Equation (114) should give the same value of V_1 for both the selected velocities.

Now to revert to the problem, which will be solved after the first method, designated A, and the steps of the solution will be numbered as in the explanation above.

We have to determine the ballistic constants for use in the velocity and pressure formulas.

Since $\mu=0$ we see from equation (110) that

$$N'=0$$

and that since λ is negative N is also negative.

Velocity formula (112) therefore becomes for this powder

$$v^2=MX_1(1-NX_0) \tag{117}$$

from which with two measured values of v and the corresponding values of u, and hence of X_1 and X_0, we may determine M and N. We must use for this purpose two values of v while the powder is burning.

1. We will take the two measured values 2794 and 3166 and determine afterwards whether we are right in the selection.

2. The X functions for $u=199.2$ corresponding to $v=2794$ are found as follows.

Equation (103), $x=6.2853$, for $u=199.2$.

From the table of X functions, using first differences only,

$$\log X_0=0.82110$$

In the same way the other functions for this value of x, and the functions for the values of x corresponding to the other given values of u, are obtained from the table.

u	x	v	$\log X_0$	$\log X_1$	$\log X_2$
199.2	6.2853	2794	0.82110	0.50606	$\bar{1}.68496$
259.2	8.1784	2940	0.86213	0.58011	$\bar{1}.71799$
409.2	12.9112	3166	0.93117	0.69774	$\bar{1}.76657$
559.2	17.6446	3284	0.97710	0.77150	$\bar{1}.79440$

In equation (117), using two values v and v' and the values of X_0 and X_1 corresponding to each, and solving for N and M, we obtain

$$N=\frac{v'^2X_1-v^2X_1'}{v'^2X_0X_1-v^2X_0'X_1'}$$

$$M=\frac{v^2}{X_1-NX_0X_1}$$

3. Making $v=2794$ and $v'=3166$, we obtain with the corresponding values of X_0 and X_1

$$\log M=6.59155$$
$$\log N=\bar{2}.75465$$

With these, as has been shown on page 91, all the other ballistic constants are determined.

4. We will first determine from the second of equations (110)

$$\log \overline{X}_0=0.94432$$

and from the table find the corresponding value of x by interpolation, using first differences only,

$$\bar{x}=14.11$$

From equation (103) $\bar{u}=447.19$, that is, the burning of the powder was completed at the instant that the shot had travelled 447.19 inches.

5. The values of u for the points selected for the determination of the constants in the equations being less than $\bar{u}$ we find ourselves justified in the selection of these points.

From equation (105) $\log M'=4.91005$
(111) $\log V_1^2=7.23484$
(106) $\log P'=5.07622$

We now have all the constants that enter the equations (112) to (116) for velocity and pressure and fraction of charge burned. These equations become for this round

$$v^2=[6.59155]X_1(1-[\bar{2}.75465]X_0) \tag{118}$$

$$p=[4.91005]X_3(1-[\bar{2}.75465]X_4) \tag{119}$$

$$v_a{}^2=[7.23484]X_2 \tag{120}$$

$$p_a=\frac{[5.07622]}{(1+x)^{\frac{4}{3}}} \tag{121}$$

$$y=[\bar{6}.20449]v^2/X_2 \tag{122}$$

With these five equations we can determine the velocity, pressure, and weight of powder burned as the projectile passes any point in the bore, by substituting the values of the X functions determined from Table I for the value of x corresponding to the travel of the projectile at the point.

In this way we find from equation (118) for $u=259.2$, for which $x=8.1784$,—(the symbol L indicates a constant logarithm in the equation),

$$\begin{array}{rl}
\log X_0 & 0.86213 \\
L & \bar{2}.75465 \\
\cline{2-2}
0.41379 & \bar{1}.61678 \\
\hline
0.58621 & \bar{1}.76805 \\
\log X_1 & 0.58011 \\
L & 6.59155 \\
\cline{2-2}
\log v^2 & 6.93971 \\
\log v & 3.46985 \\
v= & 2950 \text{ foot-seconds}
\end{array}$$

This differs from the measured velocity by 10 feet.

To find the velocity at the muzzle, for comparison with the measured velocity, we must make use of equation (114), since the powder was all burned before the projectile reached the muzzle.

$$\begin{array}{ll}
\log V_1^2 & 7.23484 \\
\log X_2 & \underline{\bar{1}.79440} \\
\log V^2 & 7.02924 \\
\log V & 3.51462 \\
\quad V = & 3270.5 \text{ foot-seconds}
\end{array}$$

This differs but 13.5 feet from the measured velocity of 3284 feet. The difference, $\frac{4}{10}$ of one per cent of the measured velocity, is negligible.

In the same way the velocity at any point may be determined and the curve v in Fig. 20 plotted.

54. Pressures.—The pressure at any point may be similarly obtained from equations (119) and (121). The pressures so obtained are plotted in the curve p, Fig. 20.

MAXIMUM PRESSURE.—As the cylindrical grain burns with a decreasing surface the maximum pressure is obtained as explained on page 89 by making $x=0.45$ in equation (119),

$$\text{for } x=0.45 \qquad \log X_3=\bar{1}.85640 \qquad \log X_4=0.48444$$

With these values we get from equation (119)

$$p_m=48{,}276 \text{ lbs.}$$

Weight of Powder Burned.—From equation (122) we obtain the curve y, Fig. 20, which shows the weight of powder burned at each point of the travel. From this curve it is seen that at the point of maximum pressure, for which $u=14.26$ inches, about 12 of the 27.5 pounds of the charge were consumed. The charge was half consumed when the travel was 18 inches, and three-quarters consumed at a travel of about 68 inches.

The following table obtained from the three equations, (118), (119), and (122), is represented by the curves v, p, and y in Fig. 20

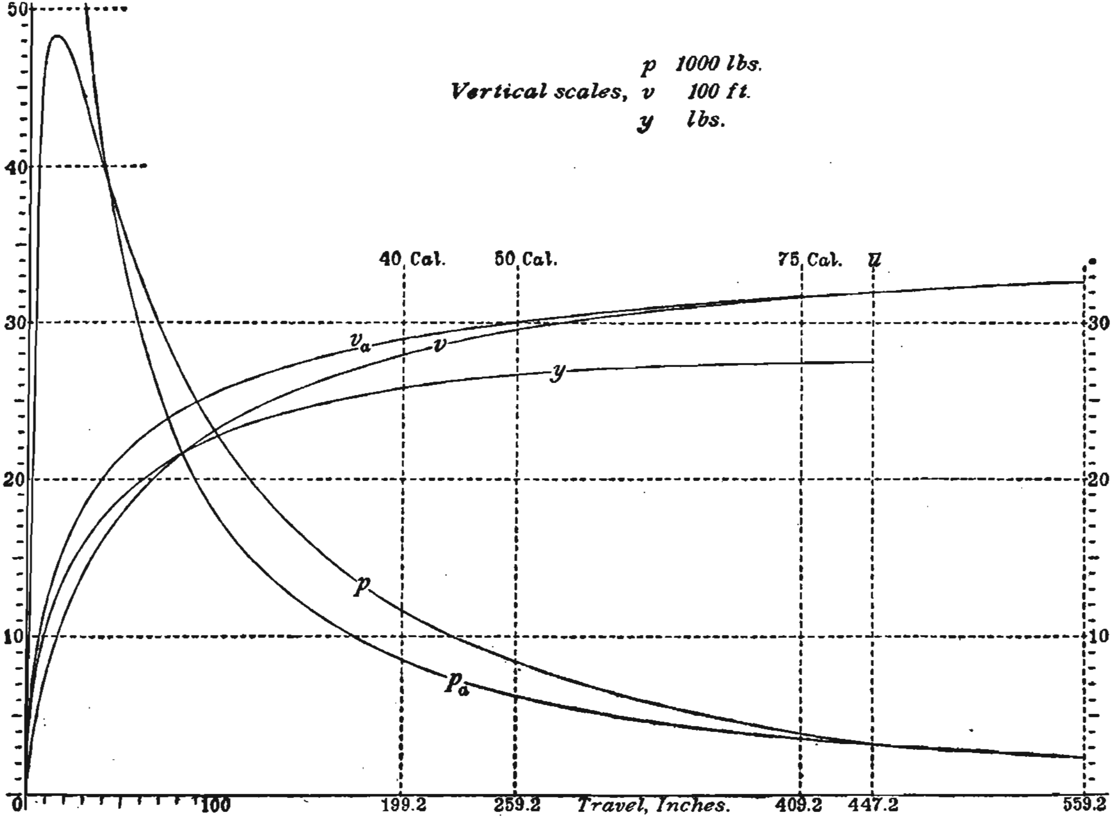

FIG. 20.—Charge, 27.5 pounds Cordite.

x	Travel u inches.	Veolcity v ft.-secs.	Pressure p pounds.	Powder burned y pounds.
0.2	6.34	564.99	43929	8.669
0.4	12.67	876.56	48183	11.597
0.6	19.02	1109.1	47558	13.584
0.8	25.36	1295.2	45569	15.097
1.0	31.69	1449.8	42895	16.315
1.5	47.54	1747.9	36632	18.589
2.0	63.38	1967.2	31386	20.209
2.5	79.24	2138.0	27158	21.442
3.0	95.08	2276.1	23738	22.419
4.0	126.77	2488.0	18600	23.873
5.0	158.46	2644.2	14975	24.898
6.2853	199.2	2794.0	11642	25.822
8.1784	259.2	2950.0	8329	26.677
12.9112	409.2	3166.0	3840	27.475
14.1100	447.2	3198.0	3191	27.500
17.6446	559.2	3271.0	2411	

In the figure the curve y stops at the travel $\bar{u}$ because equation (122) can only apply as long as the powder is burning. The powder, wholly burned at $\bar{u}$, is of course wholly burned at every point beyond $\bar{u}$.

The curves v_a and p_a in Fig. 20 are similarly obtained from equations (120) and (121). They represent the velocity and pressure under the supposition that the powder was wholly burned before the projectile moved, and from them are obtained the velocities and pressures in the gun after the powder is all burned, that is, after the travel $\bar{u}$.

The size of the page does not permit the representation of the first part of the curve p_a. This curve intersects the vertical axis at a point obtained by making $x=0$ in equation (121), for which value $p_a=119{,}180$ lbs. per sq. in. $=P'$, see (115). As explained on page 90, P' is the pressure per unit of surface exerted by $\bar{\omega}$ pounds of powder confined in a volume equal to the initial air space.

The Force Coefficient f and Constant τ.—From equation

(109) $f=2247.4$ lbs. per sq. in.

(108) $\tau=0.50486$ seconds

f was originally considered as the force of the powder or, in the units assigned, the pressure exerted by a pound of a gas occupying a cubic foot at the temperature of explosion, see equation (45). But it has been affected by whatever errors there are in the assumptions made in the deduction of the formulas. It can consequently be regarded only as a coefficient, called the *force coefficient.*

τ is the total time of burning of the grain in air. The velocity of burning in air is, therefore, for this grain,

$$l_0/\tau = 0.39615 \text{ inches per second.}$$

55. Velocity of Combustion.—The velocity of combustion of the powder at any instant may be obtained from equation (61).

$$v_c = \frac{l_0}{\tau}\left(\frac{p}{p_0}\right)^{\frac{1}{2}} \tag{123}$$

by substituting the value of p corresponding to any point in the travel of the projectile.

Thus at the moment of maximum pressure, $p_m = 48{,}276$, and

$$v_c = 22.7 \text{ inches per second.}$$

At this rate of burning the charge would be consumed in about nine one-thousandths of a second.

Thickness of Layer Burned.—Combining equations (65) and (68) we obtain

$$l = l_0 X_0/\overline{X}_0 \tag{124}$$

Substituting for any point the value of X_0 we obtain l.

Thus for $u = 199.2$, $\log X_0 = 0.82110$, and for the thickness of layer burned at this travel

$$l = 0.1506 \text{ inches.}$$

Variation in Size of Grain.—The thickness of layer burned at any travel of the projectile is evidently the half thickness of web of some whole grain of the same shape that would be completely burned at that point. We may therefore write in equation (124) l_0' for l and $\overline{X}_0'$ for X_0 and form the equation

$$2l_0' = 2l_0\overline{X}_0'/\overline{X}_0 \tag{125}$$

The web of a grain designed to be completely burned at any travel of the projectile under the same conditions of loading as in problem 1 will therefore have a thickness equal to twice the thickness of layer burned at the travel as obtained in that problem.

$$\text{For } u = 199.2, \quad 2l_0' = 0.3012 \text{ inches,}$$

which is twice the value we found for l at this length of travel.

Variation in Initial Surface of Charge for Same Shape of Grain.—From equations (19) and (125) we obtain

$$S_1' = S_1\overline{X}_0/\overline{X}_0' \tag{126}$$

For the grain whose web we have just determined the initial surface of the charge would have the following relation to the same weight of charge of the powder used in problem 1.

$$S_1' = 1.322\, S_1$$

56. Variations in Gun, Powder, or Projectile.—Having once determined the constants τ and f for any powder in a gun of any caliber, we may assume any variation in the gun except in caliber, or any variation in the powder or in the projectile, and determine the effect of the variation on the circumstances of motion. τ, the time of complete burning of the grain in air, is proportional to the web thickness. Its value for the same powder in grains of any other shape or size is equal to the determined value multiplied by the ratio of the web thicknesses of the new grain and of the grain used in the determination. For any assumed size of the chamber and fixed weight of charge or density of loading we may proceed exactly as in problem 1. For changes in the weight of the charge or of the projectile the procedure is the same as in that problem. For changes in the shape of the powder grain the method to be pursued will be best understood from an example.

Problem 2.—Suppose that the powder used in problem 1 instead of being made up into cylindrical grains was made into ribbons 0″.4 thick, 2″ wide, and 8″ long, of the same density as the cylindrical grains.

Determine the circumstances of motion with the same weight of charge, 27½ pounds, as in that problem.

The thickness of web, 0″.4, is the same as for the cordite cylinder.

The values of the constants of form for the parallelopiped grain are, see page 19,

$$\alpha=1+x+y$$

$$\lambda=-\frac{x+y+xy}{1+x+y}$$

$$\mu=\frac{xy}{1+x+y}$$

in which $x=2l_0/m$ and $y=2l_0/n$.

Making $x=0.4/8=0.05$ and $y=0.4/2=0.2$ we find for the ribbon grain assumed in this problem

$$\alpha=1.25, \quad \lambda=-0.208, \quad \mu=0.008.$$

As the initial surfaces of two charges of equal weight composed of the same powder in grains of different shapes are to each other as the values of α for the two forms of grain, see equation (19), the initial surface of this charge will be $1.25/2=5/8$ of the initial surface of the charge in problem 1, and as the maximum pressure is dependent upon the initial surface we may expect a lower maximum pressure from this charge than from the first.

The values of f and τ determined in problem 1, being constant for the same powder and gun, are applicable to this round, and it will be seen from equations (100) to (109) that Δ, a, z_0, $\bar{v}^2$, P', $\overline{X}_0$, and V_1^2 have the same values as in that problem.

Therefore from equations (110), (111), and (105) we obtain at once the values of the constants in the formulas for velocity and pressure.

$$\log M = 6.38743$$
$$\log N = \bar{2}.37374$$
$$\log N' = \bar{4}.01445$$
$$\log M' = 4.70593$$

and with these values we may write the formulas for velocity and pressure while the powder is burning.

$$v^2 = [6.38743]X_1\{1 - [\bar{2}.37374]X_0 + [\bar{4}.01445]X_0^2\},$$
$$p = [4.70593]X_3\{1 - [\bar{2}.37374]X_4 + [\bar{4}.01445]X_5\}.$$

The formula for the weight of powder burned is the same as in problem 1, equation (122), but since the value of v for any value of x is now different the weights burned at the different travels will also be different.

The formulas for velocity and pressure after the charge is all burned are the same as in problem 1, equations (120) and (121), and the velocities and pressures beyond the point of complete consumption are the same. The point of complete consumption is the same as in that problem, since $\overline{X}_0$ has the same value.

The velocities and pressures and weight of powder burned under the conditions of this problem are shown in the subjoined table and in Fig. 21.

x	Travel u inches.	Velocity v f. s.	Pressure p pounds.	Powder burned y pounds.
0.2	6.34	458.86	29584	5.718
0.4	12.67	720.16	33587	7.828
0.6	19.02	919.33	34089	9.333
0.8	25.36	1081.6	33381	10.528
1.0	31.69	1218.6	32220	11.527
1.5	47.54	1489.7	28926	13.503
2.0	63.38	1696.1	25922	15.024
2.5	79.24	1862.3	23390	16.269
3.0	95.08	2005.6	21278	17.326
4.0	126.77	2223.2	18001	19.062
5.0	158.46	2397.2	15600	20.465
6.2853	199.2	2576.0	13324	21.947
8.1784	259.2	2780.3	10977	23.697
12.9112	409.2	3131.0	7559	26.871
14.1100	447.2	3198.0	7091	27.500
17.6446	559.2	3271.0	2411	

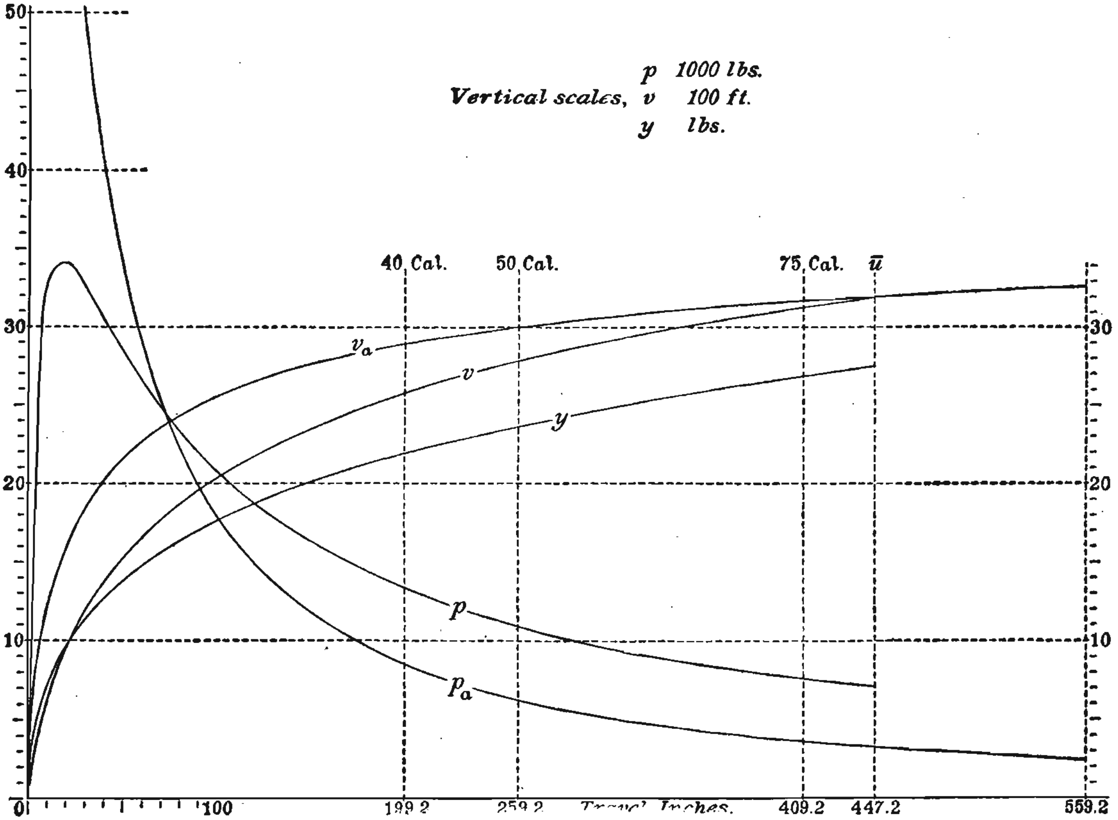

FIG. 21.—Charge, 27.5 pounds, Ribbon Grains.

Comparing this charge, by means of the tables or of the curves, with the charge in problem 1 we see that while the muzzle velocity is the same the maximum pressure is reduced from about 48,000 to about 34,000 lbs. The pressures along the chase are increased. The total area under the pressure curves, which represent the work expended upon the projectile, must be equal.

It is apparent from the powder curves that the powder burned more progressively in the second charge than in the first. This was to have been expected, for if we compare the rate of burning of the two grains in air by means of equations (9) and (7), dividing the half thickness of web into five equal parts, we find for the fraction burned in each layer:

Cordite grains........	0.36	0.28	0.20	0.12	0.04
Ribbon grains........	0.24	0.22	0.20	0.18	0.16

57. Velocities and Pressures after the Powder is Burned.—We have seen, pages 86 and 90, that equations (114) and (115) are the equations for the velocity and pressure under the supposition that the powder is all burned before the projectile moves.

