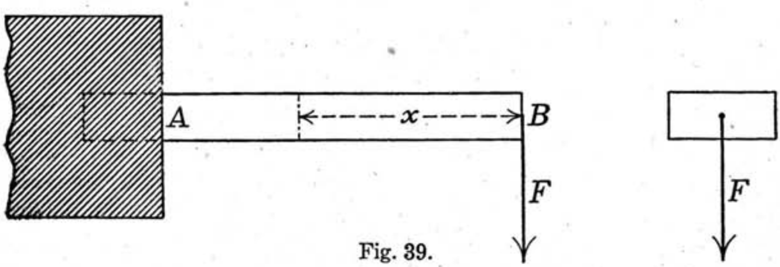

Fig. 39.

BENDING STRESS.

72. Cantilever Subjected to a Concentrated Bending Force. — Let AB, Fig. 39, be a rod or beam fixed at the end A* and acted

* See footnote, page 109.

on at the free end by a force F whose action line is perpendicular to and intersects the axis of the beam. Such a force has no torsional moment at any cross-section of the beam but produces in each section a shearing and a bending stress. The bending stress only will be considered. The force F bends the beam and rotates each section about its axis perpendicular to the action line of the force, the amount of this rotation varying directly as the moment of the force with respect to the section. This moment is called the bending moment of the force and is equal to

$$M = Fx \qquad (5)^*$$

in which M is the bending moment, F is the intensity of the force, and x the perpendicular distance in inches between it and the section under consideration.

The rotation of the sections causes the fibres on top to be extended producing in them a stress of tension, and those on the bottom to be compressed producing in them a stress of compression. If the force acted upward the upper fibres of the beam would be compressed and the lower fibres extended. The fibres intersecting the axes about which the sections rotate, called the neutral axes, are neither extended nor compressed, and the tension or compression in any other fibre at any section is directly proportional to the bending moment at that section and to the distance of the fibre from the neutral axis. A bending stress, therefore, is one in which the fibres on one side of the neutral axis are subjected to a tensile stress and those on the other side to a compressive stress.

In Fig. 39 the force acts at the free end of the beam and every section between it and the fixed end is subjected to a bending moment and resulting bending stress, but if the force did not act at the free end there would be no bending moment or bending stress in any section between the force and that end. If a number of bending forces act on the beam, each will produce a bending moment at every section affected by it, and the algebraic sum of all these moments at any section will be the total bending moment at the section.

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* The general expressions for the bending moments given by equations (5) to (23), inclusive, are taken from Merriman's Text Book on the Mechanics of Materials.

A beam such as is shown in Fig. 39 is called a cantilever. The maximum bending moment occurs at the section at the fixed end.

Intensity of Bending Stress. — Let I be the moment of inertia of a cross-section of a beam with respect to its neutral axis, M the bending moment at the section, and S''' the stress per unit of area at a unit's distance from the neutral axis; then, as shown on pages 49 and 50, Fiebeger's Civil Engineering, the stress per unit of area at any distance y from the neutral axis is

$$S'''y = My/I \qquad (6)$$

Beam Fixed at Both Ends Subjected to a Concentrated Bending Force.* — If both ends of the beam are fixed as in Fig. 40 the

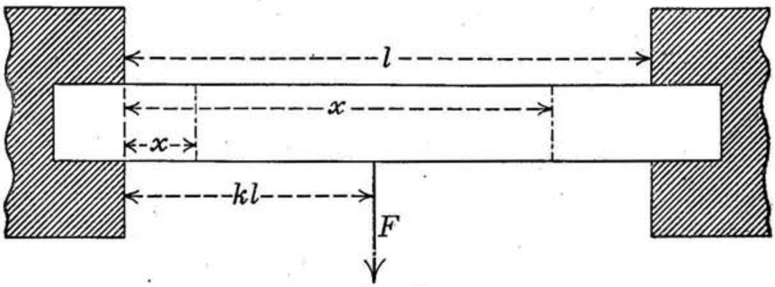


Fig. 40.

force F will produce a bending moment on each side of the section in whose plane it is contained. If l is the length of the beam in inches and kl the distance of the force from the left end, k being a fraction less than unity, the bending moment at any section on the left of the force at a distance x from the left end of the beam is

$$M = -Flk\,(1 - 2\,k + k^2) + F\,(1 - 3\,k^2 + 2\,k^3)\,x \qquad (7)$$

and the bending moment at any section on the right of the force at a distance x from the left end of the beam is

$$M = -Flk\,(1 - 2\,k + k^2) + Fk\,(l - 3\,kx + 2\,k^2x) \qquad (8)$$

If k is $\frac{1}{2}$, that is, if the force is applied at the middle of the beam,

* See footnote, page 109.

the bending moment at any section on the left of the force at a distance x from the left end is

$$M = -\frac{Fl}{8} + \frac{Fx}{2} \tag{9}$$

and at any section on the right of the force at a distance x from the left end, it is

$$M = -\frac{Fl}{8} + \frac{F}{2}(l - x) \tag{10}$$

The maximum bending moment in this case occurs at the ends of the beam and under the force, and is

$$M = \pm Fl/8 \tag{11}$$

If a number of forces parallel to F act on the beam each will cause bending moments in the various sections thereof which may be determined from equations (7) and (8), and the algebraic sum of all those moments at any section will be the total bending moment at the section.

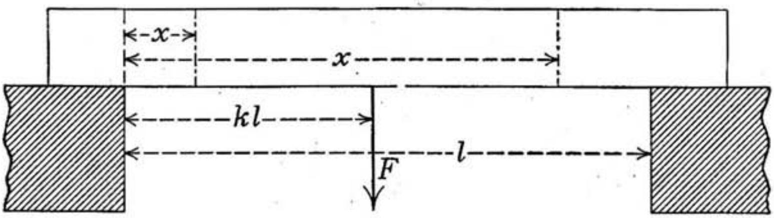


Fig. 41.

Beam Supported at Both Ends Subjected to a Concentrated Bending Force.* — If the ends of the beam are not fixed but are merely supported as shown in Fig. 41, the bending moment at any section on the left of the force at a distance x from the left support is

$$M = F(1 - k)x \tag{12}$$

and at any section on the right of the force at a distance x from the left support, it is

$$M = Fk(l - x) \tag{13}$$

in which l is the length of the beam in inches.

* The length of a beam that is supported at both ends is taken to be the distance between the supports, and the sections of the beam at the supports are referred to as the ends of the beam.

When $x = 0$ or $x = l$, $M = 0$

and, therefore, the bending moment is zero at the ends of a beam that is merely supported. The maximum bending moment occurs under the force and is

$$M = F(1 - k)kl \tag{14}$$

If the force acting at the middle of the beam k is $\frac{1}{2}$ and the bending moment at any section on the left of the force at a distance x from the left support is

$$M = Fx/2 \tag{15}$$

and at any section on the right of the force at a distance x from the left support, it is

$$M = F(l - x)/2 \tag{16}$$

The maximum bending moment occurs under the force and is

$$M = Fl/4 \tag{17}$$

If a number of forces parallel to F act on the beam, each will cause bending moments in the various sections thereof which may be determined from equations (12) and (13); and the algebraic sum of all these bending moments at any section will be the total bending moment at the section.

Cantilever Subjected to a Uniformly Distributed Bending Force. — If the force F shown in Fig. 39 were uniformly distributed over the length of the beam instead of being concentrated at the free end, the bending moment at any section at a distance x from the *free* end of the beam would be

$$M = wx^2/2 \tag{18}$$

in which w is the intensity of the force per linear inch of the beam. The maximum bending moment would occur at the fixed end of the beam and would be

$$M = wl^2/2 \tag{19}$$

l being the length of the beam in inches.

Beam Fixed at Both Ends Subjected to a Uniformly Distributed Bending Force. — If the force in Fig. 40 were uniformly distributed over the length of the beam the bending moment at any section would be

$$M = \frac{wlx}{2} - \frac{wx^2}{2} - \frac{wl^2}{12} \tag{20}$$

in which x is the distance of the section from the left end of the beam, l the length of the beam in inches, and w the intensity of the force per linear inch of the beam. The maximum bending moment would occur at the ends of the beam and would be

$$M = -wl^2/12 \tag{21}$$

Beam Supported at Both Ends Subjected to a Uniformly Distributed Bending Force. — If the force in Fig. 41 were uniformly distributed over the length of the beam between the supports, the bending moment at any section would be

$$M = \frac{wlx}{2} - \frac{wx^2}{2} \tag{22}$$

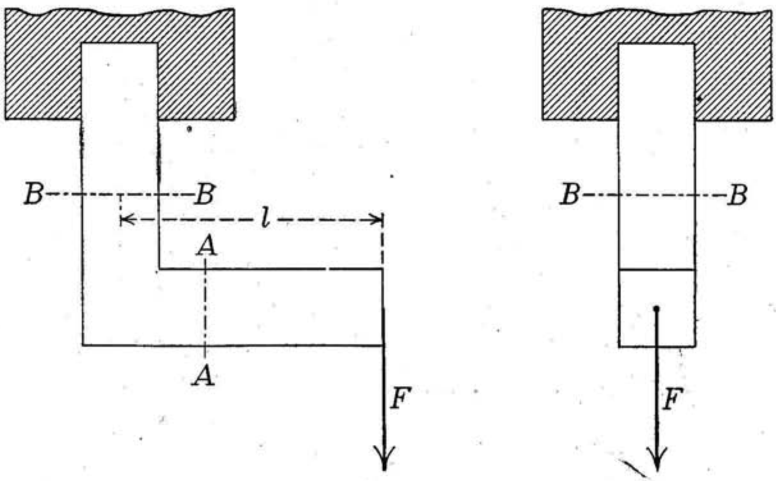


Fig. 42.

in which x, l and w have the same significance as before. The bending moment at the ends of the beam would be zero. The maximum bending moment would occur at the middle of the beam and would be

$$M = wl^2/8 \tag{23}$$

Bending Moment at a Section of a Beam Due to Any Force Having a Lever Arm with Respect to an Axis of the Section. — In the preceding discussion of bending moments the force has always been taken perpendicular to the axis of the beam and parallel to the section; but a bending moment at a section may be caused by any force that has a lever arm with respect to one of the axes of the section. Thus in Fig. 42 the force F not only has a bending moment with respect to any section such as AA in the horizontal portion of the beam but it also has one with respect to any section such as BB in the vertical part. The bending moment with respect to the section BB is Fl where l is the perpendicular distance in inches between the action line of the force and the axis of BB perpendicular to the plane of the paper.* If a number of forces parallel to F act on the beam each having a lever arm with respect to the same axis of section BB, each will cause a bending moment at the section and the algebraic sum of all the bending moments will be the total bending moment at the section.

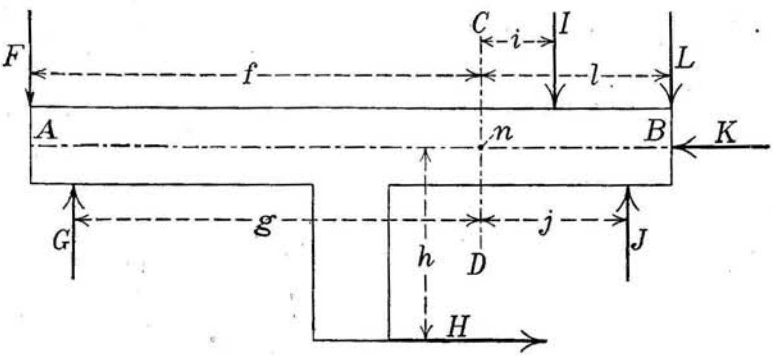


Fig. 43.

General Case of Bending Moments. — When a beam is in equilibrium under the forces acting upon it and all the bending forces either to the left or to the right of a section are known, the total bending moment at that section is obtained by taking the algebraic sum of the individual bending moments at the section due to each of the forces on its left or to each of the forces on its right. The reactions between the beam and its supports must be included among the forces considered and when this can be done it is unnecessary to consider whether the beam is a cantilever, or

* The force F also produces tension in Section BB.

one supported or fixed at both ends. If on either side of the section there are no bending forces there is no bending stress in the section. Thus in Fig. 43 AB is a beam with a projecting vertical part, which is in equilibrium under the action of the forces F, G, H, I, J, K, and L; F, G, and H acting on the left of the section CD and the others on its right. The bending moment at section CD, considering moments that produce compression in the upper fibres as positive, is

$$M = -Ff + Gg - Hh = -Ii + Jj - Ll$$

The force K does not appear in the expression for the bending moment as it intersects the neutral axis of the section, shown projected at n, and, therefore, has no lever arm with respect to it. In the ordinary case the forces F, G, J, L, and K would be the reactions of the supports on the beam.

COMBINED STRESSES.

73. — It fréquently happens in engineering structures that a piece is subjected to tension or compression, shear, bending, and torsion, or to some of these stresses, at the same time. When this is the case each stress is computed as it if were the only one existing in the piece and then all are combined in a manner to be described.

Tension or Compression and Bending. — Since a bending stress is one of tension on one side of the neutral axis of a section and compression on the other side, a stress of tension or compression in a section due to a simple tensile or compressive force is added algebraically to the stress in the section caused by a bending force. Thus if a piece subjected to a bending stress is also subjected to a simple stress of tension, the intensity of the stress in all the fibres on the tension side of the neutral axis due to the bending force will be increased by the intensity of the simple stress of tension; and the intensity of the stress in all the fibres on the compression side of the neutral axis due to the bending force will be decreased by the intensity of the simple stress of tension.

Tension or Compression and Shear. — If a piece be subjected to either tension or compression and shear the resulting stress of tension or compression in any section due to combination with the shear is

$$S_{tc} = \tfrac{1}{2} S_t + \sqrt{S_s^2 + (S_t/2)^2} \qquad (24)^*$$

and the resulting shearing stress due to combination with the tension or compression is

$$S_{sc} = \sqrt{S_s^2 + (S_t/2)^2} \qquad (25)^*$$

in which S_{tc} and S_{sc} are the resulting stresses per unit of area of tension or compression and shear, respectively; S_t is the stress per unit of area of tension or compression computed as if the shearing stress did not exist; and S_s is the shearing stress per unit of area computed as if the stress of tension or compression did not exist. If S_t is tension S_{tc} will be tension; and if S_t is compression S_{tc} will be compression.

Bending and Shear. — Since a bending stress is one causing tension in the fibres on one side of the neutral axis and compression on the other side, the formulas for combining it with a shearing stress are the same as for combining a stress of tension or compression with one of shear. S_t in this case would represent the stress of tension or compression due to the bending force computed as if the shearing stress did not exist.

Ordinarily a bending stress is not combined with a shearing stress due to the same bending force or to a parallel bending force, because such a shearing stress varies from a maximum at the neutral axis, where the bending stress is zero, to zero at the fibre farthest from the neutral axis, where the bending stress is a maximum.

Shear and Torsion. — A torsional stress is essentially one of shear differing from the ordinary shearing stress only in that its intensity per unit of area varies with the distance of the fibre being considered from the axis of the p ece under torsion. A shearing stress distributed uniformly over a cross-section may, therefore, be combined with a torsional stress by simple algebraic addition. If the particular fibre under consideration is subjected to a tors onal stress which acts in a direction opposite to that of the shear, the combined stress is numerically equal to

* From Fiebeger's Civil Engineering.

the difference of the torsional and the shearing stresses computed as if each existed alone, but if at the particular fibre under consideration the stresses act in the same direction the combined stress is numerically equal to their sum.

Tension or Compression and Torsion. — Since a torsional stress is a form of shearing stress the formulas for combining it with a stress of tension or compression are the same as for combining a shearing stress therewith. S_s in this case would represent the torsional stress computed as if the stress of tension or compression did not exist.

Bending and Torsion. — The formulas for combining bending and shear apply here also; but since each stress in this case varies in intensity with the position in the section of the fibre under consideration, care must be taken to see that the stresses combined pertain to the same fibre. The maximum stresses are what it is generally desired to determine and consequently it is customary to obtain first the maximum simple stress S_l due to bending alone, which occurs in that fibre most distant from the neutral axis, and to combine it with the stress S_s due to torsion alone which occurs in the same fibre. Ordinarily the maximum simple stresses S_l and S_s occur at the same fibre but when this is not the case the maximum value of S_s must also be found and combined with the value of S_l at the same fibre to determine which of the combined stresses is the greater. It may also happen that the maximum combined stress S_{lc} or S_{sc} occurs in a fibre in which neither S_l nor S_s is a maximum. No general rule can be laid down to cover a case of this kind but consideration of the shape of the section will generally indicate where the maximum combined stress is likely to occur.

Method of Combining Stresses. — If a piece is subjected to tension or compression, bending, shear, and torsion, at the same time, the stress of tension or compression should be combined by algebraic addition with the bending stress, and the shearing stress combined with the torsional stress in the same way. The resulting stress of tension or compression should then be combined with the resulting shearing stress by equations (24) and (25).

NEUTRAL AXIS. CENTER OF GRAVITY. MOMENT OF INERTIA.

74. Neutral Axis. — When the bending force is parallel to the plane of the section the neutral axis is the straight line through the center of gravity* of the section perpendicular to the force. When the bending force is perpendicular to the plane of the section it tends to bend it about every straight line passing through its center of gravity that does not intersect the action line of the force, prolonged if necessary. Any such line, therefore, becomes the neutral axis of the section when the bending about itself is being considered.

Irregular Sections. — The positions of the centers of gravity of a number of plane figures and the moments of inertia of these figures with respect to various axes passing through their centers of gravity are given in most works on mechanics; but in many or most instances in the design of gun carriage parts the sections upon which bending and torsional stresses occur are of complex shape for which the position of the center of gravity, and the moment of inertia, must be specially determined.

Determination of the Centers of Gravity of Irregular Sections. — The position of the center of gravity of an irregular plane figure may be determined from the principle that the moment of the total area of the figure about any axis must equal the sum of the moments about that axis of the various partial areas into which the figure may be divided. The moment of an area about an axis is the product of the area by the distance between the axis and a line parallel to it passing through the center of gravity of the area. If, therefore, the sum of the moments of the partial areas about any axis is determined the quotient obtained by dividing that sum by the total area will be the distance from the axis to a parallel line passing through the center of gravity of the figure. By repeating these operations with respect to another axis the direction and position of a second line passing through the center of gravity of the figure can be determined. The inter-

* What is here called the center of gravity of a section is really its center of figure, for strictly speaking a surface has no weight. However, as it is convenient to use the term center of gravity for center of figure this practice will be continued throughout the text.

section of the two lines so determined must necessarily be the center of gravity of the figure.

When finding the center of gravity of an irregular plane figure or section in this way, the partial areas into which it is subdivided should be of such regular forms that the area and the position of the center of gravity of each are known or can readily be obtained by the ordinary rules of mensuration. The areas and the positions of the centers of gravity of a number of regular plane figures are given in Table 5, in which will also be found the moment of inertia of each figure about an axis passing through its center of gravity. The last will be needed for determining the moments of inertia of irregular plane sections as will be described later.