The curves v_a and p_a in Figs. 20 and 21 are calculated from equations (120) and (121) for both shapes of grain. They are alike in the two figures since the weight of charge is the same. The curve v_a, from equation (120), shows what the velocities would be if the $27\frac{1}{2}$ pounds of powder were all burned before the projectile moved, and the curve p_a shows the pressures under the same condition.

We find in practice that the velocities measured beyond the point where the powder is all burned agree with the velocities obtained from the v_a formula. We are therefore warranted in using this formula for determining velocities after the powder is burned. And if the correct velocities are given by the v_a formula, the pressures obtained from the p_a formula must also be correct.

Therefore velocities and pressures after the powder is all burned are taken from the v_a and p_a curves or formulas.

From the manner of deduction of equations (112) and (114) these two equations will give the same value $\bar{v}$ for the value $\bar{u}$. The curves v_a and v therefore coincide at that value of the travel. It will be observed, however, in Fig. 21, that the curves p_a and p for the ribbon grain do not coincide at the travel $\bar{u}$.

It may be shown analytically that these curves coincide only for grains of such form that the vanishing surface is zero; such as the cube, sphere, or solid cylinder, see page 18. The vanishing surface of the ribbon grains of this problem is a finite surface that suddenly becomes zero at the travel $\bar{u}$. Coincidence of the two curves at this point could therefore not be expected.

The curves p_a and p in Fig. 20, for the cordite grain, coincide at $\bar{u}$, since the vanishing surface of the cordite grain is zero.

58. The Action of Different Powders.—In Fig. 22 the curves of velocity, pressure, and weight of powder burned, from problems 1 and 2, are shown together. This figure serves well to illustrate the action of different powders in the gun.

The curves with the subscript 1 are taken from problem 1, in which the charge was 27.5 lbs. of cordite. The curves with subscript 2 are from problem 2, in which the charge was of the same weight as in problem 1 and of powder of the same composition, but made up into ribbon-shaped grains with the same thickness of web as the cordite.

Regarding the curves y_1 and y_2 we see that the burning of the charge of powder was completed in each case at the same point of travel, $\bar{u} = 447.2$ inches. The quantity burned at any travel less than $\bar{u}$ was less for the ribbon grain than for the cordite.

The rate of emission of gas as a function of the travel of the projectile is shown by the tangents to the curves y_1 and y_2. For equal travels of the projectile the ribbons gave off gas less rapidly at first and until the projectile had traveled about 63 inches, at which point the curves y_1 and y_2 are farthest apart. From this point on the ribbon grains emitted gas more rapidly than the cordite.

We consequently find in the pressure curves lower pressures

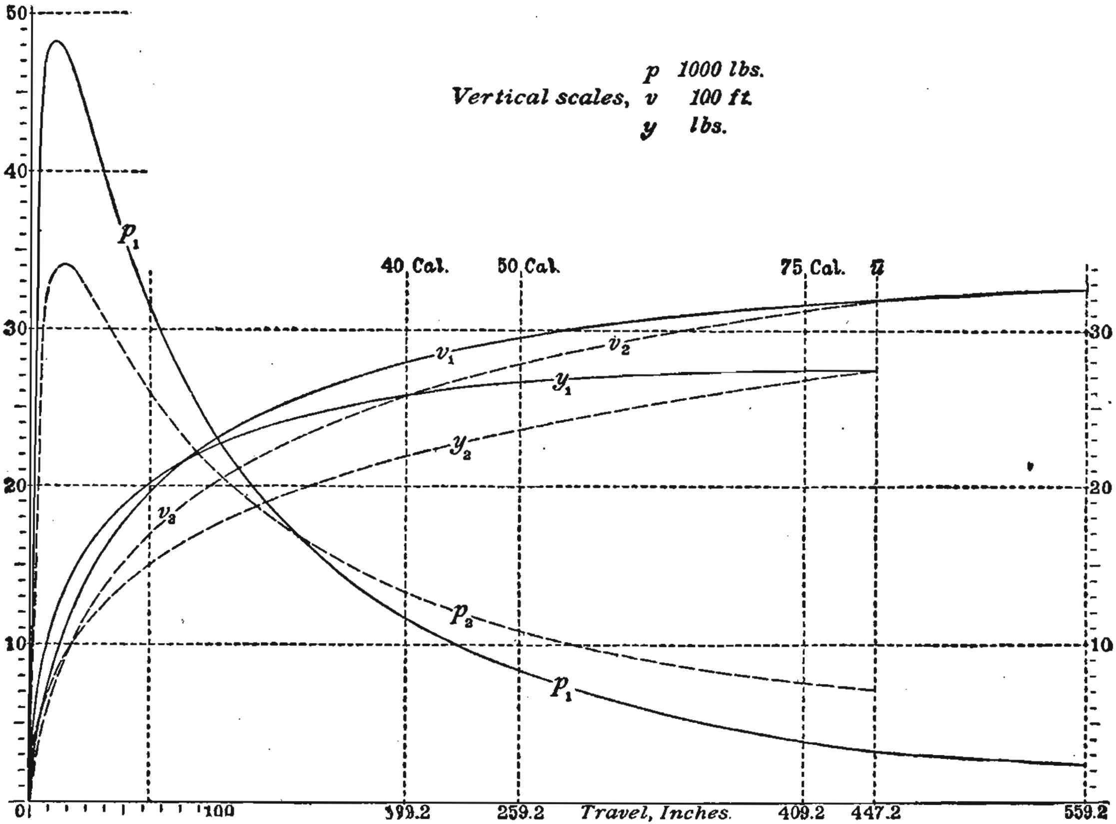

FIG. 22.—Comparison of Results with Cordite and Ribbon Grains.

from the ribbon grains over this part of the bore. The maximum pressure is lower and occurs later than the maximum pressure from the cordite. After the travel of 63 inches the pressure is better maintained by the more rapid evolution of gas from the ribbon grains and we find that the pressure curve p_2 falls off more slowly than the curve p_1, so that the two curves rapidly approach each other, and later cross at a travel of about 130 inches.

At the instant before the travel $\bar{u}$ is reached the area of the burning surface of the ribbon grains has a considerable value. It may readily be determined, from the given dimensions and density of the ribbon grains, that there are 76 of these grains in the charge of 27½ lbs. The initial surface of the charge is 3040 square inches.

The vanishing surface of each grain, determined by mensuration or by making $l=l_0$ in equation (1), is 24.32 square inches, and for the 76 grains, 1848 square inches. This is more than 6/10 of the original surface.

At the travel $\bar{u}$ this large burning area suddenly becomes zero. There is a sudden cessation of the emission of gas and a sharp drop in the pressure. As the burning surface of the cordite grain approaches zero gradually the pressure curve p_1 of this grain is continuous.

Since at the travel $\bar{u}$ the projectile has the same velocity from the two charges, the work done upon it is the same in each case, and the areas under the pressure curves to this point must be equal.

Corresponding with the sudden change in pressure at the travel $\bar{u}$ we find in the curve v_2 a sudden variation in the rate of change of the velocity of the projectile as a function of the travel, represented by the tangent to the curve.

The above considerations apply to the 100 caliber length of the gun.

Now if we consider the gun as 40, 50, or 75 calibers in length neither charge would have been wholly consumed in the bore; and we see from the curves that in each case the muzzle velocity would be less from the slower burning powder. It is therefore apparent that to produce in the gun of any of these lengths a given muzzle velocity, v_1, taken from the cordite curve, a larger charge of the slower powder would be required.

The maximum pressure from the larger charge of slow powder would remain less than that from the quicker powder, since the area under the two pressure curves must be equal and the pressure curve of the slow powder would be the higher at the muzzle.

As the gun is longer the difference in the weight of the two charges of the quick and slow powder that produce the same muzzle velocity is less, until at some length the difference becomes zero. The advantage of lower maximum pressure always remains with the slower powder.

59. Quick and Slow Powders.—It is apparent from Fig. 22 that if the maximum pressure and the muzzle velocities obtained from the cordite in the 40 and 50 caliber guns are satisfactory, the muzzle velocities produced by the same charge of powder in the form of ribbons would be too low. This powder would be too slow for guns of those lengths, while for the guns of 75 or more calibers it would be satisfactory.

The powder for a gun of any caliber and length has the greatest efficiency when in grains of such shape and dimensions that the charge of least weight produces the desired muzzle velocity within the allowed maximum pressure. The powder that produces these results may be considered the standard powder for the gun.

The maximum pressure is dependent on the initial surface of the powder charge. A powder with greater initial surface than the standard powder, that is a powder of smaller granulation, will produce a greater maximum pressure and therefore will be a quick powder for the gun, and a powder of larger granulation will be a slow powder.

In powder grains that are similar in shape but of different dimensions, the thickness of web will vary as the square root of the surface. We may therefore judge as to whether the powder is quick or slow for any gun by comparing its web thickness with that of the standard powder of the same shape.

It is also found that usually a powder that is satisfactory in a gun of a given caliber is slow for a gun of less caliber and quick for a gun of larger caliber. Therefore, as has been shown in the chapter on gunpowders, a special powder is provided for each caliber of gun and for markedly different lengths of the same caliber.

Effects of the Powder on the Design of a Gun.—In the design of a gun, the caliber, weight of projectile, and muzzle velocity being fixed, consideration must be given to the powder in order that the size of chamber, length of gun, and thickness of walls throughout the length may be determined. We have seen that to produce a given velocity in any gun we require a larger charge of a powder that is slow for the gun than of a quicker powder. The larger charge will require a larger chamber space, and will thus increase the diameter of the gun over the chamber. The maximum pressure being less than with the quicker powder the walls of the chamber may be thinner. The slow powder will give higher pressures along the chase, therefore the walls of the gun must here be thicker. The weight of the gun is increased throughout its length.

If we do not wish to increase the diameter of the chamber we must, for the slow powder, lengthen the gun in order to get the desired velocity.

On the other hand, with a powder that is too quick for the gun very high and dangerous pressures are encountered, requiring excessive thickness of walls over the powder chamber. The difficulties of obturation are increased. Excessive erosion accompanies the high pressures and materially shortens the life of the gun. The gun may be shorter and thinner walled along the chase.

It is evident from the above considerations that each gun must be designed with a particular powder in view, and that a gun so designed and constructed will not be as efficient with any other powder.

DETERMINATION OF THE BALLISTIC FORMULAS FROM A MEASURED MUZZLE VELOCITY AND MAXIMUM PRESSURE.

60. In the previous problems we determined the constants in the ballistic formulas by means of measured interior velocities. This method will usually not be available, as interior velocities can be measured only by special apparatus not usually at hand. The usual data observed in firing are the muzzle velocity and the maximum pressure.

The method of determining the constants with this data is illustrated in the following problem, and at the same time the method of applying the formulas to the multiperforated grain.

Problem 3.—Five rounds were fired from the Brown 6 inch wire wound gun at the Ordnance Proving Grounds, Sandy Hook March 14, 1905. The projectiles weighed practically 100 lbs. each. The charge was 70 lbs. of nitrocellulose powder in multiperforated grains, with two igniters, each containing 8 ounces of black powder, at the ends of the charge. The multiperforated grains weighed 89 to the pound. They were of the form described on page 22. Their dimensions, corrected for shrinkage, were

$$D_1 = 0''.512 \qquad d_1 = 0''.051 \qquad m = 1''.029$$

The mean muzzle velocity of the five rounds was 3330.4 f. s.

The measured maximum pressure was 42,497 lbs. per sq. in.

The capacity of the powder chamber was 3120 cubic inches.

The total travel of the shot was 252.5 inches.

Determine the circumstances of motion.

Before we can proceed with the solution of the problem we must determine the constants of the powder. We will make no distinction between the two different kinds of powder, but consider the weight of charge as 71 pounds of multiperforated powder.

Dimensions of grains, $D_1 = 0''.512$, $d_1 = 0''.051$, $m = 1''.029$.

Weight of grain, 89 to 1 pound.

We will first determine the constants of form of the powder grain.

From equation (13)

$$2l_0=0.08975$$

and from equations (12) we find $\alpha=0.72667$, $\lambda=0.19590$, $\mu=0.02378$. Equation (11), in which F is the fraction of grain burned when the web is burned, therefore becomes for this grain

$$F=0.72667\frac{l}{l_0}\left\{1+0.19590\frac{l}{l_0}-0.02378\frac{l^2}{l_0^2}\right\} \tag{127}$$

Making $l=l_0$,

$$F=0.85174 \tag{128}$$

the fraction of grain burned when the burning of the web is completed. The slivers therefore form 0.14826 of this particular grain.

Fictitious Multiperforated Grain.—The body of the grain burns with an increasing surface, while the slivers burn with a decreasing surface. To avoid the difficulties that would follow from the introduction of the two laws of burning into the ballistic formulas, we will substitute for the real grain a fictitious grain with such a thickness of web that when the web is burned the same weight of powder is burned as when the whole of the real grain is burned; that is, the body of the fictitious grain is equivalent to the whole of the real grain.

For the body of the fictitious grain F in the formula of the fraction burned must be unity when $l=l_0$. Making $F=1$ in equation (127) and solving the cubic equation by Horner's Method, as explained in the algebra, we obtain for l/l_0

$$l/l_0=1.1524$$

The value of l/l_0 that will make $F=1$ in equation (127) can be obtained more simply and with sufficient accuracy by trial as follows.

We have determined that when $l=l_0$ and $l/l_0=1$, $F=0.85174$. This value is less than unity by 0.148. For a first trial we will increase the value of l/l_0 by 0.148 and obtain from (127),

$$\text{with } l/l_0=1.148 \qquad F=0.99568$$

an increase in the value of F of 0.144. Therefore if we further increase l/l_0 by 0.005 we will get a value of F near unity;

$$\text{with } l/l_0=1.153 \qquad F=1.0006$$

Interpolating, by the rule of proportional parts, between these two sets of values we find that for $F=1$

$$l/l_0=1.1524$$

Substituting this value in (127) it becomes

$$1=0.837416(1+0.22573-0.031581)$$

Comparing this with equation (5), $1=\alpha(1+\lambda+\mu)$, which is derived from the formula for the fraction burned by making $l=l_0$, and which expresses the relations existing between the constants of form of the powder grain, we see that for the fictitious grain

$$\alpha=0.837416 \qquad \lambda=0.22573 \qquad \mu=-0.031581$$

The new value of l_0 must be the former value multiplied by the above ratio, $l/l_0=1{,}1524$, since we have multiplied all the quantities in equation (127) by this ratio to make $F=1$. Therefore $l_0=0.044875\times1.1524=0.051714$.

The volume of the real grain is

$$V_0=\tfrac{1}{4}\pi(D_1^2-7d_1^2)m=0.197144$$

Whence from equation (18) with $n=89$, $\delta=1.5776$.

61. Solution.—We have now all the data necessary for the solution of the problem. For convenience it is repeated here.

Constants of the Gun.	*Constants of the Powder.*
$C=3120$	$\varpi=71$
$d=6$	$\delta=1.5776$
$U=252.5$	$\alpha=0.837416$
$w=100$	$\lambda=0.22573$
Measured Data.	$\mu=-0.031581$
$V=3330.4$	$l_0=0.051714$
$p_m=42497$	

From equation (100) $\Delta = 0.6299$
(101) $\log a = \bar{1}.97940$
(102) $\log z_0 = 1.82144$
$z_0 = 66.289$

On account of the thinness of web of the powder grain, and the high pressure, we may be certain that the charge was wholly consumed in the bore. Assuming that the maximum pressure was the maximum pressure on the base of the projectile we then have a pressure while the powder was burning and a velocity after the charge was all burned. As explained on page 92, equations (92) and (91), or (114) and (113), are applicable in this case.

Method of Procedure.—The procedure is as follows.

1. Substitute in (114) the measured muzzle velocity and the value of X_2 taken from the table with the value of x corresponding to the travel of the projectile at the muzzle.

2. Determine V_1.

3. Substitute in (113) the measured value of the maximum pressure and the values of the X functions corresponding to $x = 0.8$ or $x = 0.45$, according as the grain burns with an increasing or decreasing surface.

4. Assume a value for the travel at the moment of complete combustion and determine for this travel the values of $\bar{x}$ and $\overline{X}_0$.

5. With this value of $\overline{X}_0$ and the value of V_1, previously determined, find values for N, N', and M' from (110), (111), and (105).

6. Substitute these values in the second member of (113).

7. If the second member has then the same value as the first member, which is the measured maximum pressure, our assumption of the travel $\bar{u}$ is correct. If not we must make new assumptions for $\bar{u}$ and determine new values for M, N, and N' until we find values that will satisfy equation (112).

The successive steps of the solution which follows are numbered as in the preceding paragraph.

1. For the muzzle $U = 252.5$ and, equation (103),

$$x = 3.8091$$

From the table, for this value of x

$$\log X_2 = \bar{1}.61019$$

Therefore equation (114) becomes for the muzzle

$$v_a^2=(3330.4)^2=V_1^2\,[\bar{1}.61019] \tag{131}$$

from which

2. $$\log V_1^2=7.43481$$

3. It was shown on page 90 that with a grain burning with an increasing surface the maximum pressure may be taken as occurring when

$$x=0.8$$

which for this round corresponds to a travel $u=53.03$ inches, see equation (103).

For this value of x we find from the table

$$\log X_3=9.86027 \qquad \log X_4=0.60479 \qquad \log X_5=1.17352$$

Equation (113) therefore becomes, since μ and N' are negative,

$$p_m=42497=[\bar{1}.86027]M'\{1+[0.60479]N-[1.17352]N'\} \tag{129}$$

From equation (105) we determine for this problem

$$M'=[\bar{3}.99801]\,M$$

and substituting this value of M' in equation (129) it becomes

$$p_m=42497=[\bar{3}.85828]M\{1+[0.60479]\,N-[1.17352]N'\} \tag{130}$$

4. The proper values of M, N, and N' must satisfy equation (130). But we see that equations (110) and (111) express fixed relations between these constants and V_1 at the moment of complete burning of the charge.

Therefore we will assume the travel at the moment of complete consumption, and with the corresponding value of $\bar{x}$, and therefore of $\overline{X}_0$, determine N and N' from equations (110) and M from (111).

Then substituting this set of values in equation (130) we will determine whether the values satisfy that equation. If not we will make other assumptions for $\bar{x}$ and proceed in the same way until we find satisfactory values of the constants.

The value of x at the muzzle is 3.8091. The value $\bar{x}$ must be less than this since we are assuming that the charge was all consumed in the gun. Let us assume $\bar{x}=2$.

5. Taking from the table the corresponding value of $\log X_0$ we find from equations (110) and (111) values of M, N, and N'.

6. These substituted in equation (130) make the second member equal to 45,746.

7. This is greater by 3249 pounds than the measured maximum pressure, 42,497 pounds; and we therefore conclude that we have assumed a too rapid combustion of the powder. The true value of $\bar{x}$ is therefore greater than 2.

Assume next $\bar{x}=2.3$
From the table $\log \overline{X}_0=0.65467$
From equation (111) $\log M=6.70307$
From equation (110) $\log N=\bar{2}.69892$
$\log N'=\bar{3}.19009$

With these values in equation (130) we get

$$p_m=42{,}909 \text{ pounds}$$

As this differs from the given pressure, 42,497 pounds, by less than one per cent, we may without material error use these values of the constants as the true values.

The assumed value $\bar{x}=2.3$, by means of which the constants were determined, gives, from equation (103)

$$\bar{u}=152.5 \text{ inches}$$

We have from equations (105) and (106)

$$\log M'=4.70108$$

$$\log P'=4.95570$$

We may now from equations (112) to (116) form the five equations applicable to this round.

$$v^2=[6.70307]X_1\{1+[\bar{2}.69892]X_0-[\bar{3}.19009]X_0^2\} \tag{132}$$

$$p=[4.70108]X_3\{1+[\bar{2}.69892]X_4-[\bar{3}.19009]X_5\} \tag{133}$$

$$v_a^2=[7.43481]X_2 \tag{134}$$

$$p_a=\frac{[4.95570]}{(1+x)^{\frac{6}{5}}} \qquad (135)$$

$$y=[6.41645]v^2/X_2 \qquad (136)$$

With these equations we may determine the velocity, pressure, and weight of charge burned at any point in the bore. For any travel less than 152½ inches equations (132) and (133) apply for the velocity and pressure, and equation (136) for the weight of powder burned. For any travel greater than 152½ inches, equations (134) and (135) apply.

The table and curves which will follow are derived from these equations.