Example. — To illustrate the method of determining the centers of gravity of irregular sections, let it be required to determine the center of gravity of the section shown in Fig. 44.

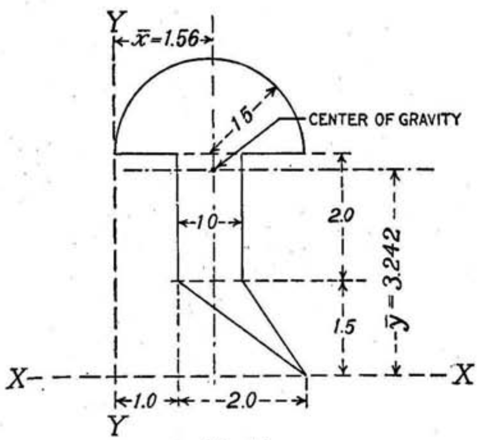
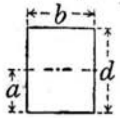
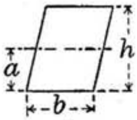
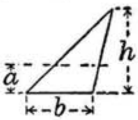
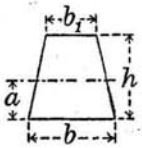
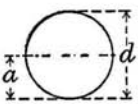
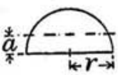
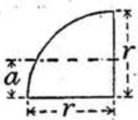
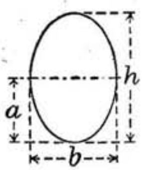
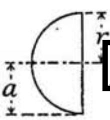


Fig. 44.

Divide the section into a semi-circle, a rectangle, and a triangle by the dotted lines as shown. Draw two axes XX and *YY* perpendicular to each other through the lowest and extreme left-hand points, respectively, of the section. Now if two lines passing through the center of gravity of the section be determined, one of them parallel to *XX* and the other to *YY*, their intersection will give the center of gravity desired. Taking moments

TABLE 5.

In the table the moment of inertia is given for the axis shown by the dotted line passing through the center of gravity of the figure. The distance *a* is measured from the axis to some prominent line or point of the figure.

Figure.		Moment of inertia.	Distance *a*.	Area of figure.
Rectangle		$bd^3/12$	$d/2$	bd
Parallelogram		$bh^3/12$	$h/2$	bh
Triangle		$bh^3/36$	$h/3$	$bh/2$
Trapezoid		$\frac{b^2+4\,bb_1+b_1^2}{36\,(b+b_1)}h^3$	$\frac{b+2\,b_1}{b+b_1}\frac{h}{3}$	$\frac{b+b_1}{2}h$
Circle		$\pi d^4/64 = .0491\,d^4$	$d/2$	$\pi d^2/4 = .7854\,d^2$
Semi-circle		$.110\,r^4$	$.4244\,r$	$\pi r^2/2 = 1.5708\,r^2$
Circular Quadrant		$.055\,r^4$	$.4244\,r$	$\pi r^2/4 = .7854\,r^2$
Ellipse		$\pi bh^3/64 = .0491\,bh^3$	$h/2$	$\pi bh/4 = .7854\,bh$
Semi-circle		[illegible]	[illegible]	$\ldots^2/2 = 1.5708\,r^2$

about XX, and consulting Table 5 for the areas of the elementary figures and the locations of their centers of gravity, we have

$$\left.\begin{array}{l}\dfrac{\pi \times (1.5)^2}{2}(.4244 \times 1.5 + 2 + 1.5) + (1 \times 2)(1+1.5) + \\ \left(1 \times \dfrac{1.5}{2}\right)\left(1.5 - \dfrac{1.5}{3}\right) = \left\{\pi \times \dfrac{(1.5)^2}{2} + 1 \times 2 + 1 \times \dfrac{1.5}{2}\right\}\bar{y}\end{array}\right\} \quad (26)$$

in which $\bar{y}$ is the distance between the axis XX and a line parallel thereto passing through the center of gravity of the section.

From equation (26) $\bar{y} = 3.242$ ins.

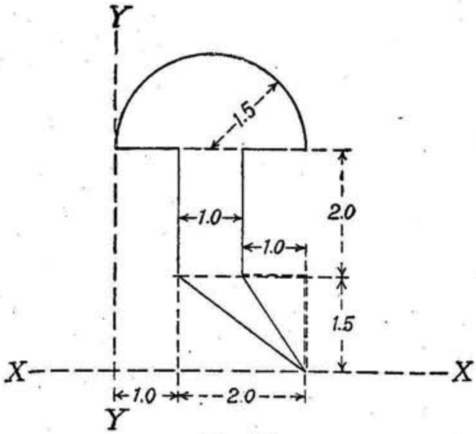


Fig. 45.

Taking moments about YY, and adding the triangle shown in double dotted lines in Fig. 45 to aid in obtaining the moment of the original triangle about this axis, we have

$$\left.\begin{array}{l}\dfrac{\pi \times (1.5)^2}{2} \times 1.5 + (1 \times 2)(.5 + 1) + \left(1.5 \times \dfrac{2}{2}\right)\left(1 + 2 - \dfrac{2}{3}\right) \\ -\left(1.5 \times \dfrac{1}{2}\right)\left(1 + 2 - \dfrac{1}{3}\right) = \left\{\dfrac{\pi \times (1.5)^2}{2} + 1 \times 2 + 1 \times \dfrac{1.5}{2}\right\}\bar{x}\end{array}\right\} \quad (27)$$

in which $\bar{x}$ is the distance between the axis YY and a line parallel thereto passing through the center of gravity of the section.

From equation (27) $\bar{x} = 1.56$ ins.

The coordinates of the center of gravity of the figure with respect to the axes XX and YY are, therefore,

$$\bar{y} = 3.242 \text{ ins.}; \quad \bar{x} = 1.56 \text{ ins.}$$

Determination of the Moments of Inertia of Irregular Sections. — The moment of inertia of an irregular section about any axis in its plane is the sum of the moments of inertia of the various partial areas into which it may be divided about the same axis. Each partial area should be of such regular form that its moment of inertia about an axis passing through its center of gravity is known or can readily be obtained by the methods of the calculus. Having the moment of inertia about an axis passing through the center of gravity of a figure, its moment of inertia about any other axis parallel to the first is obtained by adding the area of the figure multiplied by the square of the distance between the two axes to the moment of inertia of the figure about the axis passing through its center of gravity.

The moments of inertia of a number of plane figures of regular form about axes passing through their centers of gravity are given in Table 5.

Example. — To illustrate the method of determining the moment of inertia of an irregular section let it be required to determine the moment of inertia of the section shown in Fig. 44, (*a*) about the axis through its center of gravity parallel to *XX*, (*b*) about one through its center of gravity parallel to *YY*, and (*c*) about the axis passing through its center of gravity perpendicular to its plane. The moment of inertia about the last axis is called the polar moment of inertia and would be used in determining the torsional stress in the section.

Consulting Table 5 we find that the moment of inertia of the semicircular part of Fig. 44 about an axis parallel to *XX* passing through the center of gravity of the section is

$$.110 \times (1.5)^4 + 1.5708 \times (1.5)^2 \times \{1.5+2+.4244\times 1.5-3.242\}^2 = 3.385 \text{ ins.}^4$$

Similarly, the moment of inertia of the rectangular part about the same axis is

$$\frac{1 \times (2)^3}{12} + \{1 \times 2\}\{3.242 - (1.5 + 1)\}^2 = 1.768 \text{ ins.}^4$$

and the moment of inertia of the triangular part about the same axis is

$$\frac{1 \times (1.5)^3}{36} + \left\{1 \times \frac{1.5}{2}\right\}\left\{3.242 - \left(1.5 - \frac{1.5}{3}\right)\right\}^2 = 3.864 \text{ ins.}^4$$

The moment of inertia of the section about this axis is, therefore,

$$3.385 + 1.768 + 3.864 = 9.017 \text{ ins.}^4$$

The moments of inertia of the partial areas of the section about an axis parallel to YY passing through the center of gravity of the section are:

Of the semicircle,

$$.3937 \times (1.5)^4 + 1.5708 \times (1.5)^2 \times (1.56 - 1.50)^2 = 2.006 \text{ ins.}^4$$

Of the rectangle,

$$\frac{2 \times (1)^3}{12} + \{1 \times 2\}\{1.56 - (1 + .5)\}^2 = .174 \text{ ins.}^4$$

Of the triangle, (see Fig. 44.)

$$\frac{1.5 \times (2)^3}{36} + \left\{2 \times \frac{1.5}{2}\right\}\left\{1 + \left(2 - \frac{2}{3}\right) - 1.56\right\}^2 - \left[\frac{1.5 \times (1)^3}{36} + \left\{1 \times \frac{1.5}{2}\right\}\left\{1 + \left(2 - \frac{1}{3}\right) - 1.56\right\}^2\right] = .270 \text{ ins.}^4$$

The moment of inertia of the section about this axis is, therefore,

$$2.006 + .174 + .270 = 2.45 \text{ ins.}^4$$

Having obtained the moments of inertia of the section about two axes in its plane perpendicular to each other and passing through its center of gravity, the polar moment of inertia I_p may be obtained from the principle that the sum of the moments of inertia of a plane figure about two axes in its plane perpendicular to each other is the moment of inertia of the figure about an axis perpendicular to its plane passing through the point of intersection of the first two axes. Whence

$$I_p = 9.017 + 2.45 = 11.467 \text{ ins.}^4$$

Cubical Contents, Centers of Gravity, and Moments of Inertia, of Irregular Volumes. — The cubical contents and the positions of the centers of gravity of a number of regular volumes, and the moments of inertia of the volumes about axes passing through their centers of gravity, may be found tabulated in most books on mechanics. To determine these data for irregular volumes the procedure is the same as for irregular plane sections. The irregular volume is divided into a number of partial volumes of regular form or approaching some regular form sufficiently closely for the purpose. The cubical contents of the whole is then equal to the sum of the cubical contents of the various partial volumes; and the position of the center of gravity of the whole may be obtained by the principle of moments. The moment of inertia of the irregular volume about any axis is equal to the sum of the moments of inertia about that axis of the various partial volumes into which it is divided.