A convenient method of performing the work in constructing the table or curves is here shown. It is always best to assume values of x that are given in the table, rather than values of u, which would require interpolation in the table to find the values of the X functions.

The symbol L in the following work is used to designate the various constant logarithms in equations (132) to (136).

We will take for example the value $x=0.8$, corresponding to the travel at which we found the maximum pressure.

From the table:

$\log X_0=0.46075$ $\quad\log X_1=9.71100$ $\quad\log X_2=9.25025$

$\log X_3=9.86027$ $\quad\log X_4=0.60479$ $\quad\log X_5=1.17352$

$$\text{Equation (103)}\qquad \begin{array}{ll} \log x & \bar{1}.90309 \\ \log z_0 & 1.82144 \\ \hline \log u & 1.72453 \end{array} \qquad \underline{u=53.031\text{ inches}}$$

$$\begin{array}{llcllll}
\text{Equation (132)} & & & \log X_0 & 0.46075 & & \\
 & & & \log N & \bar{2}.69892 & & \\ \cline{5-5}
\log X_1 & \bar{1}.71100 & & & 1.15967 & \log X_0^2 & 0.92150 \\ \cline{5-5}
\log M & 6.70307 & +1 & & 1.14443 & \log N' & \bar{3}.19009 \\ \cline{2-2}\cline{7-7}
 & 6.41407 & & & 0.01293 \ldots\ldots\ldots\ldots & & 2.11159 \\ \cline{5-5}
 & 0.05365 \ldots\ldots\ldots\ldots & & & 1.13150 & & \\ \cline{2-2}
\log v^2 & 6.46772 & & & & & \\
\log v & 3.23386 & & & & & \\
v = & \underline{1713.4\text{ foot seconds}} & & & & &
\end{array}$$

Equation (133)		log X_4	0.60479		
		L	$\bar{2}.69892$		
log X_3	$\bar{1}.86027$		$\bar{1}.30371$	log X_5	1.17352
log M'	4.70108	+1	1.20124	log N'	$\bar{3}.19009$
	4.56135		0.02310		$\bar{2}.36361$
	0.07120		1.17814		
log p_m	4.63255				
$p_m=$	42909 lbs. per sq. in.				

Equation (136)	log v^2	6.46772	
	L	$\bar{6}.41645$	
	colog X_2	0.74945	
	log y	1.63392	$y=43.045$ lbs.

And if we desire the values of v_a and p_a,

Equation (134)	log V_1^2	7.43481
	log X_2	$\bar{1}.25025$
	log v_a^2	6.68586
	log v_a	3.34253
	$v_a=2200.5$ f. s.	

Equation (135)	log 1.8	0.25527
	×4/3	0.34036
	log P'	4.95570
	log p_a	4.61534
	$p_a=41{,}242$ lbs. per sq. in.	

These values of v_a and p_a are what the velocity and pressure would have been had the powder all burned before the projectile moved.

The calculations for velocity and pressure at any point of the bore beyond the point of complete combustion of the charge are extremely simple, being limited to the solving of the two equations (134) and (135), which require from the table the function X_2 only.

Proceeding as above for different values of x we obtain the data collected in the table on page 130, from which the curves in Fig. 23 are constructed.

62. Pressure Curves for Real and Fictitious Grains.—We have used in the deduction of the equations from which the table is produced a fictitious multiperforated grain the body of which, without the slivers, equals the whole of the real grain. The body of the real grain was, as shown by equation (128), 85.174 per cent of the whole grain, the slivers forming 14.826 per cent of the whole. The table and curve p show discontinuity of

x	Travel u inches.	Powder burned y lbs.	Velocity v f. s.	Pressure p lbs.	Velocity v_a f. s.	Pressure p_a lbs.
0.0	0.00	00.00	000.0	00000	000.0	90300
0.1	6.63	15.02	424.3	26018	922.5	79525
0.2	13.26	21.43	695.9	33698	1266.8	70814
0.4	26.52	30.48	1113.4	40316	1699.2	57657
0.6	39.77	37.35	1440.8	42500	1986.5	48254
0.8	53.03	43.05	1713.4	42909	2200.5	41242
1.0	66.29	47.98	1947.9	42500	2369.5	35836
1.3	86.18	54.43	2248.9	41223	2568.6	29742
1.6	106.06	60.04	2505.3	39659	2724.5	25258
1.9	125.95	65.04	2729.1	38052	2851.4	21836
2.3	152.47	71.00	2989.2	36002	2989.2	18380
3.1	205.50				3195.4	13761
3.8091	252.50				3330.4	11124

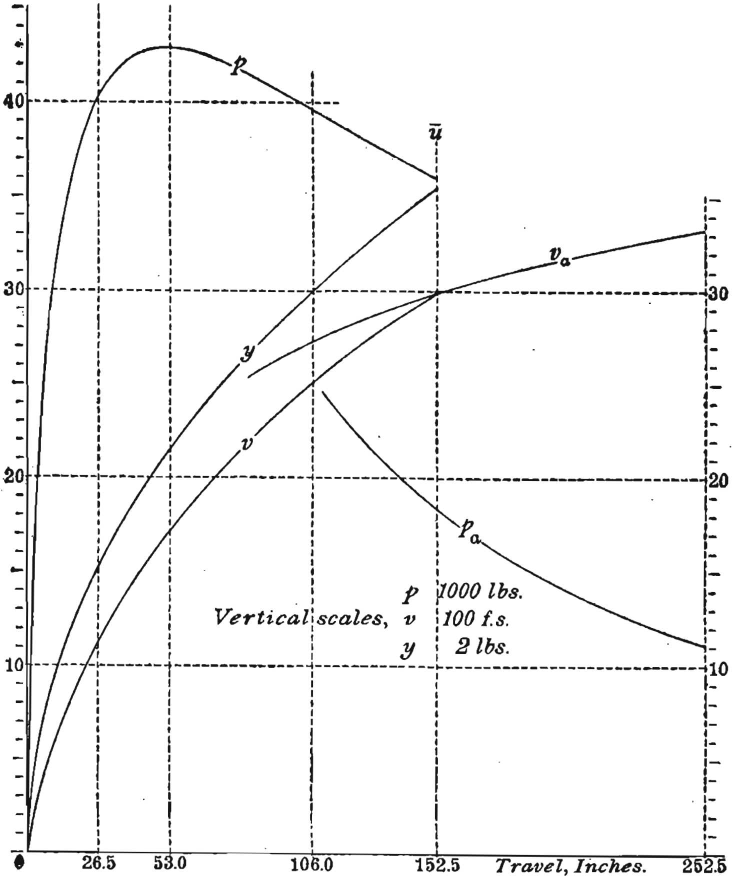

FIG. 23.—Charge, 71 pounds, Multiperforated Grains.

the pressure at the travel 152.5 inches when the burning of the whole charge is completed.

Actually there is no discontinuity in the true pressure curve. The web of the real grain was burned when 85.2 per cent of the body of the fictitious grain, or of the whole charge, was burned. This portion of the charge, 60.5 lbs., was burned at a travel of about 109 inches, as may be seen from the table. The charge burned with an increasing surface up to this point of travel and then with a decreasing surface which gradually approached the vanishing surface zero.

The pressure would therefore, at a travel of 109 inches, begin to fall off more rapidly, making a point of inflection in the true pressure curve. From this point, as the slivers burn, the pressure curve should gradually approach the curve p_a and join it at some point beyond the theoretical $\bar{u}=152.5$ inches, since the slivers, burning with a constantly decreasing surface, will require a longer time for complete consumption than the same weight in the body of the fictitious grain.

The Constant τ for this Powder.—From equation (108),

$$\tau=0.37477 \text{ seconds}$$

This is the time of burning of the whole grain in air.

The velocity of burning of this grain in air, l_0/τ, $=0.138$ inches per second.

The velocity of combustion in the gun is given by equation (123), and the thickness of layer burned at any travel by equation (124).

The Force Coefficient f.—From equation (109),

$$f=1379.5 \text{ lbs. per sq. in.}$$

It has been previously stated that f is constant for any powder in a given gun for charges not differing greatly in weight. The effective value of f, as measured in the formulas by projectile energy, must decrease as the charge decreases, for we have omitted in the formulas all consideration of the force necessary to start the projectile. It is apparent that if the charge were sufficiently reduced the projectile would not move, and f in the formula would be zero.

Therefore for any charge differing materially in weight from the charge used in the determination of f the value of f must be modified.

Ingalls adopts provisionally, this relation.

$$f=f_0\left(\frac{\bar{\omega}}{\bar{\omega}_0}\right)^{\frac{1}{3}} \tag{137}$$

in which $\bar{\omega}_0$ is the weight of charge used in the determination of f_0; f is the modified value of f_0 for the charge $\bar{\omega}$; $\bar{\omega}$ is any charge differing in weight from the charge $\bar{\omega}_0$ by 15 per cent or more.

The value of f will be modified also by a marked change in the weight of the projectile. Ingalls uses for f in this case the value

$$f=f_0\left(\frac{w}{w_0}\right)^{\frac{1}{6}}$$

and if both $\bar{\omega}$ and w change sufficiently,

$$f=f_0\left(\frac{\bar{\omega}}{\bar{\omega}_0}\right)^{\frac{1}{3}}\left(\frac{w}{w_0}\right)^{\frac{1}{6}} \tag{138}$$

With the modified value of f from equation (137) we may now determine the velocities produced by reduced charges.

63. Problem 4.—What muzzle velocities should be expected from the 6 inch gun of problem 3, with charges (including igniters) of 59 and 33¼ lbs. of the powder used in that problem?

As these charges differ in weight by more than 15 per cent of the charge of 71 lbs. used in problem 3, we will obtain the value of f from equation (137), using for $\bar{\omega}_0$ and f_0 the values of problem 3.

We have as before

$$C=3120 \qquad \delta=1.5776 \qquad U=252.5$$

The work may be conveniently performed as follows.

		Charge, 59 lbs.	*Charge, 33¼ lbs.*
Equation (137)	log $\bar{\omega}$	1.77085	1.52179
	log $\bar{\omega}_0$	1.85126	1.85126
	÷3	$\bar{1}.91959$	$\bar{1}.67053$
		$\bar{1}.97320$	$\bar{1}.89018$
	log f_0	3.13972	3.13972
	log f	3.11292	3.02990
Equation (109)	log $\bar{\omega}/w$	1.77085	1.52179
	L	4.44383	4.44383
	log V_1^2	7.32760	6.99552
Equation (100)	Δ	=0.5234	0.2950
Equation (101)	log a	=0.10605	0.44028
Equation (102)	log z_0	=1.86768	1.95285
	z_0	=73.736	89.712
Equation (103) for the muzzle,			
	x	=3.4244	2.8146
From the table	log X_2	$\bar{1}.59202$	1.55630
Equation (114)	log V_1^2	7.32760	6.99552
	log v_a^2	6.91962	6.55182
	log v_a	3.45981	3.27591
		$V=2883$ f. s.	$V=1888$ f. s.

The muzzle velocities actually obtained with charges of the above weights were 2879 and 1913 f. s. respectively. The calculated velocities show differences of 4 and 25 f. s. respectively. The latter difference, though practically not very great, shows that the modified value of f determined from the value deduced from one charge gives only approximate results when the second charge is, as in this case, less than 47 per cent of the first.

64. Problem 5.—What muzzle velocities should be expected from the 6 inch gun of problem 3, with charges (including igniters) of 68 and 75 lbs. of the powder used in that problem?

As these charges differ but little in weight from the charge of 71 lbs. used in problem 3, the value of f there determined will serve in this problem.

$f=1379.5$ $C=3120$, $\delta=1.5776$ $U=252.5$

	Charge, 68 lbs.	*Charge, 75 lbs.*
Equation (100)	$\Delta=0.6033$	0.6654
Equation (101)	$\log a=0.01016$	1.93901
Equation (102)	$\log z_0=1.83344$ $z_0=68.146$	1.80486 63.806
Equation (103)	$x=3.7052$	3.9573
Equation (109)	$\log V_1^2=7.41606$	7.45861
From the table	$\log X_2=\bar{1}.60555$	$\bar{1}.61648$
Equation (114)	$\underline{V=3242 \text{ f. s.}}$	$\underline{V=3448 \text{ f. s.}}$

The measured muzzle velocities with these charges were, respectively, 3236 and 3455 f. s. The differences between the calculated and measured velocities are immaterial.

We may make for this powder and gun any desired assumption as to the form of the powder grain, weight of charge, weight of projectile, size of powder chamber or length of gun, and with the values of f and τ from problem 3, determine the full circumstances of motion under the assumption.

Sufficient illustration has now been given of the remarkable accuracy, the simplicity and extensiveness of application of the ballistic formulas deduced by Colonel Ingalls. By their use we may obtain a more intimate knowledge of the conditions existing in the bore of a gun than has heretofore been attainable; and the knowledge so obtained will be applied in the manufacture of powder and of guns, and will result in the production of more efficient weapons.

United States Army Cannon.—A table containing data concerning the principal cannon now in service follows. The bursting charges for projectiles as given in the table are of rifle powder for the 1.457 inch and 3.2 inch guns, the 3.6 inch mortar, the 6 inch howitzer, and the two subcaliber tubes. For all other projectiles the bursting charges are of high explosive.

PRINCIPAL UNITED STATES ARMY CANNON.

Gun.	Weight.	Capacity of powder chamber.	Travel of projectile.	Weight of charge.	Weight of projectile.	Muzzle velocity.	Maximum pressure.	Density of loading.	Bursting charge.
Mountain, Field and Siege.	pounds.	cubic ins.	inches.	pounds.	pounds.	ft. secs.	pounds.		pounds.
1.457 inch pompon	410	4.86	40.68	0.075	1	1800	27000	0.4272	0.039
2.95 inch mountain gun	236	34.9	26.4	0.5	12.5	920	18000	0.3966	1.14
2.38 inch field rifle, 1905	440	34	59.05	0.72	7.5	1700	33000	0.5861	0.81
3 inch field rifle, 1905	788	66.5	74.54	1.625	15	1700	33000	0.6764	0.82
3.2 inch field rifle, 1897	830	50	75.05	1.12	13.5	1685	35000	0.6228	0.2
3.8 inch field rifle, 1905	1535	142.6	93.73	3	30	1700	33000	0.5823	2.1
3.8 inch field howitzer, 1906	391	66.34	30.01	1.2	30	900	15000	0.4966	2.1
3.6 inch field mortar, 1890	245	33.2	16.065	0.375	20	690	17000	0.3127	0.6
4.7 inch siege rifle, 1904	2688	251	114.9	5.94	60	1700	33000	0.6547	3.09
4.7 inch siege howitzer, 1906	....	....	....	....	60	900	15000	....	3.09
5 inch siege rifle, 1898	3639	402.5	119.8	5.37	45	1830	35000	0.3693	1.75
5 inch siege howitzer, 1900	1170	100	55.655	2.1	55	1000	23000	0.5660	2
6 inch siege howitzer, 1905	1660	281	62.325	4	120	900	15000	0.4	3.86
7 inch siege howitzer, 1898	3650	316.7	81.385	4.6	105	1100	28000	0.3933	7.4
7 inch siege mortar, 1892	1715	182.8	44.82	4	125	800	20000	0.6057	11.9
Seacoast.									
1.457 inch subcaliber tube	120	7.7	68.37	0.156	1.057	2000	25000	0.5617	0.1
2.95 inch subcaliber tube	236	34.9	26.4	0.44	18	750	18000	0.3470	0.81
2.24 inch rifle (6 pdr.), 1900	850	50	101.759	1.35	6	2400	34000	0.7183	0.25
3 inch rifle (15 pdr.), 1903	2692	296	139.28	6.06	15	3000	34000	0.6594	0.35
4.72 inch rifle, 50 cals	6160	496	205.1	10.5	45	2600	34000	0.5860	4.5
5 inch rifle, 1900	11120	1211	214.605	26	58	3000	36000	0.5478	2.3
6 inch rifle, 1905	21148	2122	256.285	42	106	2900	36000	0.5630	4.6
8 inch rifle, 1888	32218	3617	205.25	80	316	2200	38000	0.6122	19
10 inch rifle, 1900	76830	10040	329.62	224	604	2500	38000	0.6064	33
10 inch mortar, 1890	16734	1554	82.76	34	604	1150	33000	0.6056	33
12 inch rifle, 1900	132380	17487	395	367	1046	2500	38000	0.5697	58.3
12 inch mortar, C. I., 1886	31920	2021	91.64	33	824	1050	27500	0.4519	32.1
12 inch mortar, steel, 1890	29120	2674	98.92	54	1046	1150	33000	0.5590	58.3
14 inch rifle, 1906	111000	13363	401.4	280	1660	2150	36000	0.58	58.5
16 inch rifle, 1895	284500	29624	451.86	612	2400	2150	38000	0.6167	139.3

CHAPTER IV.

EXPLOSIVES.

65. Explosive and Explosion.—An *explosive* is a substance that is capable of sudden change from a solid or liquid state to a gaseous state, or a mixture of gases whose chemical combination, suddenly effected, results in a great increase of volume. A chemical explosion is always attended by the emission of great heat.

An explosion due to physical causes alone, as when a gas under compression is suddenly released and allowed to expand, causes cold.

Effects of Explosion.—The effects of an explosion are dependent on the quantity of gas evolved, on the quantity of heat, and on the rapidity of the reaction.

QUANTITY OF GAS. PRESSURE.—The volume of gas at the temperature of explosion determines the pressure exerted against the walls of the vessel containing the explosive.

Force.—The pressure per unit of surface exerted by the gases from unit weight of the explosive, the gases occupying unit volume at the temperature of explosion, is called the *force* of the explosive. The unit volume occupied by the gases is exclusive of the co-volume of the gases and the volume of any residue.

QUANTITY OF HEAT. WORK.—The quantity of heat determines the quantity of work that may be effected by the explosion. The bursting of the walls of the containing vessel and the projection of the fragments, or the projection of the shot from a gun, are effects produced by the conversion of the heat of explosion into work.

Potential.—The total work that can be performed by the gas from unit weight of the explosive under indefinite adiabatic expansion measures the *potential* of the explosive.

The theoretical potential of an explosive is never reached in practice. The potentials, however, afford the means of comparing the maximum theoretical quantities of work to be obtained from different explosives. The maximum practical effect obtained from explosives in firearms is from $\frac{1}{5}$ to $\frac{1}{6}$ of the potential.

RAPIDITY OF REACTION.—An explosion starts with the explosion of a single molecule, or particle, of the explosive. The heat generated and the shock developed by the explosion of the first molecule are communicated to the surrounding molecules and by the explosion of these molecules are transmitted further into the mass.

The rapidity with which the explosive reaction is transmitted through the mass varies greatly in different explosives.

The explosion of gunpowders does not differ in principle from the burning of a piece of wood or other combustible. As we have seen in the chapter on gunpowders the combustion proceeds from layer to layer and the rate of combustion, in air and in the gun, and the quantity of powder burned at any time, may be determined by means of the formulas of interior ballistics.

The explosion of nitroglycerine, of guncotton, and of other explosives of like nature is effected with very much greater rapidity than the explosion of gunpowder. The theory of Berthelot is that in these explosives the spread of the explosive reaction is not confined to the exposed surfaces, but that the explosion of the initial molecule gives rise to an explosive wave which is transmitted with great velocity in all directions through the mass and causes the almost instantaneous conversion of the whole body into gas. The velocity of propagation of the explosive wave through a mass of guncotton has been determined experimentally by Sebert to be from 16,500 to 20,000 feet per second.

The progressive emission of gas from gunpowder produces a propelling effect by causing a gradual increase of pressure on the base of the projectile, while the sudden conversion into gas of nitroglycerine or guncotton produces the effect of a blow of great intensity.

66. Orders of Explosion.—The differences in the rapidity of reaction give rise to the division of explosives into two groups, high explosives and low or progressive explosives. Explosions are designated as detonations or explosions of the first order, and progressive explosions or explosions of the second order.

The high explosives are those of great rapidity of reaction. Their complete explosions are of the first order, and produce by reason of their quickness a crushing or shattering effect on any material exposed to them.

The principal high explosives in general use are nitroglycerine, the dynamites, guncotton, picric acid and its salts, the Sprengel mixtures, and the fulminate of mercury.

The cadets of the Military Academy have studied in their course in chemistry (Descriptive General Chemistry (Tillman), pages 369 to 385) the constitution, method of production, and characteristics of the principal high explosives. It is therefore unnecessary to further describe these explosives here.

The progressive explosives are those that consume an appreciable time in the explosion. They produce explosions of the second order. The explosion is slow, comparatively, and progressive, and produces a propelling or pushing effect.

The various gunpowders are progressive explosives. Gunpowders have been fully described in Chapter I.

Nitrocellulose.—The classification by Vielle of the nitrocelluloses of various degrees of nitration is shown in the following table. The higher the degree of nitration of the cellulose the greater is the power of its explosion.

VIELLE'S CLASSIFICATION OF NITROCELLULOSES.

Formula.	Designation.	c.c. of NO_2.	Per Cent of N.	Remarks.	
$C_{24}H_{36}O_{20}(NO_2)_4$	Tetra-n.c.	108	6.76		
$C_{24}H_{35}O_{20}(NO_2)_5$	Penta-	128	8.02		
$C_{24}H_{34}O_{20}(NO_2)_6$	Hexa-	146	9.15	Only slightly attacked by acetic ether and ether-alcohol.	Friable cottons.
$C_{24}H_{33}O_{20}(NO_2)_7$	Hepta-	162	10.18	Becomes gelatinous in acetic ether and ether-alcohol.	
$C_{24}H_{32}O_{20}(NO_2)_8$	Octo-	178	11.11	Soluble in ether-alcohol. Inferior colloid.	
$C_{24}H_{31}O_{20}(NO_2)_9$	Ennea-	190	11.96	Highly soluble in ether-alcohol. Superior colloid.	
$C_{24}H_{30}O_{20}(NO_2)_{10}$	Deca-	203	12.75		
$C_{24}H_{29}O_{20}(NO_2)_{11}$	Endeca-	214	13.47	Insoluble in ether-alcohol. Soluble in acetone. Guncotton.	

It will be observed that the general formula for nitrocellulose is $C_{24}H_{40-n}O_{20}(NO_2)_n$.

The last four nitrocelluloses of the table are used in the manufacture of gunpowders.

67. Conditions that Influence Explosions.—The character of the explosion produced by any explosive is influenced by the physical condition of the explosive, by the external conditions, and by the nature of the exciting cause.

PHYSICAL CONDITION.—The influence of the physical condition of the explosive is seen in the sputtering of damp black powder when ignited, and in the insensitiveness to explosion of nitroglycerine when frozen.

EXTERNAL CONDITIONS.—External conditions influence the explosion chiefly by the amount of confinement they impose. Confinement is necessary to obtain the full practical effect of all explosives. The more rapid the reaction the less the degree of confinement required. Thus blocks of iron may be broken by the explosion of nitroglycerine upon their surfaces in the open air. In this case the air imposes sufficient confinement, as the explosion is so quick that its effect on the iron is produced before the air has time to move.

Gunpowder, on the other hand, requires strong confinement if its complete explosion is desired. Thus, in firing a large charge of gunpowder under water, unless the case is strong enough to retain the gases until the reaction is complete the case will be broken by the pressure of the gases first given off, and a portion of the charge will be thrown out unburned. Large powder grains are frequently thrown out of the gun not wholly burned.

The confinement required by the slower explosives may be diminished by igniting the charge at many points, so that less time is required for the complete explosion.

EXCITING CAUSE.—Heat is the immediate cause of all explosions, whether communicated to the explosive directly by a flame or heated wire, or indirectly through friction, or percussion, or chemical action. Each explosive has a specific temperature of explosion, to which one or more of the molecules must be raised before the explosion can begin. The heating of the initial molecule to the exploding point is not of itself sufficient to cause explosion of the entire mass, but this temperature must be transmitted from molecule to molecule throughout the mass.

The method of producing the explosion of the initial molecule has with many explosives an important influence on the character of the explosion. Nitroglycerine when ignited in small quantities burns quietly, but when struck it explodes violently. Similarly, guncotton when ignited by a flame burns progressively and the combustion may be extinguished by water, but when detonated by an explosive cap the explosion is of the first order. Most of the high explosives produce either detonations or explosions of lower order, depending upon the manner in which the explosion is initiated, and it is stated by Roux and Sarrau that even black gunpowder may be detonated by the use of nitroglycerine as an exploding charge.

Flame is sufficient to cause the complete explosion of the progressive explosives, though it may be necessary with some explosives that the flame be continuously applied. For some of the high explosives a percussive shock suffices to induce an explosion of the first order, while other high explosives are practically insensitive to shock and require for their explosion an initial explosion of some detonating substance.

68. Uses of Different Explosives.—It is apparent from what has been said concerning the differences in rapidity of reaction of the various explosives and the influences of external conditions that each class of explosives has its particular field of usefulness.

Thus the progressive explosives are more suitable for use in guns where a propelling rather than a shattering effect is desired from the explosion. A high explosive acts so quickly that if used in a gun its explosion would be completed practically before the projectile moved, with the result that the whole of its enormous force would be exerted upon the walls of the gun to produce rupture.

For the movement of masses of earth the slow explosive is better than the more rapid one, for here also a propelling rather than a shattering effect is desired.

In submarine mines the best results are obtained from dynamite No. 1, a dynamite consisting of 75 parts by weight of nitroglycerine absorbed into the pores of 25 parts of the siliceous earth called kieselguhr. The effect of the inert substance is to retard the explosion of the nitroglycerine, and the retarded explosion is of greater effect in a yielding substance like water than the more rapid explosion of pure nitroglycerine.

In hard rocks and metals the quickest explosive will give the best results, as in these hard substances the greatest intensity of blow is required to produce the shattering effect desired. Dynamite is ordinarily used for blasting purposes on account of its convenient form, its comparative safety in handling, and its ease of ignition.

Bursting Charges in Projectiles.—The explosives used as bursting charges in armor piercing projectiles must have a great shattering effect in order to break the projectile into fragments and to project the fragments with force; and at the same time the explosive must be practically insensitive to shock, so that it will not be exploded by the shock of discharge in the gun or the shock of impact on the ship's armor. The explosion of the bursting charge of an armor piercing projectile is effected by a detonating fuse so arranged as to cause the projectile to burst after it has perforated the armor.

The explosives used by the various foreign nations as bursting charges in projectiles are all composed principally of picric acid or its derivatives. The French melinite, the English lyddite, the Japanese shimose powder are examples.

Some of the picrates, as the picrates of lead, calcium, mercury, and others, are more sensitive to friction and percussion than picric acid itself. In order to prevent the formation by chemical action of any of these sensitive compounds when the bursting charge is composed of picric acid or of any of its derivatives, the walls of the projectile and all metal parts that come in contact with the bursting charge are covered with a protecting coat of rubber paint.

The walls of the cavity of the shell, the base plug, and the body of the fuse are so painted; also the screw threads of the base plug and fuse. Red or white lead or other metal lubricant must not be used on the screw threads.

69. Requirements for High Explosives for Projectiles.—The following requirements are considered essential for a high explosive to be used in filling shell. They have been found necessary as a result of a long series of tests.

SAFETY AND INSENSITIVENESS.—The explosive should be reasonably safe in manufacture and free from very injurious effects upon the operatives.

It must show a relatively safe degree of insensitiveness in an impact testing apparatus.

It must withstand the maximum shock of discharge under repeated firings in the shells for which it is intended.

It must withstand the shock of impact when fired in unfused shell, as follows:

(*a*) *Field Shell.*—With maximum velocity, against 3 feet of oak timber backed by sand. With the remaining velocity for full charge at 1000 yards range, against a seasoned brick wall.

(*b*) *Siege Shell.*—Against seasoned concrete thicker than the shell will perforate with remaining velocity for full charge at 500 yards range.

(*c*) *Armor Piercing Shell.*—Against a hard faced plate of thickness equal to the caliber of the projectile.

DETONATION AND STRENGTH.—It must be uniformly and completely detonated with the service detonating fuse.

It should possess the greatest strength compatible with a satisfactory fragmentation of the projectile. The average fragment of a projectile should be effective against the vulnerable material of a ship, such as the mechanisms of guns, gun mounts, engines, boilers, electric installations, and the like. With very quick and powerful explosives, as explosive gelatin and picric acid, the shattering effect is excessive and the fragments of the projectile are too small.

STABILITY.—It must not decompose when hermetically sealed and subjected to a temperature of 120° F. for one week.

It should preferably be non-hygroscopic, and its facility for detonation must not be affected by moisture that can be absorbed under ordinary atmospheric exposure necessary in handling.

It must not deteriorate or undergo chemical change in storage.

GENERAL CONDITIONS.—Loading must not be attended with unusual danger and should not require exceptional skill or tedious methods.

The explosive should be obtainable quickly in large quantities and at reasonable cost.

REMARKS.—The explosives used as shell fillers are more stable under severe heat treatment than the service smokeless powders. The explosives should therefore be correspondingly safer to store in large quantities.

Explosive D, used in our service, invented by Major Beverly W. Dunn, Ordnance Department, is safer to handle than black powder.

70. Exploders.—Fulminate of mercury is one of the most violent explosives. By reason of its sensitiveness to explosion by heat or percussion, and the intensity of the shock obtained by its explosion in small quantities, the fulminate of mercury is the most suitable substance for use in initiating detonations or explosions in other explosives.

It forms the principal or the only ingredient of the detonating composition in explosive caps, primers, and fuses. \Other ingredients may be potassium chlorate or nitrate, or bisulphide of antimony, the proportions differing in order to produce the best results from the particular explosive with which the exploder is to be used.

DETONATORS.—A commercial detonating cap or fuse is shown in the accompanying figure. The fulminate of mercury, or detonating composition, *B*, is enclosed in a copper case closed with a plug of sulphur through which pass the bared ends of the electric wires. A platinum bridge connects the ends of the wires, and the heating of the bridge by the electric current fires the detonator.

In order to secure the best results it is necessary that the detonator be in intimate contact with the explosive. It is therefore usually placed in the midst of the mass, and the explosive is packed closely around it.

PRIMERS FOR GUNPOWDERS.—For the ignition of charges of gunpowder a large body of flame is of more advantage than an intense shock. Consequently in small-arm primers mercury fulminate has been replaced by a less violent composition of chlorate of potash and bisulphide of antimony, which produces a larger body of flame and is at the same time less sensitive to percussion and therefore safer for use in a small-arm cartridge. In primers for cannon the large body of flame is produced by the use of black powder for the priming charge in the primer, the ignition of the black powder being effected by the explosion of a small percussion cap or by the electric ignition of a small quantity of loose guncotton.

Explosion by Influence.—The detonation of a mass of explosive may under certain circumstances induce the explosion of another mass of the same or of a different explosive not in contact with the first. The induced explosion is called an explosion by influence or a *sympathetic explosion*.

The ability of one explosive to induce the sympathetic explosion of another not in contact with it appears to depend on the character of the shock communicated by the first explosive. Abel found that while the detonation of guncotton would cause the sympathetic detonation of nitroglycerine in close proximity to it, the detonation of nitroglycerine would not cause the detonation of guncotton, although nitroglycerine is more powerful than guncotton.

In explanation of this difference in action Abel advanced the theory of *synchronous vibrations*. It is a well established fact that certain vibrations will induce the decomposition of chemical compounds whose atoms are in a state of unstable equilibrium; and according to Abel sympathetic explosion is produced when the first explosive sets up in the connecting medium vibrations that are synchronous with those that would result from the explosion of the second explosive.

This theory is questioned by later investigators, and it is now generally held that sympathetic explosion is due to the transmission of a shock of sufficient intensity.

THEORETICAL DETERMINATIONS OF THE RESULTS FROM EXPLOSIONS.

71. In the theoretical determinations of the results from explosions metric units and the centigrade thermometric scale are usually employed.

Definitions. CALORIE.—*A small calorie* is the quantity of heat required to raise the temperature of 1 gram of water (1 cubic centimeter) from 0 degrees to 1 degree centigrade.

A large calorie is the quantity of heat required to raise the temperature of 1 kilogram of water (1 liter, 1 cubic decimeter) from 0 degrees to 1 degree. A large calorie is equal to 1000 small calories.

EXOTHERMIC AND ENDOTHERMIC REACTIONS.—An exothermic reaction gives off heat, an endothermic reaction absorbs heat.

MOLUGRAM.—The term mologram is used to designate a weight of as many grams as there are units in the molecular weight of the substance. Thus, the molugram of hydrogen, H_2, is 2 grams. Water or water vapor, H_2O, has a molecular weight of 18. The molugram of water is therefore 18 grams. The molugram of nitroglycerine, $C_3H_5(NO_2)_3O_3$, is 227 grams.

The molugram of a mixture has a weight in grams equal to the sum of the molecular weights of as many molecules of each constituent as appear in the formula for the mixture. Thus, the molugram of $10KNO_3+3S+C$ is 1119 grams.

Specific Heats of Gases.—The specific heat of a gas at constant pressure is the number of calories required to heat 1 gram of the gas from 0° to 1° while the gas is permitted to expand under the constant pressure.

The specific heat of a gas at constant volume is the number of calories required to heat 1 gram of the gas from 0° to 1°, the volume of the gas remaining unchanged.

When large calories are used the unit weight of gas is 1 kilogram.

MOLECULAR HEAT.—The molecular specific heat of a gas, or more simply the molecular heat, is the number of calories required to heat a molugram of the gas from 0° to 1°.

The molecular heat is obtained by multiplying the specific heat of the gas by its molecular weight. The molecular heat may be under constant pressure or under constant volume, depending upon whether the specific heat used as a multiplier is the specific heat at constant pressure or at constant volume.

Thus, carbon dioxide, CO_2; molecular weight, 44.

At constant pressure, specific heat, 0.2169; molecular heat, $0.2169 \times 44 = 9.5436$.

At constant volume, specific heat, 0.172; molecular heat, $0.172 \times 44 = 7.568$.

72. Specific Volumes of Gases.—The specific volume of a gas is the volume in cubic decimeters (liters) of 1 gram of the gas at 0° temperature and under atmospheric pressure (barometer, 760 millimeters; pressure, 103.33 kilograms per square decimeter).

MOLECULAR VOLUME.—*The molecular volume* is the volume, at 0° and 760 mm. pressure, of a molugram of the gas. It is obtained by multiplying the specific volume by the molecular weight.

Thus, CO_2, specific volume, 0.5073, molecular volume, $44 \times 0.5073 = 22.32$ cubic decimeters or liters.

The molecular volumes of all gases are the same, 22.32 liters, as will be shown.

LAW OF AVOGADRO.—*All gases under the same conditions of pressure and temperature have the same number of molecules in equal volumes.*

It follows from this law that the single molecules of all gases, whether simple or compound, occupy equal volumes under the same conditions of pressure and temperature.

The volume of the hydrogen atom is taken as the unit volume. The molecule of hydrogen and the molecules of the other simple gases as well are composed of two atoms. A molecule of any gas therefore occupies 2 unit volumes.

In the following reaction the number of volumes appears under each of the symbols

$$\underset{\text{1 vol}}{N} + \underset{\text{3 vols}}{H_3} = \underset{\text{2 vols.}}{NH_3}$$

That is, 1 volume of N combining with 3 volumes of H forms 2 volumes of ammonia, NH_3. The volumes may be expressed in any unit, as liters or cubic feet.

The atomic weight of nitrogen is 14 and of hydrogen 1. There are therefore in the molecule of NH_3 17 parts by weight occupying the same volume as 2 parts of hydrogen alone. The specific volume of NH_3, the volume of unit weight, is therefore 1/17 of the molecular volume of hydrogen, and the molecular volume of NH_3, which is the specific volume multiplied by the molecular weight, 17 in this case, is the molecular volume of hydrogen.

As the same is true for any other gaseous compound, it follows that the product of the specific volume of a gas by its molecular weight is a constant and is equal to the molecular volume of hydrogen.

The molecular volume of all gases is 22.32 liters.

By means of the molecular volume we may determine the volume of any weight of gas, or the weight of any volume, since we know that a molugram of any gas occupies 22.32 liters.

The specific volume, the number of liters occupied by 1 gram, is equal to 22.32 divided by the molecular weight.

The specific weight, the number of grams occupying one liter, is the reciprocal of the specific volume, or the molecular weight divided by 22.32.

Classification of Gases.—Compound gases such as CO_2, NH_3, C_2H_4, whose molecules contain more than two atoms, are called *gases with condensation*, as in their formation more than two atoms are condensed into the volume of two simple atoms. Compound gases such as CO, HCl, whose molecules contain two atoms, are called *gases without condensation.* Oxygen, hydrogen, and nitrogen are simple or perfect gases.

In the following determinations of the effects of explosion we will follow the methods described by Leon Gody in his work entitled *Matières Explosives.*

73. Quantity of Heat.—The heat given off in explosions can be measured experimentally by means of special calorimeters. Roux and Sarrau made use of a very strong cylindrical bomb, similar to the apparatus of Noble and Abel, illustrated on page 67. The bomb, charged with a few grams of explosive, was immersed in a known volume of water. After the explosion of the charge, effected electrically, the increased temperature of the body of water was noted and the quantity of heat necessary to produce the rise in temperature calculated.

The theoretical determination of the quantity of heat resulting from an explosion involves the application of certain principles of thermochemistry established by Berthelot.

PRINCIPLE OF THE INITIAL AND FINAL STATE.—The heat liberated (or absorbed) in any modification of a system of simple or compound bodies, effected under constant pressure or at constant volume and without any external mechanical effect, depends solely on the initial and final states of the system. It is completely independent of the series of intermediate transformations.

From this principle it follows that the heat liberated in any transformation accomplished through successive reactions is the algebraic sum of the heats liberated in the different reactions.

We may consider the formation of an explosive as an intermediate reaction in the formation of the products of explosion from simple elements. If we then subtract from the total heat of formation of the products of explosion the heat of formation of the explosive, the difference will be the heat liberated in the reaction of explosion.

PRINCIPLE OF MAXIMUM HEAT.—All chemical changes effected without the intervention of external energy tend toward the formation of the body or the system of bodies that liberates the most heat.

The quantity of heat liberated or absorbed in a reaction is independent of the time occupied in the reaction.

74. Heats of Formation.—The heats of formation *at constant pressure* of the principal explosives and of the gases resulting from explosion are given in Table II at the end of the volume. The heats are given in large calories for the molugram of each substance. Thus hydrochloric acid gives off in its formation 22 large calories; that is, 1 gram of hydrogen and 35.5 grams of chlorine in combining give off sufficient heat to raise the temperature of 22 kilograms of water from 0° to 1°. The heat of formation of 36.5 grams of HCl is therefore 22 large calories.

The heats of formation of endothermic bodies are preceded by the minus sign in the table.

The atomic and molecular weights in Tables II, III, and IV are those that were in use at the time these tables were formed. Atomic weights according to the latest determinations are given in Table V. In the examples which follow, involving the use of Tables II, III, and IV, the atomic and molecular weights as given in those tables are used.

Quantity of Heat at Constant Pressure.—In order to determine the quantity of heat given off in any chemical change the chemical reaction must be known. The composition of explosives is generally known and the products of explosion can be predicted, under the principle of maximum heat, when the body undergoes complete combustion; that is, when it contains sufficient oxygen to form stable compounds of the maximum oxidation.

The sum of the heats of formation of the products of explosion that appear in the formula for the reaction, minus the heat of formation of the explosive, is the quantity of heat liberated by the explosion.

Example 1.—As an example we will find the heat given off in the explosion of nitroglycerine under constant pressure, as in the open air.

The equation of the reaction is as follows:

$$\underset{454}{2C_3H_5(NO_2)_3O_3} = \underset{264}{6CO_2} + \underset{90}{5H_2O} + \underset{84}{3N_2} + \underset{16}{\tfrac{1}{2}O_2}$$

With the heats of formation from Table II for the molugram of each substance we obtain, for the numbers of molecules in the reaction,

$2C_3H_5(NO_2)_3O_3$, $2 \times 98 = 196$,
$6CO_2$, $6 \times 94.3 = 565.8$,
$5H_2O$, $5 \times 58.2 = 291$.

The nitrogen and oxygen being simple elements add no heat.

We therefore have for the heat given off by the explosion under constant pressure of 2×227 grams of nitroglycerine

$$(565.8 + 291) - 196 = 660.8 \text{ l. cal.,*}$$

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* In other works the abbreviation used to designate a large calorie is *cal. k. d.* (kilogram-degree), and for a small calorie, *cal. g. d.* (gram-degree). The abbreviations *l. cal.* and *s. cal.* are used here, as they more plainly indicate the words abbreviated.

and for the heat given off by 227 grams of the explosive, a molugram,

$$Q_{mp}=660.8/2=330.4 \text{ l. cal.}$$

For the heat given off by a kilogram of the explosive,

$$Q_{kp}=\frac{330.4\times1000}{227}=1455.5 \text{ l. cal.}$$

75. When Solid Products are Formed.—If the explosion produces solid products the heats of formation of these bodies are added to the heats of formation of the gases in the determination of Q_{mp} and Q_{kp}.

Example 2.—A mixture of nitrobenzol with sufficient potassium chlorate to make the combustion of the nitrobenzol complete is exploded.