When the moment of inertia of a volume about an axis passing through its center of gravity is known, its moment of inertia about any other axis parallel to the first is obtained by adding the cubical contents of the volume multiplied by the square of the distance between the two axes to the moment of inertia of the volume about the axis passing through its center of gravity.

Weight and Mass of any Volume Composed of a Given Material. Moment of Inertia of the Mass in any Volume. — The weight of a volume of any material is equal to the cubical contents of the volume multiplied by the weight of a unit volume of the material of which it is composed.

The mass in a volume of any material is equal to the cubical contents of the volume multiplied by the number of units of mass in a unit volume of the material of which it is composed; or the mass in a volume of any material is equal to the weight of the volume divided by the acceleration due to the force of gravity.

The moment of inertia about any axis of the mass in a volume of any material is equal to the moment of inertia of the volume about that axis multiplied by the number of units of mass in a unit volume of the material of which it is composed.

75. Permissible Stresses in Gun Carriage Parts. — Printed specifications are published by the Ordnance Department, U. S. Army, in which are given the elastic limit in tension, the tensile strength, the elongation per unit of length at rupture, and the contraction of area at rupture demanded by the department for the most important materials used for parts of gun carriages. In some materials such as cast iron, copper, and bronze, the elastic limit is not clearly defined in the testing machine and on this account it is omitted in the specifications. Sometimes also the contraction of area at rupture is omitted.

The elastic limit in compression is assumed to be equal to the elastic limit in tension, and the elastic limit in shear is taken as four-fifths of the elastic limit in tension.

When the elastic limit is specified for a given material the dimensions of the part to be made thereof should be so regulated that the stresses developed in it shall not exceed one-half of the elastic limit. When the tensile strength only is specified the dimensions of the part should be such that the stresses therein shall not exceed one-fifth of the tensile strength. The object of prescribing a certain percentage of elongation per unit of length and of contraction of area at rupture for materials used in parts of gun carriages is to prevent their being unduly brittle and, therefore, likely to break under shock.

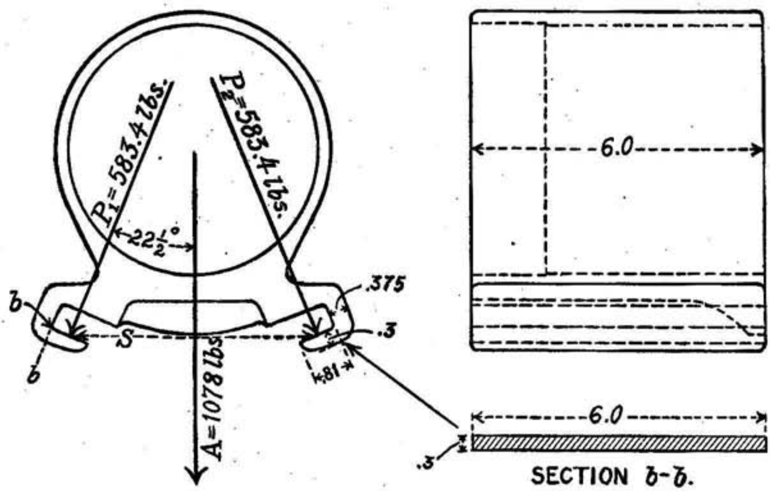


Fig. 46.

CALCULATION OF STRESSES IN PARTS OF GUN CARRIAGES. 3-INCH FIELD GUN.

76. Bending and Shearing Stresses in the Front Clip of the Gun. — The front clip of the gun is shown in Fig. 46. The force A = 1078 lbs., see Figs. 15 and 16, acts vertically downward on the clip. It is the resultant of two equal forces P_1 and P_2 whose action lines are normal to the surfaces S of the clip as shown in the figure. The parts of the clip on which the forces

P_1 and P_2 act make an angle of $22\frac{1}{2}°$ with the horizontal and the intensities of P_1 and P_2 are, therefore,

$$P_1 = P_2 = \frac{1078}{2 \cos 22\frac{1}{2}°} = 583.4 \text{ lbs.}$$

These parts of the clip are .81 in. long and each may be considered as a cantilever over which the force is uniformly distributed, and which tends to break under the action of the force through the section *bb* shown in the figure. The bending moment at this section is, from equation (19),

$$M = \frac{583.4 \times (.81)^2}{.81 \times 2} = 236.3 \text{ in. lbs.}$$

Section *bb* is 6 ins. wide and .3 ins. deep and its neutral axis is midway between the top and the bottom. *I*, the moment of inertia of the section with respect to its neutral axis, is

$$bd^3/12 = 6 \times (.3)^3/12 = .0135 \text{ in.}^4$$

and $$y = .15 \text{ in.}$$

The stress in the fibre most distant from the neutral axis is, therefore, from equation (6),

$$S'''y = 236.3 \times .15/.0135 = 2626 \text{ lbs. per sq. in.}$$

tension at the top of the section and compression at the bottom.

Considering the shearing stress as uniformly distributed over section *bb* its intensity per unit of area is

$$\frac{583.4}{6 \times .3} = 324 \text{ lbs. per sq. in.}$$

The force P_1 or P_2 also causes a combined bending and tensile stress in the part of the clip above section *bb* where the dimension .375 is placed. The bending moment with respect to the section there may be obtained by considering the force as concentrated at the middle of the length of .81 in., whence

$$M = 583.4 \left\{\frac{.81}{2} + \frac{.375}{2}\right\} = 345.7 \text{ in. lbs.}$$

This section is 6 ins. wide and .375 in. deep and its moment of inertia with respect to its neutral axis, half way between the inner and outer edges of the section, is

$$I = 6 \times (.375)^3 / 12 = .0264 \text{ in.}^4$$

The maximum bending stress is, therefore,

$$S'''y = \frac{345.7 \times .375/2}{.0264} = 2455 \text{ lbs. per sq. in.}$$

tension on the inside and compression on the outside of the section.

The tensile stress in the section is

$$\frac{583.4}{6 \times .375} = 259 \text{ lbs. per sq. in.}$$

The maximum stress in the section is, therefore, one of tension which occurs on the inner edge and is equal to

$$2455 + 259 = 2714 \text{ lbs. per sq. in.}$$

So far as the stresses in the clip are concerned its length might safely be reduced to .75 in. since the elastic limit of the material of which it is made is required to be not less than 53000 lbs. per sq. in. But when one part slides upon another as in this case it is desirable to keep the pressure between the sliding surfaces as low as possible to prevent wear and consequent loose fitting of the parts after they have been in service for a considerable time.

STRESSES IN THE RECOIL LUG OF THE GUN.

77. Maximum Intensity of the Force *P*. Force Required to Accelerate the Recoil Cylinder. — When the gun is fired the recoil lug draws the cylinder filled with oil to the rear compressing the counter-recoil spring and reducing its length from 70 to 25 inches. The resistance to drawing the cylinder to the rear including the compression of the spring is the force P, see Figs. 15 and 16, which when the gun is at its extreme position to the rear during recoil was found, equation (32), page 50, to be 3849 lbs., corresponding to a total resistance to recoil of 4077 lbs.

Evidently, however, the value of P will be considerably greater than this when the gun is just commencing to recoil for then the total resistance to recoil, equation (12), page 44, is 4923 lbs. The larger value of P will, therefore, be calculated and used in determining the stresses in the recoil lug. Fig. 47 shows the forces on the gun when it is at 15° elevation and just commencing to recoil.*

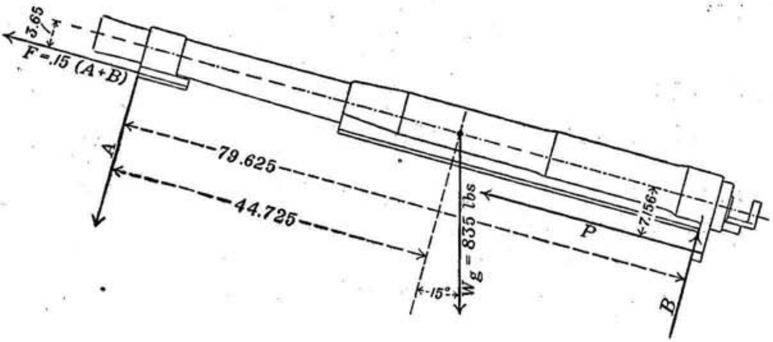


Fig. 47.

Taking moments about the center of mass of the gun, the equations expressing the relations between the forces (see article 37) are as follows:

$$P - 835 \sin 15° + .15\,(A + B) = R_1 = 4923 \qquad (28)$$

$$A - B + 835 \cos 15° = 0 \qquad (29)$$

$$P \times 7.156 + .15\,(A + B) \times 3.65 - B \times 34.9 - A \times 44.725 = 0 \qquad (30)$$

From which

$$A = 101 \text{ lbs.} \quad (31) \qquad F = 151 \text{ lbs.} \quad (33)$$

$$B = 908 \text{ lbs.} \quad (32) \qquad P = 4988 \text{ lbs.} \quad (34)$$

It will be seen that at the beginning of recoil the force P is considerably greater than at the end, but that the contrary is the case with the forces A, B, and F.