The reaction is

$$\begin{array}{lllll} 2C_6H_5NO_2+\frac{25}{3}KClO_3 & =12CO_2 & +5H_2O & +N_2 & +\frac{25}{3}KCl \\ \quad 1266.8 & \quad 523 & \quad 90 & \quad 28 & \quad 620.8 \end{array}$$

A molugram of a mixture is the sum of the molecular weights in grams of as many molecules of each of the constituents as appear in the reaction. The molugram of this explosive mixture is therefore $2\times123+\frac{25}{3}\times122.5=1266.8$ grams.

Heats of formation:

$$\begin{array}{ll} 12CO_2, & 12\times\ 94.3=1131.6 \\ 5H_2O, & 5\times\ 58.2=\ 291 \\ \frac{25}{3}KCl, & \frac{25}{3}\times105\ =\ 875 \\ \hline & \qquad\qquad 2297.6 \end{array}$$

$$\begin{array}{ll} 2C_6H_5NO_2, & 2\times\ \ 4.2=\quad 8.4 \\ \frac{25}{3}KClO_3, & \frac{25}{3}\times\ 94.6=\ 788.3 \\ \hline & \qquad\qquad 796.7 \end{array}$$

$$Q_{mp}=2297.6-796.7=1500.9 \text{ l. cal.}$$

$$Q_{kp}=\frac{1500.9\times1000}{1266.8}=1184.8 \text{ l. cal.}$$

Incomplete Combustion.—When an explosive does not contain sufficient oxygen for complete combustion the products formed vary with the temperature, the pressure, and the density of loading. Therefore no fixed formula can be written for the reaction. The products of combustion of these explosives are determined by analysis, and the heat given off may then be determined as above.

The explosion of guncotton under atmospheric pressure gives the following reaction.

$$C_{24}H_{29}O_{20}(NO_2)_{11} = 15CO + 9CO_2 + 9H_2O + 5.5H_2 + 5.5N_2$$

Under high pressure the reaction is as follows.

$$C_{24}H_{29}O_{20}(NO_2)_{11} = 12CO + 12CO_2 + 6H_2O + 8.5H_2 + 5.5N_2$$

76. Quantity of Heat at Constant Volume.—If the decomposition takes place at constant volume, for instance in a closed vessel, the heat developed is greater than in the open air under constant pressure. The gases developed in the open air perform the work of driving back the air, and this work absorbs some of the heat.

Let Q_{mp} be the heat given off by the molugram of the substance in the reaction at *constant pressure* at the surround-temperature t,

Q_{mv} the heat given off by the molugram of the substance in the reaction at *constant volume* at the surrounding temperature t,

W the work of expansion at constant pressure,

E the mechanical equivalent of heat, 425 kilogram-meters.

Then W/E is the heat expended in performing the work of driving back the air, and

$$Q_{mv} = Q_{mp} + W/E \qquad (1)$$

But the work W due to the pressure of the gas against the constant pressure p is, as shown by equation (40), page 65,

$$W = \int_{v_b}^{v_1} p\,dv = p \int_{v_b}^{v_1} dv$$

v_b and v_1 representing the volumes of the gas before and after expansion.

Performing the indicated integration,

$$W = p(v_1 - v_b) \tag{2}$$

Taking the molecular volume at 0° and 760 mm., 22.32 liters, as the unit volume,

Let n_b represent the number of unit volumes before expansion,
n_1 the number of unit volumes after expansion to normal atmospheric conditions.

n_1 *will also represent the number of gaseous molecules,* since after expansion to the normal atmospheric conditions of temperature and pressure each unit volume is occupied by a molugram.

Then from Gay-Lussac's law, page 58, we have at the temperature t

$$v_1 = 22.32 n_1 (1 + \alpha t)$$
$$v_b = 22.32 n_b (1 + \alpha t)$$

Substituting these values in equation (2) we have

$$W = p\, 22.32 (n_1 - n_b)(1 + \alpha t)$$

Whence
$$\frac{W}{E} = \frac{p}{E} 22.32 (n_1 - n_b)(1 + \alpha t) \tag{3}$$

The value 425 for E, the mechanical equivalent of heat, is expressed in kilogrammeters. We must therefore express p, the normal atmospheric pressure in kilograms per square meter, 103.3×100, and the volume 22.32 liters (cubic decimeters) in cubic meters, 22.32/1000.

Equation (3) then becomes

$$\frac{W}{E} = \frac{10330 \times 22.32}{425 \times 1000} (n_1 - n_b)(1 + \alpha t)$$

or
$$W/E = 0.5424 (n_1 - n_b)(1 + \alpha t)$$

$$\alpha = 1/273 \quad \text{and} \quad 1/273 \times 0.5424 = 0.002, \text{ nearly}$$

Therefore
$$W/E = 0.5424 (n_1 - n_b) + 0.002 (n_1 - n_b) t \tag{4}$$

In the case of explosives the volume v_b is generally negligible with respect to v_1. v_b represents the volume of the explosive for those explosives that are completely converted into gas. n_b is therefore negligible with respect to n_1, and equation (4) becomes

$$W/E=0.5424n_1+0.002n_1t$$

Substituting this value in equation (1)

$$Q_{mv}=Q_{mp}+0.5424n_1+0.002n_1t$$

We will make $t=15°$, since the heats of formation in Table II have been determined for that temperature, and Q_{mp} and Q_{mv} in the above equation will be determined from the table. We have, then, finally,

$$Q_{mv}=Q_{mp}+0.5724n_1 \qquad (5)$$

for the quantity of heat given off at constant volume by the molugram of the explosive.

77. Example 3.—Take, for example, nitroglycerine,

$$\underset{454}{2C_3H_5(NO_2)_3O_3}=\underset{264}{6CO_2}+\underset{90}{5H_2O}+\underset{84}{3N_2}+\underset{16}{\tfrac{1}{2}O_2}$$

We have found at constant pressure, example 1,

$$Q_{mp}=330.4 \text{ l. cal.}$$

From the reaction we see that 2 molugrams of the explosive give off $6+5+3+0.5=14.5$ molecular volumes of gas. 1 molugram, therefore, gives

$$n_1=7.25 \text{ volumes}$$

Substituting in equation (5) we obtain

$$Q_{mv}=330.4+0.5724\times7.25=334.5 \text{ l. cal.}$$

For 1 kilogram of the explosive, example 1,

$$Q_{kv}=\frac{334.5}{227}\times1000=1473.6 \text{ l. cal.}$$

We found at constant pressure

$$Q_{kp}=1455.5 \text{ l. cal.}$$

Potential.—The potential has been defined as the total work that can be performed by the gas from unit weight of the explosive under indefinite adiabatic expansion. The kilogram is taken as the unit weight in the determination of the potential, and the meter as the unit of length. The work unit is therefore the kilogrammeter. The total work from one kilogram of the explosive is equal to the maximum quantity of heat given off by one kilogram multiplied by the mechanical equivalent of heat.

The mechanical equivalent of heat is 425 kilogrammeters. Therefore representing the potential, the total work from a kilogram of the explosive, by W_k we have

$$W_k = Q_{kv} \times 425 \text{ kilogrammeters} \tag{6}$$

78. Volume of Gases.—The volume of gases produced by explosion may be measured experimentally, the gases being drawn off from the calorimetric bomb for this purpose.

The volume of the gases may also be determined theoretically from the reaction.

As previously explained, the molecular volume (the volume of the molugram) of any gas, simple or compound, is 22.32 liters. Therefore in any reaction the molecular volume, at standard temperature and pressure, of the evolved gases is very simply obtained by multiplying the number of gaseous molecules in the formula for the reaction by 22.32.

Example 4.—A formula for the explosion of black gunpowder is

$$\begin{array}{ccccccccccccc} 10KNO_3 & + & 3S & + & 8C & = & 3K_2SO_4 & + & 2K_2CO_3 & + & 6CO_2 & + & 5N_2 \\ 1010 & & 96 & & 96 & & 522 & & 276 & & 264 & & 140 \end{array}$$

The first two products of the reaction are solid. The gaseous products are 6 molecules of CO_2 and 5 of N. Therefore the molecular volume of the gases from 1202 grams of the explosive is, at 0° and 760 mm.,

$$V_m = 11 \times 22.32 = 245.52 \text{ liters}$$

and from 1 kilogram of explosive

$$V_k = \frac{245.52 \times 1000}{1202} = 204.26 \text{ liters}$$

The volumes at any other pressure or temperature may be obtained by means of equations (31) and (34), Chapter III.

79. Temperature of Explosion.—The method of Mallard and Le Chatelier for calculating the temperature of explosion at constant volume in a closed vessel is as follows.

The quantity of heat liberated by the explosion of the molugram of the explosive would, if the specific heat of the products were constant, be equal to the molecular specific heat multiplied by the *rise in temperature.* We would then have

$$Q_{mv} = C_{mv} \times t_1 \tag{7}$$

from which t_1, the rise in temperature, could be obtained. Assuming 15°, an ordinary temperature, as the temperature of the explosive when fired, the temperature of explosion would then be

$$t = t_1 + 15 \tag{8}$$

But it is known that the specific heat increases with the temperature. Assuming that the specific heat varies with the temperature in the manner represented by the linear expression,

$$C_{mv} = a + bt_1 \tag{9}$$

the values of a and b, and the consequent values of C_{mv}, have been deduced for certain gases as follows. The values are given in *small calories.*

	a	b	
For CO_2, SO_2,	6.26	0.0037	$C_{mv} = 6.26 + 0.0037\, t_1$
For H_2O,	5.61	0.0033	$C_{mv} = 5.61 + 0.0033\, t_1$
For gases without condensation,	4.80	0.0006	$C_{mv} = 4.80 + 0.0006\, t_1$

The values of a are the molecular heats of the gases in small calories at the temperature 15°, and the values of b are the increments of the molecular heats for each degree of rise in temperature.

Suppose that the products of an explosion are as follows:

$$\alpha\, CO_2 + \beta\, H_2O + \delta\, P$$

P representing a molecule of a perfect gas. The coefficients a and b for the products of explosion will then be

$$a = 6.26\,\alpha + 5.61\,\beta + 4.8\,\delta \tag{10}$$

$$b = 0.0037\,\alpha + 0.0033\,\beta + 0.0006\,\delta \tag{11}$$

Combining equations (7) and (9) and multiplying Q_{mv} by 1000, since it has been determined in large calories, and a and b are now in small calories, we obtain

$$1000Q_{mv} = at_1 + bt_1^2$$

Solving this equation for t_1 and substituting the resulting value in equation (8), we obtain, for the temperature of explosion

$$t = \frac{-a + \sqrt{a^2 + 4000bQ_{mv}}}{2b} + 15 \tag{12}$$

80. Example 5.—Nitroglycerine. $Q_{mv} = 334.5$ l. cal. (see example 3).

$$\underset{454}{2C_3H_5(NO_2)_3O_3} = \underset{264}{6CO_2} + \underset{90}{5H_2O} + \underset{84}{3N_2} + \underset{16}{0.5O_2}$$

Since the products, as given in the formula, are from two molecules of the explosive,

$$2a = 6.26 \times 6 + 5.61 \times 5 + 4.8 \times 3 + 4.8 \times 0.5 = 82.41$$
$$2b = 0.0037 \times 6 + 0.0033 \times 5 + 0.0006 \times (3 + 0.5) = 0.0408$$
$$a = 41.205 \qquad\qquad b = 0.0204$$

and from equation (12)

$$t = \frac{-41.205 + \sqrt{41.205^2 + 4000 \times 0.0204 \times 334.5}}{2 \times 0.0204} + 15 = 3178°$$

81. Temperature when Solid Products are Formed.—If the explosion gives rise to solid products the heat absorbed in raising the temperature of these products must be considered. In equation (7) C_{mv} must be the mean specific heat of the products of the explosion of a molugram of the explosive.

Suppose that in addition to the gaseous products assumed above, page 155, we have x molugrams of a solid product having a specific heat h referred to its molecular weight. Then a, equation (10), becomes

$$a=6.26\alpha+5.61\beta+4.8\delta+hx \tag{13}$$

The specific heat of a solid product is assumed not to vary with the temperature, therefore the value of b as given by equation (11) is not affected.

The specific heats of substances will be found in Table III at the end of the volume.

Example 6.—Determine the temperature of explosion of the mixture of nitrobenzol and potassium chlorate of example 2.

The reaction is

$$\underset{1266.8}{2C_6H_5NO_2+\tfrac{25}{3}KClO_3}=\underset{528}{12CO_2}+\underset{90}{5H_2O}+\underset{28}{N_2}+\underset{620.8}{\tfrac{25}{3}KCl}$$

From example 2, $Q_{mp}=1500.9$ l. cal.
equation (5), $Q_{mv}=Q_{mp}+0.5724n_1$
page 152, $n_1=12+5+1=18$
$Q_{mv}=1511.2$

From Table III, molecular specific heat of KCl, 12.89
eq. (13), $a=6.26\times12+5.61\times5+4.8+12.89\times25/3=215.39$
eq. (11), $b=0.0037\times12+0.0033\times5+0.0006=0.0615$

eq. (12), $$t=\frac{-215.39+\sqrt{215.39^2+4000\times0.0615\times1511.2}}{2\times0.0615}+15$$

$t=3521°.$

82. Pressure in a Closed Chamber.—The pressure of the gases produced by explosion is a function of the volume occupied by the gases. In a closed chamber the volume available for the gases depends upon whether the products of explosion are wholly gaseous or whether they contain *non-gaseous* matter as well.

PRODUCTS WHOLLY GASEOUS.—We have deduced in equation (47), Chapter III, the following value for the force of an explosive.

$$f = p_0 v_0 T/273 \tag{14}$$

in which, in the metric units that have been chiefly used in the previous calculations, the kilogram and the decimeter,

f is the pressure per square decimeter of the gases from 1 kilogram of explosive, the gases occupying at the temperature of explosion a volume of 1 cubic decimeter.

p_0 the normal atmospheric pressure, 103.3 kilograms per square decimeter,

v_0 the specific volume of the gas, now taken as the volume in cubic decimeters occupied by 1 kilogram of the gas at 0° and 760 mm.,

T the absolute temperature.

The volume V_k, as determined on page 154, is the volume in cubic decimeters, or liters, of the gaseous products from 1 kilogram of the explosive. Therefore

$$v_0 = V_k \tag{15}$$

The absolute temperature $T = 273 + t$, in which t, the temperature of explosion, is taken as the rise in temperature due to the explosion plus 15°, which is the assumed temperature of the explosive when fired.

Substituting the values of p_0, v_0, and T in equation (14) we obtain for the force of the powder

$$f = \frac{103.3 V_k (273 + t)}{273} \text{ kilograms per sq. dec.} \tag{16}$$

RELATION BETWEEN PRESSURE, FORCE OF EXPLOSIVE, AND DENSITY OF LOADING.—We have, equation (49), Chapter III, for the pressure from unit weight of gas confined in the volume v,

$$p = \frac{f}{v - \alpha} \tag{17}$$

in which α is the *covolume* of the gas.

By the process followed in Chapter III in deducing equation (46) from equation (45) this equation may be put in the form

$$P=\frac{f\Delta}{1-\alpha\Delta} \tag{18)*$$

in which P is the pressure per unit of surface of the gases from ω units of weight of explosive,

Δ is the density of loading.

According to Sarrau the covolume is 1/1000 of the specific volume of the gases. Therefore when the products are wholly gaseous we have from equation (15)

$$\alpha=V_k/1000 \tag{19}$$

83. Non-gaseous Products.—When solid or liquid products result from the explosion, these products occupy part of the volume in the chamber and diminish the volume occupied by the gases.

Let y be the weight of gas from unit weight of explosive,

w_0 the volume at 0° and 760 mm., occupied by the gas from unit weight of explosive,

α' the volume, at temperature and pressure of explosion, of the non-gaseous residue from unit weight of explosive.

In this case if we consider as the specific volume of the gas the volume w_0 occupied by the gas from unit weight of the explosive instead of the volume v_0 occupied by unit weight of the gas, f, equation (14), becomes for the new specific volume

$$f=p_0w_0T/273 \tag{20}$$

And if we consider that α, the subtractive term in equation (14), includes the volume of the residue from unit weight of explosive as well as the covolume of the gases for the new specific volume,

$$\alpha=\alpha'+w_0/1000 \tag{21}$$

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[*] This equation is identical with equation (46), Chapter III, deduced by Noble and Abel. They considered α as the volume of the solid residue from unit weight of powder, but later investigations show, as explained in Chapter III, that the covolume of the gases must appear in the equation. When solid products result the value of α must be modified to include the volume occupied by the solid products.

By definition we have

$$w_0 = v_0 y \tag{22}$$

With these new values of f and α equation (17) gives the pressure due to the gases from unit weight of the explosive, and equation (18) may be deduced from it as before.

Therefore when non-gaseous products result from the explosion the pressure is obtained from equation (18) by substituting for f and α the values given in equations (20) and (21).

The volume of the solid matter is easy to calculate, as from the formula of the decomposition we may obtain the weight of the residue from 1 kilogram of the explosive, and it is only necessary to divide this weight by the density.

The densities of substances are given in Table IV at the end of the volume.

84. Example 7.—What is the pressure in a closed chamber of a charge of the explosive of example 6, the density of loading being 0.9?

The reaction is

$$\underset{1266.8}{2C_6H_5NO_2 + \tfrac{25}{3}KClO_3} = \underset{528}{12CO_2} + \underset{90}{5H_2O} + \underset{28}{N_2} + \underset{620.8}{\tfrac{25}{3}KCl}$$

From example 6, $Q_{mp} = 1500.9$

$$Q_{mv} = 1511.2$$
$$t = 3521$$
$$T = 273 + t = 3794$$

Following example 4,

$$V_k = 18 \times 22.32 \times 1000/1266.8 = \underline{317.15 = v_0}, \text{ equation (15)}$$

	KCl.	Gas.
1266.8 kilos explosive produce, kilos	620.8	646
1 kilo explosive produces, kilos	0.49	$\underline{0.51 = y}$
Divide by density KCl, 1.94, Table IV	$\underline{0.2526 = \alpha'}$	

Eq. (22), $w_0 = 317.15 \times 0.51 = 161.75$

Eq. (21), $\alpha = 0.2526 + 0.1617 = 0.4143$

Eq. (20), $f = 103.3 \times 161.75 \times 3794/273 = 232210$ kilos per sq. dec.

Eq. (18), $P = \dfrac{232210 \times 0.9}{1 - (0.4143 \times 0.9)} = 333240$ kilos per sq. dec.

For $\Delta = 1$, $P = 396460$ kilos per sq. dec.

SPECIFIC HEATS AND DENSITIES OF NON-GASEOUS PRODUCTS.—It is assumed in the above discussion that the specific heats and densities of the non-gaseous products remain constant. This assumption is generally inaccurate, and the calculated values of force and pressure for explosives that yield non-gaseous products are therefore uncertain. For these explosives the most satisfactory determinations are made by experiment. Two or more charges of the explosive are exploded in a closed chamber and the values of P measured. Substituting these with the corresponding known values of Δ in equation (18) the values of f and α are determined.

85 Complete Calculation of the Effects of Explosion.—The formula of the reaction for the complete combustion of Sprengel's explosive acid, a mixture of picric acid and nitric acid, is as follows.

$$\underset{1145}{5C_6H_2(NO_2)_3OH} + \underset{819}{13HNO_3} = \underset{1320}{30CO_2} + \underset{252}{14H_2O} + \underset{392}{14N_2}$$

The molecular weight is $1145 + 819 = 1964$.

In the work that follows, the number of the page on which the process is explained, or the number of the equation from which the value is derived, appears on the left.

146, $Q_{mp} = (30 \times 94.3 + 14 \times 58.2) - (5 \times 49.1 + 13 \times 41.6)$
$= 2857.5$ l. cal.

150, $Q_{kp} = 2857.5 \times \dfrac{1000}{1964} = 1454.9$ l. cal.

(5) $Q_{mv} = 2857.5 + 0.5724(30 + 14 + 14) = 2890.7$ l. cal.

153, $Q_{kv} = 2890.7 \times \dfrac{1000}{1964} = 1471.8$ l. cal.

(6) $W_k = 1471.8 \times 425 = 625515$ kgm.

154, $V_m = (30+14+14)22.32 = 1294.56$ liters

154, $V_k = 1294.56 \times \frac{1000}{1964} = 659.14$ liters

(10) $a = 6.26 \times 30 + 5.61 \times 14 + 4.8 \times 14 = 333.54$

(11) $b = 0.0037 \times 30 + 0.0033 \times 14 + 0.0006 \times 14 = 0.1656$

(12) $$t = \frac{-333.54 + \sqrt{\overline{333.54}^2 + 4000 \times 0.1656 \times 2890.7}}{2 \times 0.1656} + 15$$
$$= 3306°$$

(16) $f = \frac{103.3 \times 659.14(273+3306)}{273} = 892650$ kgm. per sq. dec

(19) $\alpha = \frac{659.14}{1000} = 0.65914$

(18) $P = \frac{892650\Delta}{1-0.65914\Delta}$ kilograms per sq. dec.

For $\Delta = 0.8$, $P = 1510700$ kilograms per sq. dec.

CHAPTER V.

METALS USED IN ORDNANCE CONSTRUCTION.

86. Stress and Strain.—A proper understanding of these terms will be helpful in what follows.

When a force is applied to a body the effect produced depends upon whether or not the body is free to move. A force applied to a free body produces motion of the body. A force applied to a fixed body produces change of form of the body.

Stress is the name given to any force or part of a force that produces change of form of the body. The component forces or pressures induced in the interior of the body are also called stresses.

Strain is the effect of the force as measured by the change in form of the body to which the stress is applied.

Stresses are of different kinds, depending on the manner of application of the force; as tensile stress, compressive stress, torsional stress. A torsional stress is a compound stress and may be resolved into a tensile stress on some elements of the material and a compressive stress on others.

Each kind of stress produces a corresponding strain, or effect on the material, the tensile stress producing elongation, the compressive stress compression. As all stresses may be resolved into tensile and compressive stresses, all strains may be resolved into elongation and compression.

Physical Qualities of Metals.—The following qualities of metals are those with which we are most concerned in ordnance construction.

Fusibility.—The property of being readily converted into the liquid form by heat.

Malleability.—The property of being permanently extended in all directions without rupture when hammered or rolled.

Ductility.—The property of being permanently extended without rupture by a tensile stress, as in wire-drawing.

Hardness.—The property of resisting change of form under a compressive stress. A hard metal offers great resistance to such a stress, while a soft metal yields readily and changes its form without rupture. The terms hardness and softness are of course comparative only.

Toughness.—The property of resisting fracture under change of form produced by any stress.

Elasticity.—The property of resisting permanent deformation under change of form. This is one of the most important properties of gun metals, which under the high stresses due to the explosion are subjected to extensive deformation. Through this property the deformations disappear on the cessation of the stress and the metal resumes its original dimensions.

Strength of Metals.—The strength of metals is ordinarily determined by physical tests in a testing machine. As the tensile strength of metals is less than the compressive strength, usually a tensile test only is applied. A test specimen is cut from the metal to be tested and is prepared in suitable form to be inserted in the machine. The area of the cross section of the test specimen is usually a square inch or some aliquot part of a square inch.

In the machine the test piece is subjected to a tensile stress, the amount of which is recorded by a sliding weight on a scaled beam. The test piece stretches under the applied stress. With elastic metals it will be found that up to the application of a certain stress the test piece will resume its original length if the stress is removed, but on the application of a stress greater than this the test piece will remain permanently elongated. When permanent distortion occurs the metal is said to have a permanent set.

ELASTIC LIMIT.—The stress per square inch applied at the moment that the permanent set occurs is called the elastic limit of the metal. Within this limit the metal has practically perfect elasticity and does not suffer permanent deformation.

As the stress increases beyond the elastic limit the metal stretches permanently and more rapidly, the cross section at the weakest point reduces, and finally the test piece ruptures.

The elastic strength of metals will be found more fully treated in the discussion of the elastic strength of guns in Chapter VI.

87. **TENSILE STRENGTH.**—The stress per square inch that produces rupture of the metal is called the tensile strength.

ELONGATION AT RUPTURE AND REDUCTION OF AREA.—In ordnance structures the stresses are not expected to exceed the elastic limit of the metal, but should they by any chance exceed this limit the tensile strength of the metal and its capacity to permanently elongate before rupture become of importance. The permanent elongation will serve as a warning that the elastic strength has been exceeded. The reduction of area of cross section is intimately connected with the elongation. In the tests of metals for ordnance purposes these particulars are therefore always noted and limits are prescribed. For the measurement of the elongation at rupture the parts of the ruptured test piece are brought together and the distance is measured between two punch marks that were made on the test piece before insertion in the testing machine.

The tensile test therefore includes the determination of the elastic limit, the tensile strength, the elongation at rupture, and the reduction of area of cross section. The last two are recorded in percentages of the original dimensions.

The following table shows the physical requirements demanded by the Ordnance Department in the principal metals used in ord-

	Elastic Limit.	Tensile Strength.	Elongation at Rupture.	Contraction of Area.
	lbs. per sq. in.	lbs. per sq. in.	per cent.	per cent.
Copper	......	32,000	22.0	....
Bronze, No. 1	......	28,000	....	....
Bronze, No. 4	......	60,000	20.0	....
Tobin bronze	......	60,000	25.0	....
Cast iron, No. 1	......	22,000	....	....
Cast iron, No. 2	......	* 28,000	....	....
Wrought iron	22,000	50,000	25.0	35.0
Cast steel, No. 1	25,000	60,000	16.0	24.0
Cast steel, No. 3	45,000	85,000	12.0	18.0
Forged steel, No. 1	27,000	60,000	28.0	40.0
Forged steel (caps)	......	† 60,000	30.0	45.0
Forged steel (tubes)	46,000	86,000	17.0	30.0
Forged steel (jackets)	48,000	90,000	16.0	27.0
Forged steel (hoops)	53,000	93,000	14.0	20.0
Forged steel, D	100,000	120,000	14.0	30.0
Nickel steel	65,000	95,000	18.0	30.0
Steel wire (guns)	100,000	160,000	....	....

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* Cast iron No. 2 must not show a tensile strength of more than 39,000 pounds per square inch.

† The tensile strength of steel used in caps for armor piercing projectiles must not exceed 60,000 pounds.

nance construction, the requirements varying for each kind of metal according to the use to which it is destined.

Testing Machine.—The standard government testing machine is at Watertown Arsenal, Mass. It has a testing capacity of 800,000 lbs.

A smaller testing machine, with a capacity of 50,000 lbs., is shown in Fig. 26. The specimen of the metal to be tested is turned to the shape shown by the piece marked 1. The ends of the test specimen are grasped by clamps fixed in the upper fixed head, *f*, of the machine and in the lower movable head *m*. Four heavy vertical screws pass through the corners of the movable head, and by their means the movable head is moved toward or from the fixed head, exerting on the specimen held between the clamps a force of compression or of extension as desired. The amount of this force is measured by a sliding weight, *w*, on a scaled beam in the same manner as a weight is determined on an ordinary scale. The total force divided by the area of cross section of the test specimen gives the force exerted per square inch.

A graphic representation of the relation between the force exerted and the change in length of the test specimen is made on the indicator card, *c*. An indicator card, showing the results of tensile tests on specimens of several metals, is shown in Fig. 25. Within the elastic limit of the metal the elongation of the test piece is proportional to the tensile stress. Up to this point, therefore, the line made by the indicator will be a straight line. At the elastic limit, where the bends occur in Fig. 25, permanent set occurs, and the test piece thereafter elongates more rapidly than the stress increases.

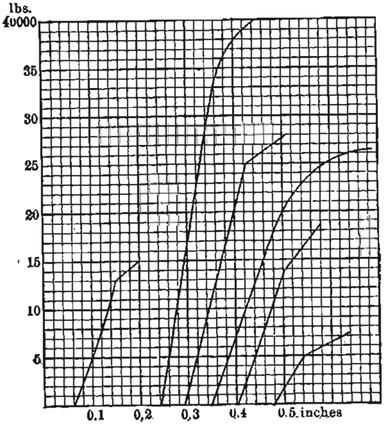

FIG. 25.

To prevent injury to the indicating apparatus by the shock that occurs when the test piece breaks, the indicator is usually disconnected after the elastic limit has been registered.

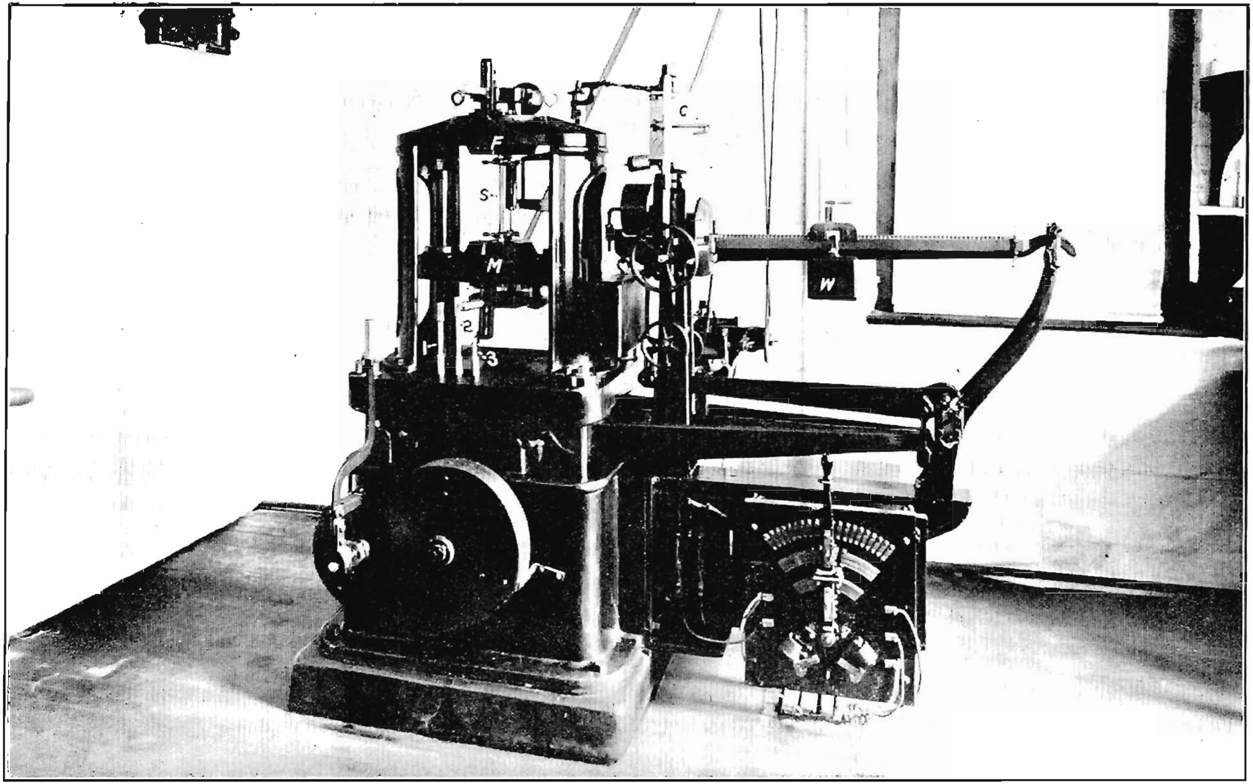

Fig. 23.—Riehlé Testing Machine. Capacity, 50,000 lbs.

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Broken test-pieces are shown at 2 and 3 in Fig. 26. Comparing these with test piece 1, the effects of the test, the elongation at rupture, and the contraction of area are apparent.

88. Copper, Brass, Bronze.—Pure copper is used for the bands of projectiles. In alloys, as brass and bronze, it enters into the construction of parts of guns and gun carriages not usually subjected to great stress. In this form, too, it is extensively employed in the manufacture of cartridge cases, fuses, primers, gun sights, and instruments. Brass is an alloy of copper with zinc. Bronze is an alloy of copper with tin and usually a small quantity of zinc. The addition of zinc or tin produces a harder and stronger metal better suited than the soft copper for the uses to which these alloys are applied. By the addition of aluminum or manganese in the alloy the strong hard bronzes known as aluminum bronze and manganese bronze are produced.

Iron and Steel.—When iron ore is melted in the furnace the product obtained, called pig iron, is an alloy of iron with carbon, the carbon content being about 5 per cent. This alloy may be readily fused and cast, and is then called *cast iron.* By various processes in the furnace the amount of carbon in the iron may be reduced. When the quantity of contained carbon is between two per cent and two tenths of one per cent the product is *steel.* When there is less than two tenths of one per cent of carbon we have *wrought iron.*

As the amount of carbon is reduced the qualities of the metal change in a marked degree. Cast iron is easily fusible, is hard and not malleable or ductile, cannot be welded, and has a crystalline structure. Wrought iron, on the other hand, is practically infusible, is soft, and possesses great malleability and ductility. It is easily welded and has a fibrous structure.

The properties of steel lie between those of wrought iron and cast iron, and the steel partakes of the characteristics of one or the other according to the percentage of carbon contained. Thus low steel, that is, steel low in carbon, is comparatively soft and may be readily welded or drawn into wire, while high steels are harder and more brittle and weld with difficulty.

Chief Constituents.—When examined under the microscope iron and steel are found to be conglomerate in structure, consisting of microscopic particles chiefly of the following substances in widely varying proportions.

1. Pure or nearly pure metallic iron, called *ferrite;* soft, weak, and very ductile.

2. A definite iron carbide, Fe_3C, called *cementite,* which is extremely hard and brittle, but probably very strong under a tensile stress.

The character of the iron or steel depends upon the proportions of these two chief constituents. The steels which are especially soft and ductile, as rivet and boiler plate steels, consist chiefly of the soft ductile ferrite, the proportion of cementite in these steels not exceeding perhaps 1 per cent. The harder steels, like rail steels, which are called upon to resist abrasion, contain a much larger percentage of cementite, about 7 per cent, and about 93 per cent of ferrite. As the proportion of cementite increases and that of ferrite decreases the hardness increases and the ductility diminishes. The tensile strength increases to a maximum when the cementite amounts to about 15 per cent of the whole, and then decreases.

The percentage of carbon in the metal is $\frac{1}{15}$ the percentage of cementite the molecular weight of Fe_3C being 180, of which 12 parts are carbon.

GRAPHITE. CAST IRON.—In *gray cast iron* there is present, in addition to the ferrite and cementite, a quantity of nearly pure carbon in the form of graphite. The graphite is in thin flexible sheets which form a more or less continuous skeleton running through the mass of gray cast iron. The graphite makes the metal weak and brittle.

White cast iron contains but little graphite, but has a much higher percentage of cementite than either gray cast iron or steel. The large percentage of cementite, over 60 per cent, brings the carbon content to about 4½ per cent, making the iron extremely hard and brittle.

SLAG. WROUGHT IRON.—Wrought iron contains, in addition to the matrix of ferrite and cementite common to all irons, a small quantity of slag, a silicate of iron formed in the process of puddling. The presence of this slag forms the chief difference between wrought iron and the low carbon steels.

89. Hardening and Tempering Steel.—The distinguishing characteristic of steel when compared with cast or wrought iron is the property it possesses of hardening when cooled quickly after being heated to a red heat, and of subsequently losing some of its added hardness when subjected to a lower heat.

There is more or less confusion in the use of the terms applied to the two processes. By some the first process, quick cooling from a high heat, is called *tempering*, and the second process, reheating to a lower heat, is called *annealing*. By others the first process is called *hardening* or *quenching*, and the second process, which mitigates or lets down the hardness, is called *tempering*. The more recent tendency is toward the use of the latter terms, and following what is perhaps the better practice, we will call the first process hardening and the second process tempering.

EFFECT OF HEAT.—In order to get a comprehensive idea of the processes of hardening and tempering it will be necessary to go somewhat further into the constitution of steel and to learn how its constitution is affected by heat. As before stated, the chief constituents of steel are ferrite (iron) and cementite (Fe_3C). These exist in different proportions, and the behavior of the metal under heat treatment is dependent to a certain extent on the proportions of these substances. The amount of carbon in the steel depends on the proportion of cementite. The results attending the application of heat to steel are chiefly due to the effect of the heat on the condition of the carbon.

Austenite.—When steel is heated to a temperature of from 700 to 1000 degrees centigrade, depending on the quantity of carbon contained, the ferrite and cementite of which it is composed are converted into a substance called *austenite*, which, according to Howe, Professor of Metallurgy in Columbia University and an eminent writer on steel, is a solid solution of carbon in iron. He defines a solid solution as a solid that bears the same relation to the definite solid chemical compounds that a liquid solution, salt water for instance, bears to the definite liquid chemical compounds, as water.

Austenite is a distinct substance with properties of its own. When it contains 0.75 per cent or more of carbon it is extremely hard and brittle. Its hardness and brittleness are approximately proportional to the percentage of carbon contained.

The temperature at which austenite forms depends upon the proportions of ferrite and cementite in the metal. When these proportions are such that there is 9/10 of 1 per cent of carbon in the metal, that is when the metal consists of $0.9 \times 15 = 13.5$ per cent of cementite and 86.5 per cent of ferrite, the transformation of these constituents into austenite takes place at a lower temperature than when they are present in any other proportions.

Pearlite. Eutectoid.—The mixture of ferrite and cementite containing 0.9 per cent of carbon is given a specific name, *pearlite,* and is characterized as a *eutectoid,* which means a solid mixture in the particular proportions that give to the mixture the lowest transformation point under the action of heat. The corresponding term applied to a liquid solution is *eutectic.* Thus the eutectic solution of salt in water contains 23.6 per cent of salt. When this percentage of salt is present the solution forms at the lowest temperature, and conversely the salt remains longest in solution as the temperature is lowered.

Steel containing less than 0.9 of one per cent of carbon is considered to be composed of pearlite and an excess of ferrite, while the steels higher in carbon contain pearlite and an excess of cementite.

Now referring to Fig. 27 we will see at what temperature the various mixtures are transformed into austenite. The proportions of carbon and iron in the metal are shown on the horizontal axis. The curves are worded to show the transformations that occur as the metal cools from the molten state.

When there is 0.9 per cent of carbon in the metal we have pearlite, which is converted into austenite at a temperature of about 680° C., as shown in the figure by the intersection of the line A_1 at the point S. In the steels lower in carbon, which are composed of pearlite and an excess of ferrite, the pearlite is transformed at the same temperature as before, but the excess of ferrite requires a higher temperature, as shown by the line SA_3, so that the transformation is not complete for any particular composition until that temperature is reached which is indicated by the intersection of the ordinate representing the composition with the line SA_3.

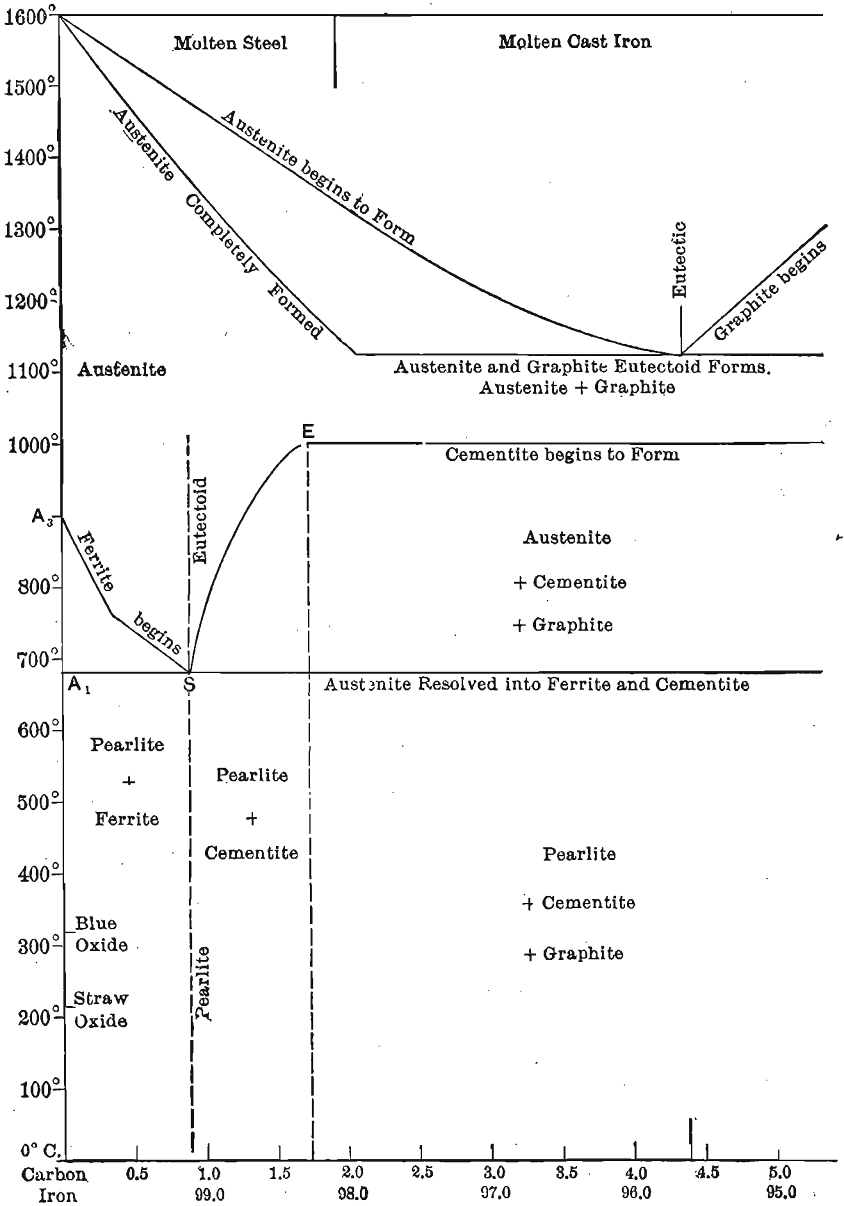

FIG. 27.—Effect of Heat on Iron and Steel.