The force P, however, is not the only one acting on the recoil lug of the gun, for at the instant of maximum powder pressure the recoil cylinder being required to move with the gun to the rear

* In determining the forces brought into action between the gun and the carriage by a given total resistance to recoil, R, the force of the powder gases does not have to be considered.

has the same acceleration as the gun, and the force required to produce this acceleration of the cylinder must be transmitted to it through the recoil lug. The total weight of the recoiling parts is 960 lbs., and since the maximum pressure in the gun is 33,000 lbs. per sq. in. the total pressure tending to move the gun to the rear is

$$33000 \times \pi (3)^2/4 = 233264 \text{ lbs.*}$$

and the corresponding acceleration is

$$233264 \times 32.16/960 = 7814 \text{ ft. per sec. per sec.}$$

The weight of the cylinder filled with oil is 61.5 lbs. and the force required to give it this acceleration is

$$\frac{61.5}{32.16} \times 7814 = 14944 \text{ lbs.}$$

The counter-recoil spring is not capable of transmitting a force sufficient to give a material acceleration to any considerable part of its mass, and, therefore, at the instant of maximum acceleration of the gun the first and possibly the second and third coils at the front end of the spring will be very much deflected while the balance of the spring will have undergone practically no displacement. On this account the acceleration of the spring will not be considered.

The total force transmitted through the recoil lug because of the force P and the acceleration of the recoil cylinder is, then,

$$4988 + 14944 = 19932 \text{ lbs.}$$

The recoil lug of the gun and the point of application of this force are shown in Fig. 48. The weakest section of the lug to resist the shearing action of the force is AB and the weakest section to resist the bending moment is CD. The lever arm of the force with respect to section CD is 3 ins.

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* Theoretically the resultant force that causes the recoil of the gun is the total powder pressure diminished by the total resistance to recoil. The latter, however, is so small compared with the maximum powder pressure that it is not worth while to consider it in computing the maximum acceleration of the recoiling parts of this carriage.

Shearing Stress in Section *AB*. — This section is shown in Fig. 48. The portion of the recoil lug below it weighs about 1.18 lbs. and to give this weight the acceleration of the gun requires a force of 286 lbs., which theoretically should be added to the

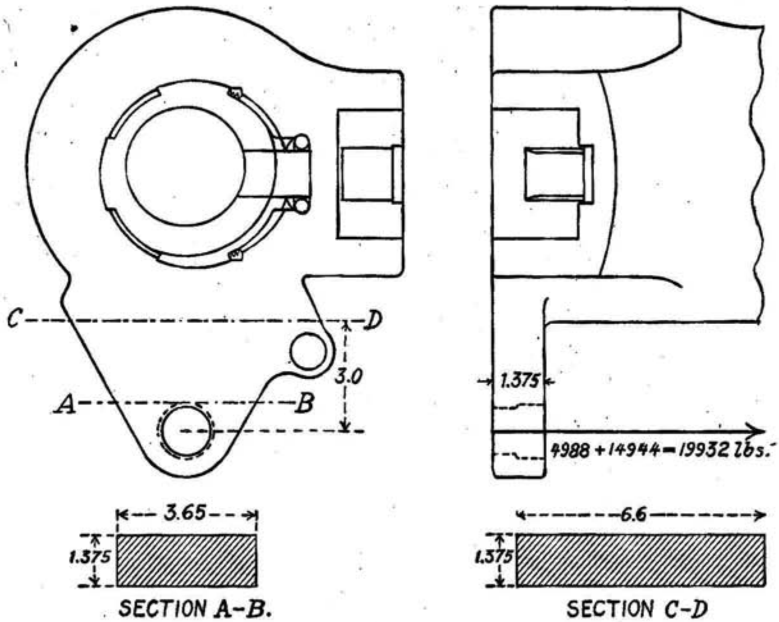


Fig. 48.

force of 19932 lbs. to determine the shearing stress in the section, but on account of its insignificance when compared with the latter force it will be neglected in these computations.

The shearing stress in section *AB* is, therefore,

$$\frac{19932}{3.65 \times 1.375} = 3972 \text{ lbs. per sq. in.}$$

Bending Stress in Section *CD*. — This section is shown in Fig. 48. The portion of the recoil lug below it weighs about 5.9 lbs. and to give this weight the acceleration of the gun requires a force of 1434 lbs. The point of application of this force is at the center of gravity of the portion of the lug below the section and its lever arm with respect to the section is about 1.25 ins.

While its bending moment with respect to section *CD* is relatively unimportant compared with that of the force of 19932 lbs. it will nevertheless be considered in determining the bending stress in the section. The total bending moment at the section is, therefore,

$$19932 \times 3 + 1434 \times 1.25 = 61589 \text{ in. lbs.}$$

and the maximum bending stress is

$$S'''y = \frac{61589 \times 1.375/2}{6.6 \times (1.375)^3/12} = 29615 \text{ lbs. per sq. in.}$$

tension at the rear edge of the section and compression at the front edge. The elastic limit of the material in the recoil lug is not permitted to be less than 65000 lbs. per sq. in.

STRESSES IN THE RECOIL CYLINDER.

78. Tensile Stress in the Rear End of the Recoil Cylinder.— The method of connecting the recoil cylinder to the recoil lug of the gun is shown in Fig. 49. The rear end of the cylinder is closed

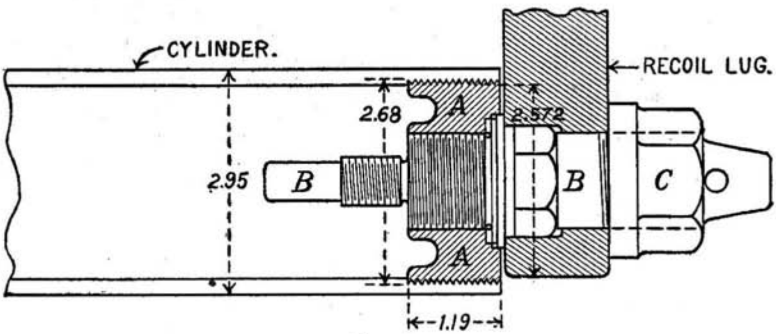


Fig. 49.

by the cylinder end *AA* screwed into it as shown. A cylinder end stud *BB* in turn screws into the cylinder end and passing through a hole in the recoil lug is held thereto by the nut *C*. The thinnest section of the cylinder occurs through the bottom of the threads into which the cylinder end is screwed. At this section the outer diameter is 2.95 ins. and the inner diameter 2.68 ins. The force of 19932 lbs., consisting of the resistance *P* and the force required to accelerate the cylinder, must be transmitted through this section, and the tensile stress therein is, therefore,

$$\frac{19932}{\pi[(2.95)^2 - (2.68)^2]/4} = 16694 \text{ lbs. per sq. in.}$$

Shearing Stress in the Threads of the Cylinder End. — The force of 19932 lbs. also tends to shear the threads connecting the cylinder end to the cylinder. The U. S. standard thread used by the Ordnance Department is of the shape shown in Fig. 50

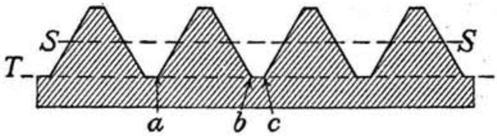


Fig. 50.

and the distance *ab* is seven times the distance *bc*. If shearing occurs along any line *SS* above the line *TT* at the bottom of the threads, the threads on both parts of the pieces screwed together will have to be shorn through but, if it occurs along the line *TT*, only seven-eighths as much metal will be shorn through as in the former case. It will, therefore, be assumed that shearing if it occurs would take place along the bottom of the threads and, since the cylinder end has the smaller diameter, that its threads would shear before those of the cylinder. The diameter at the bottom of the threads of the cylinder end is 2.572 ins. and the length of the threaded surface is 1.19 ins. Of this length only seven-eighths is effective on account of the flats *bc* at the bottom of the threads. The total area resisting the shearing action of the force of 19932 lbs. is, therefore, equal to seven-eighths of the outer surface of a cylinder whose length is 1.19 ins. and outer diameter 2.572 ins., and the shearing stress in the threads is

$$\frac{19932}{.875 \times \pi \times 2.572 \times 1.19} = 2369 \text{ lbs. per sq. in.}$$

Stress in the Walls of the Recoil Cylinder Due to the Interior Hydraulic Pressure. — The pressure in lbs. per sq. in. of the oil in the recoil cylinder must be such that it will exert a total pressure on the front end of the cylinder, tending to prevent its movement to the rear, equal to P minus the force exerted by the counter-recoil spring.

The intensity of P when recoil is just beginning, equation (34), is 4988 lbs. and at this time the force exerted by the spring is 516 lbs., whence the total pressure on the end of the cylinder is 4472 lbs. The effective area at the front end of the cylinder is equal to the effective area of the piston, which is the area of a circle whose diameter is that of the piston, minus the sectional area of the piston-rod, minus the area of the slots in the piston

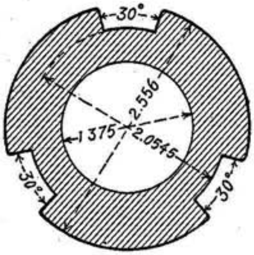


Fig. 51.

for the throttling bars. Fig. 51 shows a cross-section of the piston and piston-rod.