And similarly for the higher carbon steels containing an excess of cementite; and for the cast irons, which, containing more than 2 per cent of carbon, are composed of pearlite, cementite, and graphite.

90. Hardening.—It will now be easy to understand the process of hardening steel by means of high heat followed by quick cooling. The high heat causes the formation of austenite in the metal. If the metal is allowed to cool slowly the austenite is retransformed into ferrite and cementite. This transformation requires an appreciable time, and if the metal is suddenly cooled from its high temperature the retransformation is prevented, and the hard austenite is preserved in the cold metal.

The change in the character of steel being due principally to the change in the condition of the carbon between its states in pearlite and cementite and in austenite, the effect of the heat treatment is greater as the proportion of carbon in the metal is greater. Thus the low-carbon steels containing from 0.06 to 0.10 per cent of carbon are in general but little affected by heat treatment and are practically incapable of being hardened, while the high-carbon steels and some cast irons are greatly affected and may be given extreme hardness.

The hardness and brittleness induced increase with the rapidity of cooling without limit, but they are apparently nearly independent of the temperature from which the sudden cooling begins, provided that this temperature is above the line of complete transformation, the line A_3SE, Fig. 27. If the metal is suddenly cooled from temperatures between the beginning and end of the transformation, that is at temperatures between the lines A_1 and A_3SE, the hardening increases as the quenching temperature rises. The range of temperature between the lines A_1 and A_3SE is called the *critical range.* In this range the hardness increases with the quenching temperature, but above the critical range the hardness is independent of the temperature.

The hardening of steel greatly increases its tensile strength and elastic limit, but it makes the steel brittle, thus reducing its toughness, as shown in test pieces by reduced elongation at rupture and diminished contraction of area of cross section.

The tensile strength of low-carbon steels increases with the rapidity of cooling without limit. In high-carbon steels the tensile strength at first increases with the rapidity of cooling, but reaches a maximum and then declines; that is, there is a certain rapidity of cooling that will give to any one of these steels its maximum tensile strength. This may be due to the fact that rapid cooling induces internal strains that may become so great as to be destructive.

The following table, taken from *Iron, Steel, and other Alloys*, by Henry Marion Howe, LL.D., well shows the effects of differences in the rapidity of cooling of steel containing 0.21 per cent of carbon.

Cooled in	Tensile Strength.	Elastic Limit.	Elongation.	Contraction of Area.
	lbs. per sq. in.	lbs. per sq. in.	per cent in 2 in.	per cent.
Iced brine.	237,555	237,170	2.0	1.30
Cold water.	216,215	. . . .	1.5	1.67
Oil.	174,180	. . . .	2.9	1.403
Air.	86,797	54,342	27.76	57.829
In furnace.	80,103	44,221	28.15	54.749

91. Tempering.—Hardened steel is tempered by slight reheating, say to 200° or 300° C. This process lessens the hardness and brittleness of the steel, and thus increases its toughness. The austenite of the hardened steel is in a stable condition only when above the transformation temperature. As the temperature of the steel diminishes the austenite tends to change into ferrite and cementite. In the hardening process this tendency is resisted by the frictional resistance due to the sudden cooling, and the austenite is retained in an abnormal condition in the cold metal. The reheating of the metal in tempering appears to lessen the molecular rigidity of the austenite, and to afford opportunity for part of the austenite to follow the course that it would have taken in slow cooling through the transformation range and thus to be converted into pearlite. The higher the reheating the more does the change occur.

The rate of cooling after tempering has no effect on the steel, since the highest temperature of reheating has determined how far the change from austenite to pearlite may proceed, and no further change can occur at a lower temperature. It is therefore immaterial whether the cooling after tempering be slow or rapid.

Tempering has the effect of reducing somewhat the tensile strength and elastic limit of hardened steel, while it increases its toughness, as shown in test specimens by increased elongation at rupture and increased contraction of area of cross-section.

It will be seen that by proper regulation of the temperatures in the processes of hardening and tempering an extensive control of the properties of the metal is obtained, permitting the production of metal of the quality best suited to any particular purpose.

The tempering temperatures may be judged within limits by the color given to the steel, as it is heated, by the various oxides that form successively on the surface. The following table shows the temperatures at which the colors appear, and the tempering points for steels for various purposes.

220° C.,	straw; razors, surgical instruments.
245	yellow; penknives, taps, dies.
255	brown; cold chisels, hatchets.
265	brown with purple spots; axes.
275	purple; table knives, shears.
295	violet; swords, watch springs.
320	blue; saws.
525	incipient red.
700	dark red.
950	bright red.
1100	luminous yellow.
1300	incipient white.
1500	white.

Gun steel is tempered at temperatures between 600° and 675° C.

Annealing.—If the steel after being hardened is reheated to the critical temperature and then cooled slowly the austenite is completely converted into pearlite and ferrite or cementite, and the steel reverts to its original condition, losing all its added hardness and brittleness. This process is called *annealing*.

92. Other Substances.—In addition to the carbon in the metal, there are other substances, some of which are always present and others that may be added, that affect the qualities of steel.

Sulphur, phosphorus, manganese and silicon are usually present to a greater or less extent in all steels. If present in too large a percentage sulphur produces what is called *hot shortness* in the metal, that is brittleness when hot, while phosphorus makes the metal *cold short*, or brittle when cold. Manganese and silicon when present in proper percentages improve the qualities of the metal.

Chromium and *tungsten* give hardness to the steel without brittleness.

Nickel also greatly increases the toughness of the steel. Nickel steel for guns contains about 3½ per cent of nickel.

Uses.—Cast iron, wrought iron, cast steel and forged steel are all used in ordnance constructions. Cast iron on account of its cheapness and ease of manufacture in irregular shapes is used when practicable wherever great strength is not required, as in projectiles for the smaller guns and in parts of carriages not subject to wear or to high stresses.

Wrought iron is not now extensively used in ordnance constructions. The older seacoast carriages were almost wholly made of this metal.

Wherever great strength is required steel is employed. Cast steel is used in those parts that do not require the greater strength of forged steel, or that on account of their irregular shapes cannot be readily produced as forgings, such as the chassis and top carriages of seacoast gun carriages. Cast steel has also been used for projectiles and for guns, but without great success.

In structures or parts of structures requiring great strength, or subject to wear, forged steel only is used. Guns and armor and armor-piercing projectiles are now made of forged steel only, and the operative parts of gun carriages and of other structures are principally of this metal.

Gun Steel.—Of two steels, one high in carbon and the other low in carbon, the steel with the higher percentage of carbon will, with similar treatment, have the higher elastic limit. Since the elastic limit of the metal is the limit of the strength considered in the construction of guns, it would appear that the metal with the highest elastic limit would be the most desirable. But high steel is more difficult to manufacture than low steel, and in large pieces there is much greater liability to flaws, strains, and incipient cracks. After passing the elastic limit the hard steel has little remaining strength and breaks without warning, while the low steel, due to its greater toughness, yields considerably without fracture. For these reasons a low steel containing about one half of one per cent of carbon is used in the manufacture of guns.

MANUFACTURE OF STEEL FORGINGS FOR GUNS.

93. Open Hearth Process.—All gun steel at the present day is made by the open hearth process, which derives its name from the fact that the receptacle in which the steel is melted is open at the top and exposed to the flame of the fuel, which plays over the surface and performs a principal part in the formation of the steel. The product is called Siemens or Siemens-Martin steel, according to the ingredients contained.

The open hearth furnace, invented by Dr. Siemens, consists of the following essential parts:

1. The gas producer;
2. The regenerators;
3. The furnace proper.

THE GAS-PRODUCER.—The fuel used in the Siemens furnace is gaseous, and is obtained from ordinary fuel by subjecting the fuel to a preliminary process in the gas producer. This apparatus, Fig. 28, consists of a rectangular chamber of fire-brick, one side, *B*, being inclined at an angle of from 45 to 60 degrees. *A* is the grate. The fuel, which may be of any kind, is fed into the producer through the hopper *C*. As the fuel slowly burns, the CO_2 rises through the mass above it and absorbs an additional portion of C, becoming converted into 2CO. This gas passes out of the opening *D* into a flue. In order to cause it to flow toward the furnace it is led through a long pipe, *E*, where it is partially cooled, and then descends the pipe *F* leading to the furnace. The gas in *F* being cooler than that in *E* and *D*, a constant flow of gas from producer to furnace is maintained.

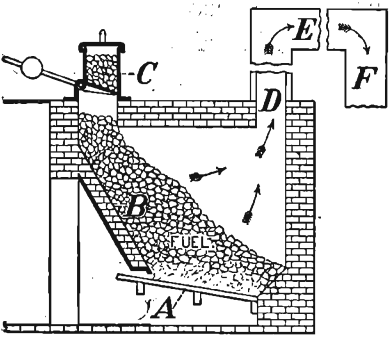

FIG. 28.

THE REGENERATORS.—The gas entering the furnace is, as has been stated, CO. To burn it to CO_2, air must be mixed with it. This mixture is made in the furnace proper, the CO and air being kept separate till they reach the point where they are to burn. The CO is cooled to some extent, as shown, before being admitted to the furnace.

To heat both air and CO before they are mixed and burned, and to accomplish this economically, and raise the gases to a high temperature, the waste heat of the furnace is employed. The heating of the gases is accomplished by means of the regenerators, Fig. 29. They consist of four large chambers, usually placed below the furnace, filled with fire-brick. The fire-brick is piled so that there are intervals between the bricks to allow the passage of gas and air. When the furnace is started, CO is admitted through *A* and air through *B*, both *A* and *B* being cold. The gases pass between the fire-bricks in *A* and *B* and through flues at the top, and flow into the furnace proper, where they are lighted. The products of combustion are caused to pass through *C* and *D*, which are similar chambers. In doing so these products heat the fire-bricks in *C* and *D*. After some time—about one hour generally—by the action of valves controlled by the workmen, the CO and air are caused to enter the furnace through *D* and *C* respectively, and the products of combustion to pass out through *A* and *B*. In this case the CO and air, entering the heated chambers *D* and *C*, are raised to a high temperature before ignition, and the temperature of the furnace is thereby greatly increased. It is also

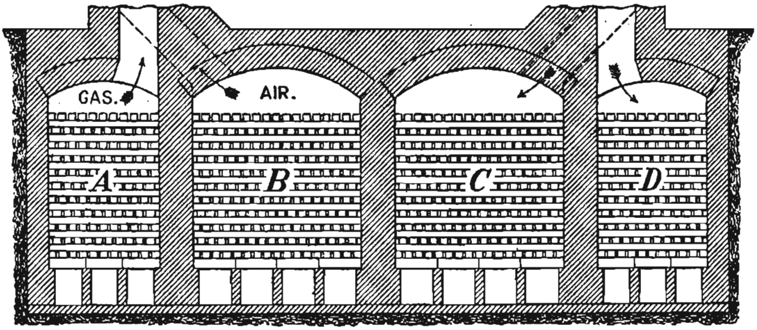

FIG. 29.

evident that A and B will be more highly heated than C and D were, and therefore when the next change is made, the gas and air passing through A and B will be more highly heated than when they passed through D and C, and so on.

The action of the furnace is therefore *cumulative*, and its only limit in temperature is the refractoriness of the material. By regulating the proportions of gas and air, which is readily done, the temperature may be kept constant.

94. THE FURNACE.—The furnace proper, Fig. 30, is a chamber situated above the regenerating chambers. The dish-shaped cast iron vessel D, lined with refractory sand S, constitutes the hearth of the furnace. The iron vessel is supported in such a manner that the air may circulate freely around it and keep it from melting. The iron that is to be converted into steel is piled on the hearth of the furnace.

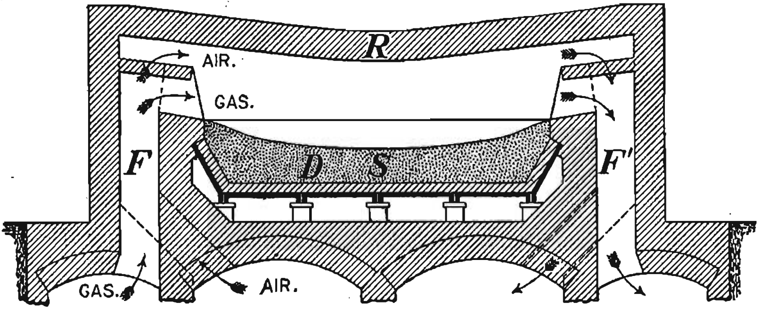

FIG. 30.

The gaseous fuel and air enter by the flues F, and the products of combustion escape by the flues F', or the reverse, according to the position of the regulating valves. The gases are ignited as they enter the furnace. The sloping roof R, lined with fire-brick, deflects the flames over the metal on the hearth.

At opposite ends of the furnace are a charging door for admission of the metal, and a tap hole, closed with a plug of fire-clay, for drawing off the finished steel.

OPERATION.—The process consists in the decarbonizing of cast iron to the point at which the metal contains only that percentage of carbon that is desired in the steel, and in the partial removal from the iron of those impurities, such as silicon, manganese, and phosphorus, that are injurious to the steel if present in too large quantities.

Pig cast iron heated to a red heat in a separate furnace is piled on the hearth of the Siemens furnace, and a quantity of steel or wrought iron scrap is usually added to the charge to reduce the percentage of carbon in the mass.

By the action of the furnace the charge is soon melted. Under the influence of the heat the carbon oxidizes to carbonic oxide gas, which escapes; the silicon oxidizes to silica and the manganese to manganous oxide. The silica and manganous oxide unite with the slag which floats in a thin layer on the molten metal.

The percentage of carbon in the steel at any stage of the process is determined by taking samples from the metal, cooling them, and observing their fracture on breaking; and by dissolving portions of the specimens in nitric acid and comparing the color with the colors of standard solutions of steel containing different percentages of carbon. In this way the composition of the steel can be exactly regulated, for the metal can be kept in a melted state without damage for a considerable time, and the character of the flame can be made oxidizing or reducing at will, according to the relative amounts of air and CO admitted.

The decarbonizing process is often continued until the percentage of carbon remaining in the steel is less than the percentage desired. The desired percentage is then obtained by the addition of pig iron containing a known percentage of carbon, such as spiegeleisen or ferromanganese, or by the addition of ore.

The lining of the hearth, *S* Fig. 30, is of sand when the iron to be reduced does not contain a harmful percentage of phosphorus. The process is then called the *acid process*, from the silicious or acid nature of the slag. When the iron contains a larger percentage of phosphorus a basic lining, as magnesia or calcined dolomite, is required for the removal of the phosphorus. The slag formed in the basic process is strongly retentive of phosphorus and removes the excess of this substance from the metal.

The reduction of a charge of metal in the Siemens furnace ordinarily takes about eight hours.

When the steel has attained its desired composition the furnace is tapped and the metal cast into ingots.

95. Other Processes.—*The crucible process* is used to some extent by Krupp in the production of gun steel. The ingredients of the steel are melted together in crucibles, and the resulting steel is poured from the crucibles into a common reservoir from which the ingots are cast.

The Bessemer Process, though important and producing large quantities of steel, is not used in making gun steel.

Casting.—The molten metal is drawn into an iron ladle which depends from a crane in front of the furnace. The ladle, Fig. 31, is lined with refractory sand. It is provided with trunnions, T', so that it may be tipped for pouring the metal into the mold, or it may have a tap hole, T, in the bottom, closed with a plug of fire-clay. The plug is lifted and replaced by means of a rod R also encased in refractory sand. There is an advantage in drawing the metal from the bottom of the ladle in that the scoria and impurities that float on the surface may be kept out of the mold. The metal if very hot is poured slowly into the mold in a thin stream, thus allowing opportunity for escape of the gases that it contains. If at a lower temperature it may be poured more quickly. It is frequently allowed to cool to the desired temperature in the ladle.

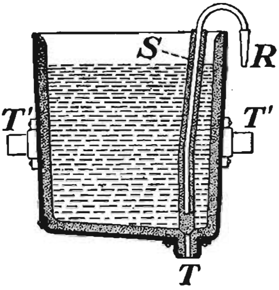

FIG. 31.

Molds.—In the casting of ordinary ingots, the iron or steel molds into which the metal is poured from the ladle are slightly conical in shape, Fig. 32, to facilitate their removal from the ingot. They are of various cross sections, depending on the shape of the ingot desired. The interior surface is covered with a wash of clay or plumbago.

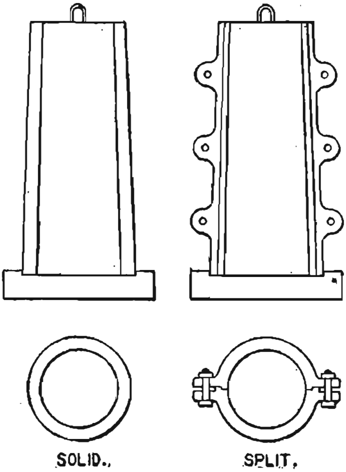

FIG. 32.

Sinking Head.—In all castings, whether of iron, steel, or other metal, an excess of metal, called the sinking head, is left at the top of the mold. The pressure due to the weight of this metal gives greater density to the casting. The sinking head also serves to collect the scoria and impurities which rise to the top, and it provides metal to fill any cracks or cavities that may form in the cooling of the ingot.

Defects in Ingots. *Blow Holes.*—The gases in the melted metal, unable to escape from the mold, form holes in the ingot, called blow holes. These cannot be detected, nor can they be removed by forging. Forging changes their form only without giving continuity to the metal. Blow holes are more prevalent in Bessemer than in open hearth steel and are more apt to occur at low temperatures of casting, when the metal hardens before the gas can escape.

Pipes.—The metal in contact with the molds cools first and solidifies. As the cooling and consequent contraction extends toward the center, the liquid metal is drawn away from the center, leaving cavities called pipes along the axis of the ingot. Pipes most frequently occur when the metal is cast too hot. Thus on the one hand too low a temperature causes blow holes and too high a temperature pipes.

Segregation.—As the various constituents of the steel (iron, silicon, manganese, etc.) solidify at different temperatures, it frequently happens that they separate from each other as the ingot cools, forming what is called segregation. This gives a different structure to the metal and greatly weakens it. Segregation is more likely to be found toward the center of the ingot and in the upper portions.

96. Whitworth's Process of Fluid Compression.—The purpose of this process, invented by Sir Joseph Whitworth of England, is to remove as far as possible the blow holes, pipes, and other defects from the ingot and to give the metal greater solidity and uniformity of structure than can be obtained in the ordinary method of casting. The object is accomplished, to a large extent, by the application of enormous pressure on the metal while in the fluid state in molds so constructed as to allow free escape of the gases.

The flask, *f* Fig. 33, made of cast steel, is of great strength to withstand the great pressure. It is built up of cylindrical sections which are bolted together to the desired length. The interior of the flask is lined with vertical wrought iron bars, *b*, whose long edges are cut away or beveled to form channels, *a*, by means of which the gas may escape: the interior and exterior channels thus formed being connected by grooves, *c*, cut in the sides of the bars at short intervals. The cylinder formed by the interior surfaces of the bars is lined with refractory sand. A cast iron plate, *d*, through which are continued the longitudinal gas channels closes the mold at the bottom. The mold rests on a car in the bottom of a pit.

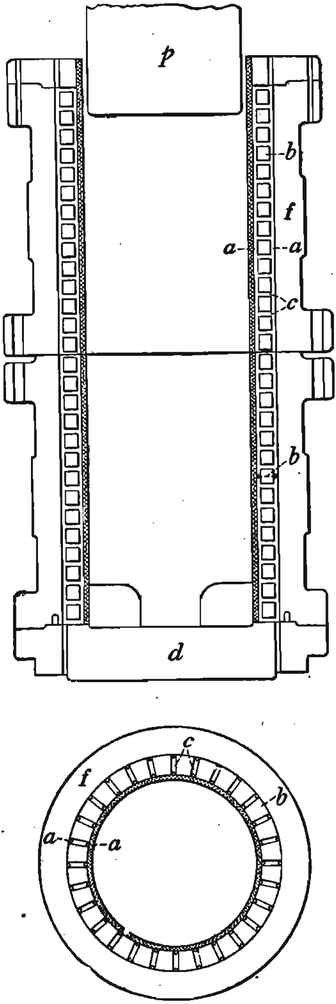

FIG. 33.