The shaded part is the effective area of the piston, the unshaded central part representing the piston-rod. The effective area is equal to

$$\frac{\pi}{4}\left\{(2.556)^2-(1.375)^2\right\}-\frac{\pi}{4}\left\{(2.556)^2-(2.0545)^2\right\}\frac{3\times 30}{360}=3.192 \text{ sq. ins.}$$

and the pressure per sq. in. is

$$4472/3.192 = 1401 \text{ lbs. per sq. in.}$$

This pressure occurs in front of the piston and between it and the front end of the cylinder. The thinnest section of this part of the cylinder occurs in front of the place where the throttling bars begin and has an outer diameter of 2.95 ins. and an inner diameter of 2.5717 ins.

Applying the formula for the maximum stress produced by the application of an interior pressure to a simple cylinder we obtain

$$\theta = \frac{2\,(1.28585)^2 + 4\,(1.475)^2}{3\,[(1.475)^2 - (1.28585)^2]} \times 1401 = 10740 \text{ lbs. per sq.in.}$$

tension at the inner surface.

This stress is not the resultant tension in the section for the force P produces a tensile stress therein, in the direction of the axis of the cylinder and at right angles to the stress produced by the interior pressure, equal to

$$\frac{4988}{\pi[(2.95)^2 - (2.5717)^2]/4} = 3040 \text{ lbs. per sq. in.}$$

and, as has been shown in the deduction of the formulas used in calculating the stresses in a gun, a force of tension or compression produces a stress in a direction perpendicular to its action line equal to one-third of the stress produced in that direction but of opposite sign. The force P, therefore, produces a compression of

$$3040/3 = 1013 \text{ lbs. per sq. in.}$$

in the direction of the tension due to the interior pressure, reducing that tension from 10740 to 9727 lbs. per sq. in.

This discussion relates to periods after the acceleration of the gun and recoiling parts has ceased for, as has been shown, at the instant of maximum acceleration the tensile stress in the cylinder due to the force required to accelerate it is largely in excess of that due to the force P or to the interior pressure.

STRESSES IN THE CRADLE.

79. Maximum Value of the Forces Acting on the Cradle.— It was shown in determining the stresses in the recoil lug that when the gun is just commencing to recoil the force P is considerably greater than it is when recoil is about to end, while the contrary is the case with forces A and B. Before computing the stresses in some of the parts of the cradle, therefore, the forces acting on it when the gun is just commencing to recoil will be determined so that the larger forces occurring under either one of the two conditions may be used in the computations.

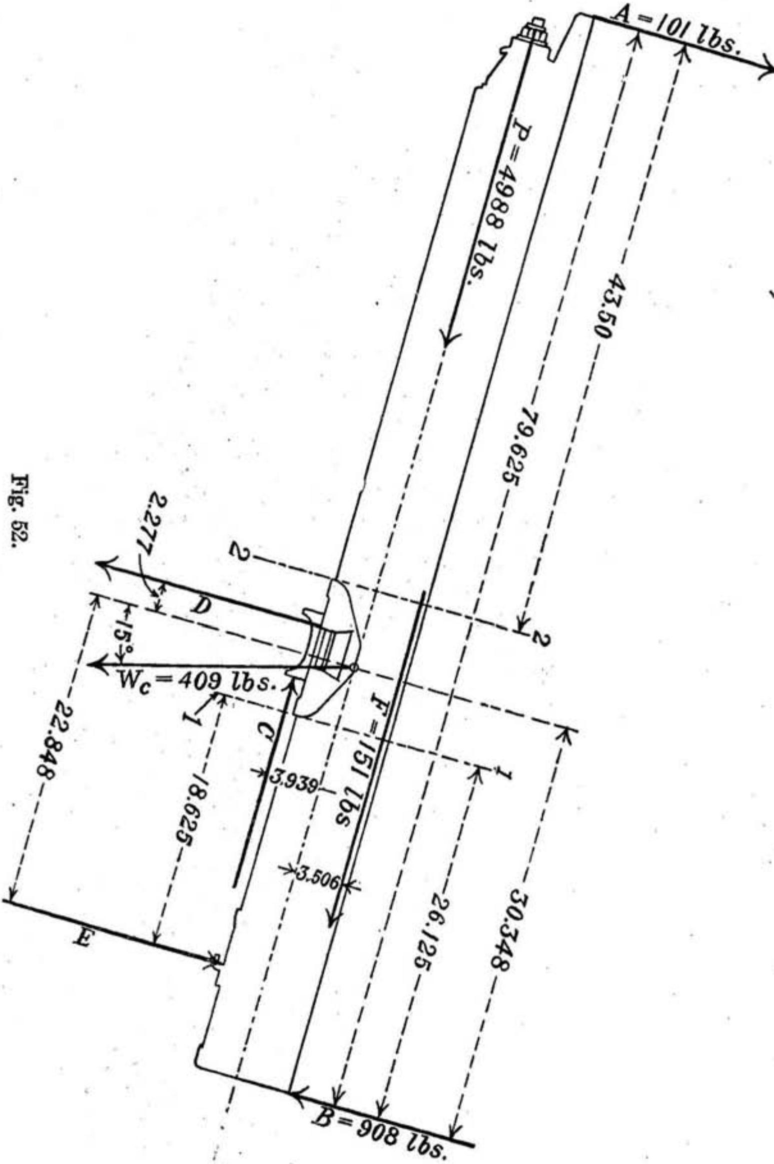
Fig. 52 shows the position of the forces acting on the cradle when the gun is just commencing to recoil, and the values of the forces A, B, F, and P already determined in article 77, page 133.

Taking moments about the center of mass of the cradle, the equations representing the relations between the forces are

$$D - E + 409 \cos 15° - 101 + 908 = 0 \qquad (35)$$

$$C - 409 \sin 15° - 4988 - 151 = 0 \qquad (36)$$

$$D \times 2.277 + E \times 22.848 - C \times 3.939 - 101 \times 49.277 - 151 \times 3.506 - 908 \times 30.348 = 0 \qquad (37)$$



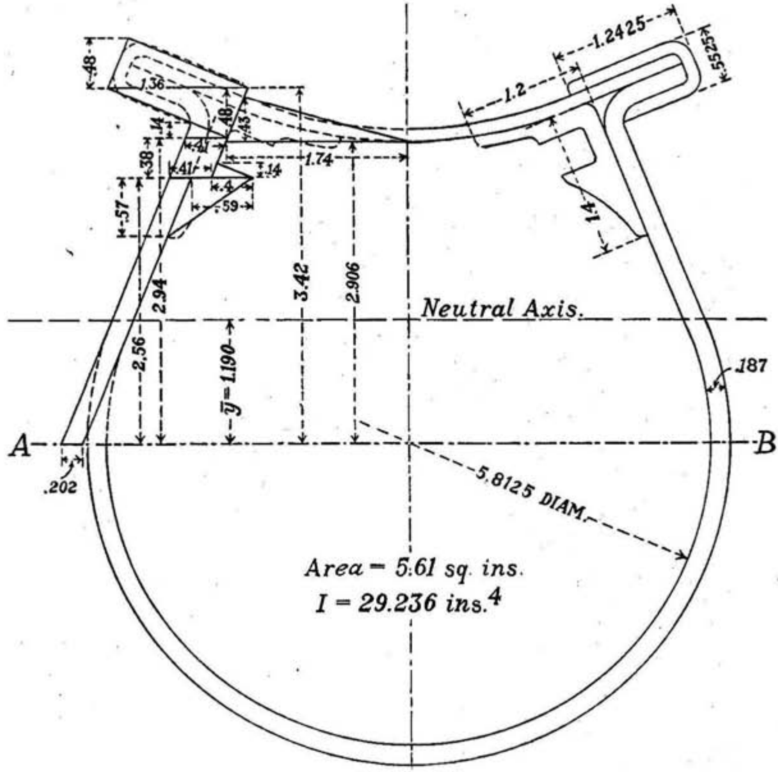
A = 101 lbs.
P = 4988 lbs.
43.50
79.625
2.277
D
2
2
15°
W_c = 409 lbs.
1
1
C
F = 151 lbs
3.939
18.625
22.848
3.506
26.125
30.348
E
B = 908 lbs.

Fig. 52.

Whence
$$C = 5245 \text{ lbs.} \quad (38)$$
$$D = 1046 \text{ lbs.} \quad (39)$$
$$E = 2248 \text{ lbs.} \quad (40)$$

Stresses in Section 1–1 of the Cradle. — This section is indicated in Figs. 16 and 52. It is shown in Fig. 53.

Moment of Inertia of Section 1–1. — As this section is subjected to a bending stress due to the forces B, E, part of F equal



Section 1–1.
Fig. 53.

to .15 B, and a component of the weight of that part of the cradle in rear of the section, it will be necessary to determine its moment of inertia about a horizontal axis passing through its center of gravity. The true shape of the section is given in the right half of Fig. 53 and, while quite irregular, it may be divided approximately into a number of regular parts whose bases are horizontal as shown in the left half of the figure. Such approximations as these are generally necessary to determine the moments of inertia of irregular sections. It will be observed that the approximate figures include in some cases small areas not found in the real section and also that some small areas included in the real section are omitted in the approximate figures. When this is necessary the areas lost and gained should balance each other both in extent and position so far as possible, in order that the moment of inertia computed from the approximate regular figures shall vary as little as possible from that of the real section.

To determine the position of a horizontal line passing approximately through the center of gravity of the real section consider only the left half of the figure, since the real section is symmetrical with respect to its vertical axis, and take moments with respect to the horizontal line AB.

Whence

$$1.36 \times \frac{.48}{2}\left\{3.42 + \frac{.48}{3}\right\} + 1.36 \times \frac{.48}{2}\left\{3.42 - \frac{.48}{3}\right\}$$
$$+1.74 \times \frac{.43}{2}\left\{2.906 + \frac{.43}{3}\right\} + .41 \times \frac{.14}{2}\left\{2.94 + \frac{.14}{3}\right\}$$
$$+.41 \times .38\left\{2.56 + \frac{.38}{2}\right\} + .4 \times \frac{.14}{2}\left\{2.56 + \frac{.14}{3}\right\}$$
$$+.59 \times \frac{.57}{2}\left\{2.56 - \frac{.57}{3}\right\} + .202 \times 2.56 \times \frac{2.56}{2} -$$
$$\frac{\pi\,[5.8125 + 2 \times .187]^2}{16} \times .4244 \times \frac{5.8125 + 2 \times .187}{2}$$
$$+ \frac{\pi\,(5.8125)^2}{16} \times .4244 \times \frac{5.8125}{2} = (\text{Area of half section})\bar{y}$$

The total area of the half section is

$$1.36 \times .48 + 1.74 \times \frac{.43}{2} + .41 \times \frac{.14}{2} + .41 \times .38 + .4 \times \frac{.14}{2}$$
$$+.59 \times \frac{.57}{2} + .202 \times 2.56 + \frac{\pi\,[5.8125 + 2 \times .187]^2}{16}$$
$$- \frac{\pi(5.8125)^2}{16} = 2.806 \text{ sq. ins.}$$

and $$\bar{y} = 1.190 \text{ ins.}$$

The area of the whole section is 5.61 sq. ins.

The moment of inertia of the half section with respect to the horizontal axis passing through its center of gravity is

$$
\begin{aligned}
&\frac{1.36 \times (48)^3}{36} + 1.36 \times \frac{.48}{2}\left\{3.42 + \frac{.48}{3} - 1.190\right\}^2 \\
&+ \frac{1.36 \times (.48)^3}{36} + 1.36 \times \frac{.48}{2}\left\{3.42 - \frac{.48}{3} - 1.190\right\}^2 \\
&+ \frac{1.74 \times (.43)^3}{36} + 1.74 \times \frac{.43}{2}\left\{2.906 + \frac{.43}{3} - 1.190\right\}^2 \\
&+ \frac{.41 \times (.14)^3}{36} + .41 \times \frac{.14}{2}\left\{2.94 + \frac{.14}{3} - 1.190\right\}^2 \\
&+ \frac{.41 \times (.38)^3}{12} + .41 \times .38\left\{2.56 + \frac{.38}{2} - 1.190\right\}^2 \\
&+ \frac{.4 \times (.14)^3}{36} + .4 \times \frac{.14}{2}\left\{2.56 + \frac{.14}{3} - 1.190\right\}^2 \\
&+ \frac{.59 \times (.57)^3}{36} + .59 \times \frac{.57}{2}\left\{2.56 - \frac{.57}{3} - 1.190\right\}^2 \\
&+ \frac{.202 \times (2.56)^3}{12} + .202 \times 2.56\left\{\frac{2.56}{2} - 1.190\right\}^2 \\
&+ \frac{.110}{2}\left\{\frac{5.8125 + 2 \times .187}{2}\right\}^4 + \frac{\pi}{16}[5.8125 + 2 \times .187]^2 \\
&\times \left\{.4244 \times \frac{5.8125 + 2 \times .187}{2} + 1.190\right\}^2 - \frac{.110}{2} \times \left\{\frac{5.8125}{2}\right\}^4 \\
&- \frac{\pi}{16}(5.8125)^2 \times \left\{.4244 \times \frac{5.8125}{2} + 1.190\right\}^2 = 14.618 \text{ ins.}^4
\end{aligned}
$$

The moment of inertia of the whole section with respect to the horizontal axis through its center of gravity is, therefore,

$$I = 29.236 \text{ ins.}^4$$

Bending Moment and Stress in Section 1–1. — The bending moment at this section is obtained by taking the algebraic sum of the moments of the forces acting on the right (to the rear) of it. In doing this it will be found that while the forces are greater when recoil of the gun is about to end, the resultant bending moment is slightly less than when the gun is just commencing to recoil. The stresses under the latter assumption will, therefore, be computed. In the computations which follow the weight of the cradle will be neglected as only that part of it in rear of the section produces a bending moment. This bending moment is slight and neglecting it increases the resultant bending moment at the section, which is an error on the safe side.

In determining the bending moment it should be noted that only that part of F which is due to the friction produced by B must be considered since the part due to the friction produced by A acts on the left (in front) of the section. Furthermore the lever arm of .15 B must be taken with reference to the horizontal axis through the center of gravity of the section which, from Fig. 53, is 1.190 ins. above the action line of the force P, or the axis of the lower, cylindrical, part of the cradle. The bending moment at the section is, then, see Figs. 52 and 53,

$$M = E \times 18.625 - B \times 26.125 - .15\,B\,(3.506 - 1.190) = 17832 \text{ in. lbs.}$$

since

$$E = 2248 \text{ lbs. and } B = 908 \text{ lbs.}$$

The lowest point of the section is at the greatest distance from the neutral axis, this distance being

$$1.190 + \frac{5.8125}{2} + .187 = 4.283 \text{ ins.}$$

and the bending stress at that point is

$$S'''y = 17832 \times 4.283/29.236 = 2612 \text{ lbs. per sq. in. tension.}$$

The force .15 B produces a tensile stress distributed uniformly over the section equal to $136/5.61 = 24$ lbs. per sq. in. which increases very slightly the tension below the neutral axis and decreases by the same amount the compression above it.

The forces B and E produce a shearing stress in the section which is greatest when recoil is about to end. At this time its intensity is, see equations (40) and (31), chapter II,

$$[3511 - 1885]/5.61 = 290 \text{ lbs. per sq. in.}$$

Stresses in Section 2–2 of the Cradle. — This section is indicated in Figs. 16 and 52. It is practically identical in shape and area with section 1–1, Fig. 53, and, therefore, the area, position of neutral axis, and moment of inertia determined for that section will also serve for it. The stresses in the section are greatest when the gun is just commencing to recoil.

The forces P and $.15\,A$, see Fig. 52, produce a stress of compression uniformly distributed over the section which is equal to

$$[4988 + 15]/5.61 = 892 \text{ lbs. per sq. in.}$$

The section is also subjected to a bending moment which may be found by taking the algebraic sum of the moments of the forces acting on the left (in front) of it. Neglecting the weight of the cradle as in the case of section 1–1 and noting by reference to Fig. 53 that the force P has a lever arm of 1.190 ins. with respect to the neutral axis of the section, we obtain, see Fig. 52,

$$M = A \times 43.50 + .15\,A\,(3.506 - 1.190) - P \times 1.190 = -1507 \text{ in. lbs.}$$

since

$$A = 101 \text{ lbs. and } P = 4988 \text{ lbs.}$$

The maximum bending stress in the section occurs at its lowest point and is

$$S'''y = 1507 \times 4.283/29.236 = 221 \text{ lbs. per sq. in. compression.}$$

The latter stress increases the compressive stress uniformly distributed over the section so that the maximum resultant stress is

$$892 + 221 = 1113 \text{ lbs. per sq. in. compression}$$

and occurs at the lowest point of the section.

The force A also produces a shearing stress in the section which is greatest when the gun is about .5 in. from the end of its recoil. At this time the intensity of A is practically 1078 lbs. and the corresponding shearing stress is

$$1078/5.61 = 192 \text{ lbs. per sq. in.}$$

It will be observed that the cradle, so far as the stresses in it due to the firing of the gun are concerned, could be made much lighter, but its walls are now only .187 in. thick and if made much thinner they would be likely to be deformed and injured in rapid transportation or maneuvering of the battery.

Stresses in the Pintle and in the Rivets fastening it to the Cradle. — Fig. 54 shows the pintle of the 3-inch field carriage with the forces C and D acting thereon.

The pintle is fastened to the cradle by twenty-three .375-in. rivets through the rivet holes shown in the figure. The

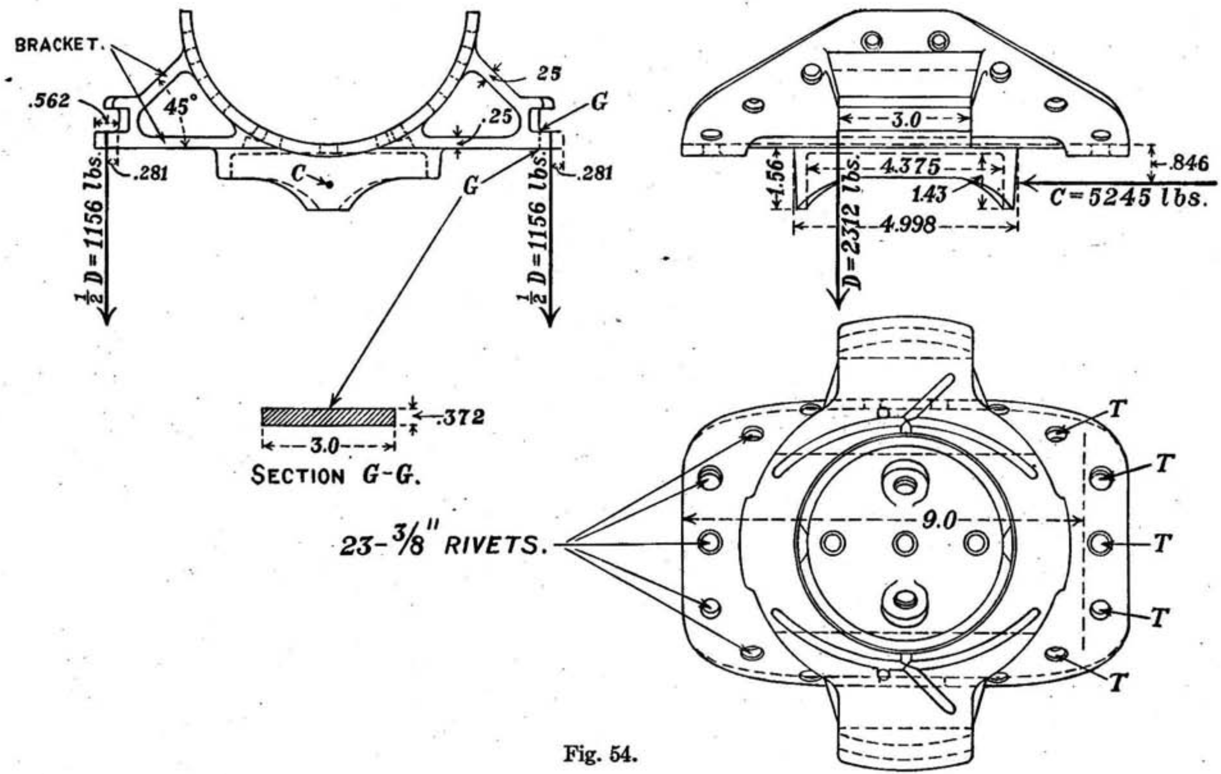

BRACKET.
.562
45°
25
.25
G
.281
C
$\frac{1}{2}$ D=1156 lbs.
$\frac{1}{2}$ D=1156 lbs.
.281
G
3.0
.846
4.375
1.56
1.43
C=5245 lbs.
4.998
D=2312 lbs.
.372
3.0
SECTION G-G.
23-3/8" RIVETS.
9.0
T
T
T
T
T

Fig. 54.

greatest value of the force C (5245 lbs.) occurs when the gun is just commencing to recoil while that of the force D (2312 lbs.) occurs when recoil is about to end. These values will be used in the computations that follow.

Stresses Caused by the Force C. — The lower projecting part of the pintle against which the force C acts is practically cylindrical in shape (it has a very slight taper) and forms a cantilever 1.56 ins. long over which the force is uniformly distributed. The dangerous section, see Fig. 54, occurs at a distance of 1.43 ins. above the lower edge of the pintle and the bending moment there is, from equation (18),

$$M = \frac{5245}{1.56} \times \frac{(1.43)^2}{2} = 3438 \text{ in. lbs.}$$

The dangerous section is a circular ring, its outer diameter being approximately 4.998 ins. and its inner diameter 4.375 ins. Its moment of inertia about its neutral axis is, therefore,

$$.0491 \{(4.998)^4 - (4.375)^4\} = 12.651 \text{ ins.}^4$$

and the bending stress at the extreme fibre is

$$S'''y = \frac{3438 \times 4.998/2}{12.651} = 679 \text{ lbs. per sq. in.}$$

compression at the front of the section and tension at the rear.

The shearing stress in the section is

$$\frac{5245 \times 1.43/1.56}{.7854 \{(4.998)^2 - (4.375)^2\}} = 1049 \text{ lbs. per sq. in.}$$

The force C tends to shear the twenty-three rivets fastening the pintle to the main body of the cradle. These rivets are .375 in. in diameter and consequently .11045 sq. in. in area. The shearing stress in them is

$$\frac{5245}{23 \times .11045} = 2065 \text{ lbs. per sq. in.}$$

C also tends to crush the rivets and the walls of the rivet holes in the pintle and the cradle. As the thickness of the cradle is less than that of the pintle the greater crushing stress will occur between the rivets and the walls of the rivet holes in the cradle. The crushing stress between a rivet and the walls of the rivet hole is considered to be uniformly distributed over an area equal to the thickness of the plate multiplied by the diameter of the rivet. Under this assumption the crushing stress due to the force C is

$$\frac{5245}{23 \times .187 \times .375} = 3252 \text{ lbs. per sq. in. compression,}$$

since there are twenty-three rivets having a diameter of .375 in., and the thickness of the cradle wall is .187 in.

The force C also tends to rotate the pintle about the line cut from the surface of contact with the cradle by a plane perpendicular to the axis of the cradle passing through the center of the pintle. This tendency to rotation is mainly resisted by the compression between the contact surfaces of the cradle and pintle in front of the central plane, and by the stress in the rivets behind that plane. The manner of distribution of the stress in these rivets is not unlike that of the tensile stress on one side of the neutral axis of a section of a beam subjected to a bending force. The greatest stress will occur in the rivets at the rear of the pintle and will be mainly a tensile stress. It can be obtained approximately with the error on the safe side in the following manner: Assume that the tendency of the pintle to rotate under the action of the force C is resisted entirely by the pressure along the line of contact at the front edge of the pintle and by the tension in the five rear rivets marked T in Fig. 54. Assume also that the mean distance of the five rivets from the front edge of the pintle is 9 ins. From Fig. 54 the lever arm of C with respect to the line of contact at this edge is .846 in. Now since the pintle is in equilibrium, the moments of the forces about the line of contact at the front edge must be zero and, calling T the force which the five rivets oppose to the rotation of the pintle, we have

$$T \times 9 = 5245 \times .846$$

or

$$T = 493 \text{ lbs.}$$

and the tensile stress in each of the five rivets is

$$\frac{493}{5 \times .11045} = 893 \text{ lbs. per sq. in.}$$

Stresses Caused by the Force D/2. — The force $D/2 = 1156$ lbs. acts downward on the lower lip of each wing of the pintle as shown in Fig. 54. These lips may be considered as cantilevers .562 in. long, approximately 3 ins. wide, and .372 in. deep with the forces uniformly distributed over them. The area of the dangerous section *GG* shown in Fig. 54 is 1.116 sq. ins. and its moment of inertia with respect to its neutral axis is .01287 in.^{4} The bending moment at this section is, from equation (19),

$$\frac{1156}{.562} \times \frac{(.562)^2}{2} = 324.8 \text{ in. lbs.}$$

and the maximum bending stress is

$$S'''y = 324.8 \times .186 / .01287 = 4695 \text{ lbs. per sq. in.}$$

The shearing stress in the section is

$$1156 / 1.116 = 1036 \text{ lbs. per sq. in.}$$

The bracket connecting the wing to the rest of the pintle, see Fig. 54, acts as a triangular frame supporting the force $D/2$. The upper arm of the bracket makes an angle of 45° with the lower arm, and the force is perpendicular to the lower arm. The total stress in the upper arm is, therefore, one of tension equal to

$$\tfrac{1}{2} D / \cos 45° = 1635 \text{ lbs.}$$

The greatest stress per unit of area in the upper arm occurs at the smallest section thereof which is .25 in. thick and 3 in. wide. This stress is

$$\frac{1635}{.25 \times 3} = 2180 \text{ lbs. per sq. in.}$$

The total stress in the lower arm is one of compression equal to $D/2 = 1156$ lbs. The area of cross-section of the lower arm being

$$.25 \times 3 = .75 \text{ sq. in.}$$

the stress per unit of area is

$$1156 / .75 = 1541 \text{ lbs. per sq. in.}$$

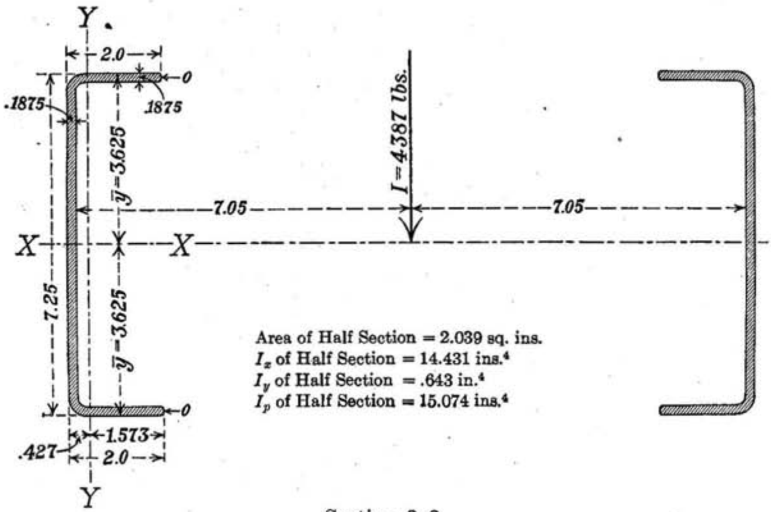
The total tensile stress of 1635 lbs. in the upper arm of the bracket puts a tensile stress in a number of the rivets fastening the pintle to the main body of the cradle. This stress is mainly confined to the four rivets near the junction of the upper arm with the body of the pintle and, assuming it to be entirely so confined and uniformly distributed over the four rivets, its intensity is

$$\frac{1635}{4 \times .11045} = 3701 \text{ lbs. per sq. in.}$$

Maximum Resultant Stress in the Rivets. — To obtain the maximum resultant stress in the rivets the simple stresses occurring in them should be combined in accordance with the methods already explained, but on account of the low intensities of the simple stresses this is unnecessary.

STRESSES IN SECTION 3-3 OF THE TRAIL.

80. Forces Causing Stress in the Section. — Area and Moment of Inertia of the Section. — This section is indicated in Fig. 18 and shown in Fig. 55. It is taken through the elevating screw



Section 3-3.
Fig. 55.

bearings and, as the force $I = 4387$ lbs. is contained in its plane, it is the dangerous section of the trail. The stresses in it are greatest when recoil is about to end and they will be determined under that condition.

The stresses in the section, see Figs. 18 and 55, are compression and bending due to the force $S = 3938$ lbs., bending and shear