When the mold is filled with metal the car is run into a hydraulic press with an adjustable head. The head, *p*, of the press, of diameter only slightly less than the interior of the mold, is brought down against the molten metal and locked in that position. The metal wells up around the head of the press and, quickly cooling, forms a solid mass which with the head completely closes the top of the mold.

The press is constructed with its piston at the bottom so that the pressure may be applied on the bottom of the car that carries the mold.

By the pressure on the bottom of the car, gradually applied until a pressure of six tons to the square inch is reached, the car and mold are slowly forced upward. The molten metal is compressed by the applied pressure, and the gas, forced through the sand lining and the channels between the lining bars, issues from the top and bottom of the mold in a violent flow of flame. The pressure is continued until the column of metal has shortened one eighth of its length. A uniform pressure of about 1500 pounds to the square inch is then left on the ingot while it cools, to follow up the metal as it contracts and prevent the formation of cracks.

97. **Processes After Casting.**—The specifications for gun forgings require that the forgings be made from that part of the ingot that remains after 30 per cent by weight has been cut from the top of the ingot and 6 per cent from the bottom. These parts are cut off, as they are likely to contain most of the defects in the ingot.

For hollow forgings the center of the part selected is then bored out in a heavy lathe, or punched out if the ingot is short.

Heating.—The ingot is then heated preparatory to forging. The heating is accomplished in a furnace erected near the forging hammer or press, and is conducted with great care. The cooling of the ingot in the mold has left in the metal strains due to the successive contraction of the interior layers. Assisted by unequal expansion in heating the strains may cause cracks to develop in the ingot. Great care is therefore exercised that the heating shall proceed slowly and uniformly, thus avoiding the overheating of the exterior layers of metal before the heat has thoroughly penetrated to the interior.

Forging.—The heated ingot is forged either by blows delivered by a steam hammer or by pressure delivered by a hydraulic forging press. Under the slow pressure of the forging press the metal of the forging has more time to flow, the effect of the treatment is more evenly distributed, and the metal is more uniformly strained. This process is therefore preferred in the manufacture of gun forgings.

Fig. 34 is a reproduction from a photograph of a 5000-ton hydraulic forging press at the works of the Bethlehem Steel Co. The print shows a bored ingot for the tube of a 12-inch gun being forged on a mandrel. The outer diameter of the ingot is reduced by the forging and the length of the ingot increased. The diameter of the bore remains practically unchanged. The outer end of the ingot is supported from an overhead crane.

The ingot is turned on the anvil of the press, and advanced when desired, by means of the chain seen through the press. The method of turning is better shown in the plate following.

The movements of the head of the press are controlled by means of levers situated at a short distance to the right of the press. The operator at the lever sees recorded on the dial the pressure exerted at any instant.

Fig. 35 shows a 10-ton steam hammer forging a solid ingot for a 3-inch gun. The ingot is supported from an overhead crane and is nearly balanced in the sling chain by the bar of iron, called a porter bar, clamped to the ingot and extending to the rear. By bearing down on the porter bar the ingot is lifted off the anvil and may then be moved by the crane back and forth under the hammer. The ingot is turned under the hammer from the crane by means of the gearing shown in the upper part of the picture.

The movements of the hammer are controlled by the man at the left through the levers shown at his hand.

98. Hollow Forgings.—In forging bored ingots a solid steel shaft called a mandrel is passed through the bore of the heated ingot, and the method pursued in forging depends upon whether the length of the ingot is to be increased without change of interior diameter, as in forging a gun tube, or whether the diameters of the ingot are to be enlarged, as in forging hoops. In the first case the ingot, on a mandrel of proper diameter, is placed directly on the anvil of the press, as shown in Fig. 34. The effect of forging is then to increase the length of the ingot and decrease the outer diameter while maintaining the interior diameter unchanged. The mandrel is withdrawn from the forging by means of a hydraulic press.

In forging hoops, the mandrel rests on two supports on either side of the head of the press, Fig. 36, and is itself the anvil on which the forging is done. By turning the mandrel new surfaces of the hoop are presented to the press. The walls of the hoop are reduced in thickness by the forging, the diameters of the hoop being increased, while the length is not materially changed.

The specifications for gun forgings require that the part of a solid ingot used for a tube forging shall have before forging an area of cross section at least four times as great as the maximum

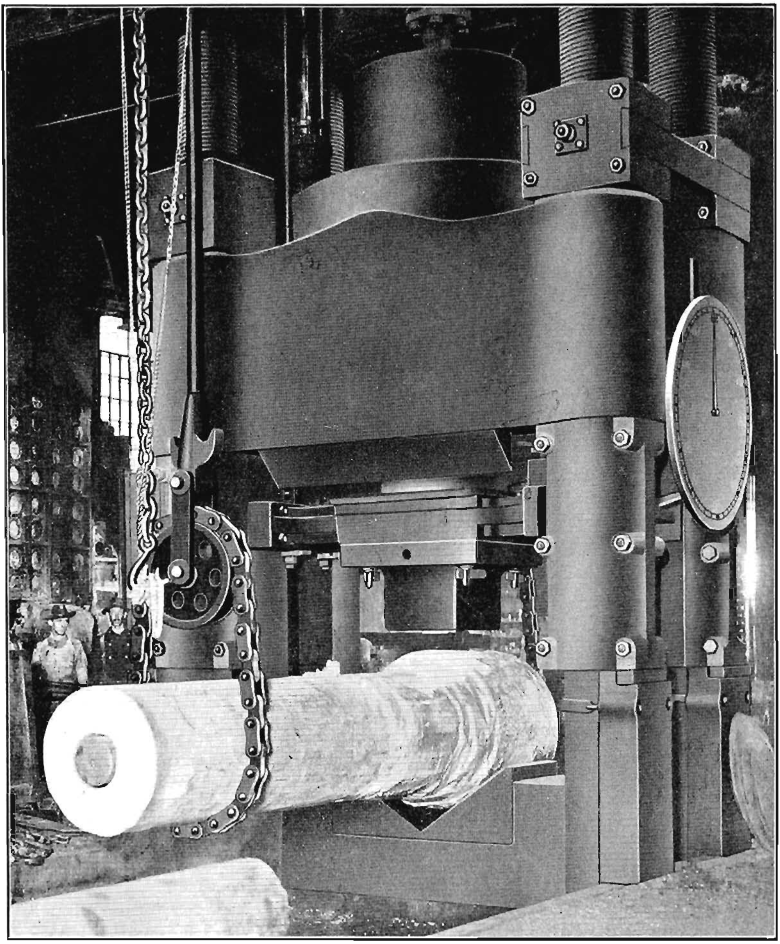

FIG. 34.—5,000-ton Hydraulic Forging Press.

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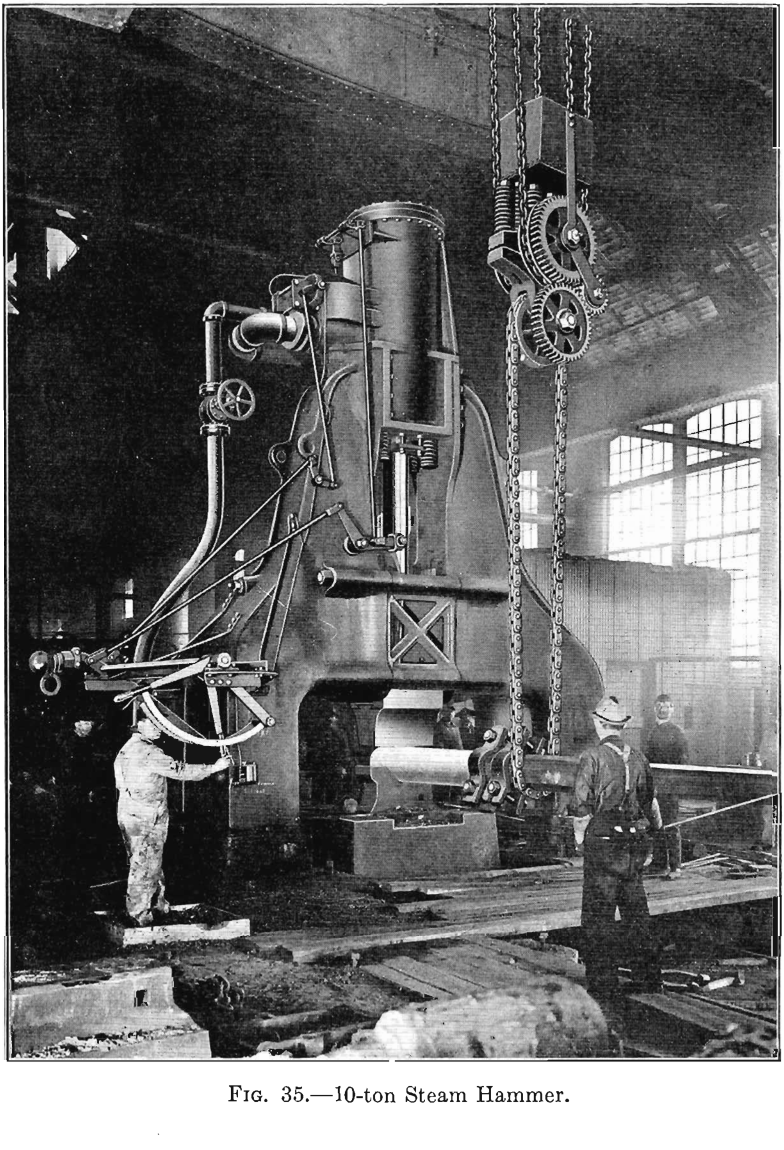

FIG. 35.—10-ton Steam Hammer.

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area of cross section of the rough forging when finished, and for a jacket forging 3½ times as great. For forgings for guns 12 inches or more in caliber these figures are reduced to 3½ and 3 respectively. Forgings for lining tubes must be reduced 6 times in area.

If bored ingots are used the wall of the ingot must be reduced at least one half in thickness.

Annealing.—The working of the ingot in forging and the irregular cooling leaves the metal in a state of strain. The strains are removed by the process of annealing. For this purpose the forging is usually laid in a brick-walled pit or furnace, and slowly and uniformly heated by wood fires, the burning logs being distributed along the pit as required to heat the forging uniformly. When the proper heat, usually a bright red, has been attained, the fires are allowed to die out, or are drawn, and the ingot remains in the furnace until both are cold. Three or four days may be required for the slow cooling of a large forging.

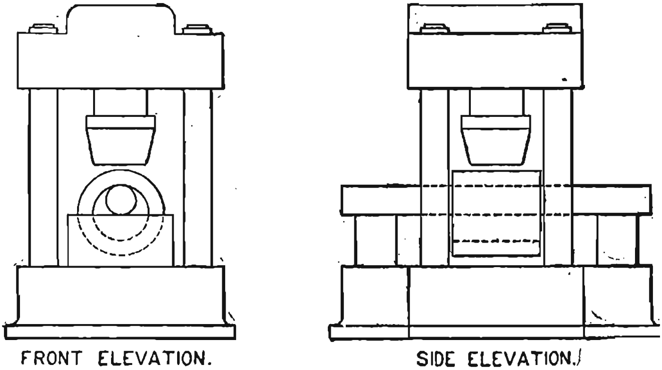

FIG. 36.

99. Hardening in Oil or Water.—Annealing removes the internal strains that exist in the forging, but, as before explained, it greatly reduces the tensile strength and elastic limit of the metal. To restore the strength to the metal and to produce in it the qualities required in gun forgings, the forging is next subjected to the process of hardening. Before hardening it is machined in a lathe nearly to finished dimensions. Specimens for tests are cut from the ends, and from their behavior in the testing machine the requirements of the subsequent treatment are determined.

The forging is then slowly and uniformly heated throughout. Large forgings, such as tubes and jackets, are heated in vertical furnaces, great care being exercised that the heating shall be uniform throughout the length of the piece in order that undue warping may not occur in the subsequent cooling. When the forging is at a uniform red heat the side of the furnace is opened and the forging is lifted out by a crane and immersed in a deep tank of oil or of water alongside the furnace. The oil tank is surrounded by another tank through which cold water is constantly running. The heat of the forging passes to the oil and thence to the water, and is thus gradually conducted away.

The Bethlehem Steel Co. of Bethlehem, Penn., and the Midvale Steel Co. of Philadelphia, the two principal manufacturers of gun forgings in this country, use different oils for oil tempering. The Bethlehem Co. uses petroleum oil once refined. The Midvale Co. uses cottonseed oil with flashing point not less than 360 degrees centigrade.

The temperature of the forging when immersed is very high compared with that of the oil. The cooling is therefore sudden at first, but as oil is a poor conductor of heat the heat of the forging is carried away slowly, leaving the metal with greater toughness than it would have if hardened in water and cooled more quickly.

Oil is customarily used in the hardening of gun forgings. Occasionally the qualities of the metal are such that better results are obtained by the quicker cooling in water.

Tempering.—The process of hardening greatly increases the elastic strength of the metal but reduces its toughness. At the same time it produces internal strains due to contraction in cooling. The strains are removed, the hardness reduced, and the toughness restored by the process of tempering, conducted in the same manner as the previous annealing, but at a lower heat, so that the gain in elastic strength is reduced but slightly and not entirely lost. The tempering temperature for gun forgings lies between 600 and 675 degrees centigrade, 1100 to 1250 degrees Fahrenheit.

Specimens are again taken from the ends of the forging and broken in the testing machine. If the specimens do not fulfil the requirements of the specifications the forging is again hardened and tempered, the temperature and conduct of the processes being so regulated as to improve those qualities in which the metal has proved defective in the tests.

Strength of Parts of the Gun.—The requirements in steel forgings for guns over 8 inches in caliber are shown in the table on page 165. It will be observed that the strength of the metal increases as we proceed outward from the center of the gun. Thus the elastic limit of the tube is 46,000 lbs., of the jacket 48,000, and of the hoops 53,000. It would be better if the strongest metal were in the tube, which has to endure the greatest strain. But the production of the high qualities required is much more difficult in large forgings than in smaller ones, and for this reason the requirements for the tubes and jackets must be lower than for the hoops. An additional reason for the difference in requirements is found in the fact that the metal of the tube has the advantage of greater elongation before rupture, as may be seen in the table on page 165. The greater elongation is difficult to produce with the higher elastic limit.

The tubes and jackets of guns under 8 inches in caliber have an elastic limit of 50,000 lbs.

Forged steel that has an elastic limit of over 110,000 lbs. is now produced.

CHAPTER VI.

GUNS.

ELASTIC STRENGTH OF GUNS.

100. The Elasticity of Metals.—In the chapter on metals the elastic limit of a metal has been defined as the minimum stress per unit of area of cross section that will produce in the metal a permanent set. For each kind of stress, whether of extension or compression, the metal has a distinct elastic limit. The elastic limit of extension, or the tensile elastic limit, is usually less than the elastic limit of compression. In gun steels the difference is not great and the two are considered equal. The tensile elastic limit is ordinarily used, as it is the limit usually measured.

Hooke's Law.—A tensile stress applied to a bar of metal causes elongation of the bar, and it is found by experiment that under stresses less than the elastic limit of the metal the elongation is proportional to the stress. In other words, within the elastic limit of the metal the ratio of the stress to the strain is constant. This law is known as Hooke's law, and is often expressed *ut tensio sic vis*.

Modulus of Elasticity.—If we measure the elongation of a bar caused by a tensile stress, and divide the measured elongation by the original length of the bar, we will obtain the elongation per unit of length, expressed as a numerical fraction.

Now if we divide any stress per unit of area within the elastic limit of the metal by the elongation per unit of length the result will be the constant ratio of stress to strain within the elastic limit. This ratio is called the modulus of elasticity.

Let E be the modulus of elasticity of the metal,

θ the elastic limit of the metal,

γ the elongation per unit of length at the elastic limit.

By definition we have

$$E = \theta / \gamma \qquad (1)$$

If we assume that the elasticity of the metal continues indefinitely we see, by making γ equal to unity in the above equation, that the modulus of elasticity is the stress per unit of area that would extend a bar to twice its length.

When the elastic limit is expressed in pounds per square inch the modulus of elasticity of steel may, without sensible error, be taken as 30,000,000.

The modulus of elasticity is really a stress per unit of area, but it had best be considered as the abstract ratio between stress and strain.

Since by Hooke's law the ratio of the stress to the strain is constant within the elastic limit, we may write for θ and γ in equation (1) any other stress within the elastic limit and its corresponding strain.

Let S be a stress per unit of area within the elastic limit,
l the strain per unit of length due to the stress.

Then

$$E = S/l \quad \text{and} \quad l = S/E \qquad (2)$$

That is, the strain per unit of length due to any stress per unit of area within the elastic limit is equal to the stress divided by the modulus of elasticity.

101. Strains Perpendicular to the Direction of the Stress.—In the previous paragraphs we have considered only the strain produced in the direction of the stress. But we have seen in the chapter on metals that a tensile stress produces a reduction in area of cross section, and it is found by experiment that, for steel, the strain at right angles to the direction of a stress within the elastic limit of the metal is equal to one third of the strain in the direction of the stress. If the cube in Fig. 37 is subjected to the tensile stress represented by p, the edges, aa, bb, etc., parallel to the direction of the stress will be elongated, and the edges, ab, ac, etc., perpendicular to this direction will be shortened by an amount equal to one third the elongation of the parallel edges.

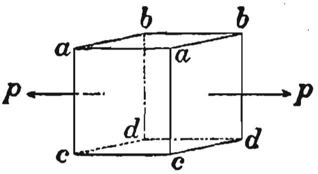

FIG. 37.

Equations of Relation between Stress and Strain.—If we consider that the cube is subjected at once to tensile stresses applied in the three directions perpendicular to its faces, the strain in each direction due to the stress in that direction will be diminished by the contrary strains due to the perpendicular stresses.

Let X, Y, and Z be three independent extraneous tensile forces perpendicular to the faces of the cube;
l_x, l_y, and l_z the strains in the directions of X, Y, and Z respectively.

The strain in the direction X due to the force X is from equation (2) X/E. It is diminished by $\frac{1}{3}\frac{Y}{E}$ and by $\frac{1}{3}\frac{Z}{E}$. Therefore, for the total strains in the three directions, we have

$$\left.\begin{aligned} l_x &= \frac{1}{E}\left(X-\frac{Y}{3}-\frac{Z}{3}\right) \\ l_y &= \frac{1}{E}\left(Y-\frac{X}{3}-\frac{Z}{3}\right) \\ l_z &= \frac{1}{E}\left(Z-\frac{X}{3}-\frac{Y}{3}\right) \end{aligned}\right\} \qquad (3)$$

Problems. 1. A steel test specimen has an elastic limit of 59,000 lbs. What will be its elongation per unit of length at the elastic limit? 0.00197

2. The original diameter of the specimen being 0.505 inches, what is its diameter when the piece is stretched to its elastic limit? 0.5047 inches.

3. A vertical steel rod 20 feet long and ½ inch square sustains at its lower end a load of 6000 lbs. The elastic limit of the steel is 72,000 lbs. What will be the elongation caused by the load? 0.192 inches.

4. Taking the modulus of elasticity of copper as 16,000,000, what will be the elongation of a copper bar 1 inch square and 10 feet long supporting a load of 5000 lbs.? 0.0375 inches.

102. Principal Stresses and Strains.—Since every stress applied to a solid produces stresses in directions perpendicular to the direction of the applied stress, at any point in a solid under stress there are always three planes at right angles to each other upon each of which the stress is normal. Thus in the cube we have just considered, the stresses at any point in the cube are normal to three planes parallel to the faces of the cube. The normal stresses are called the principal stresses at the point; and it may be shown by the ellipsoid of stress that one of the principal stresses is the greatest stress at the point. The corresponding strains are called the principal strains.

Stresses and Strains in a Closed Cylinder.—The following discussion of the elastic strength of cylinders is based on the theory of Clavarino, published in 1879, and modified through the results of experiments by Major Rogers Birnie, Ordnance Department, U. S. Army.

Consider a hollow metal cylinder, closed at both ends, to be subjected to the uniform pressure of a gas confined in the cylinder. The pressure acting perpendicularly to the cylindrical walls will tend to compress the walls radially. If we consider a longitudinal section of the cylinder by any plane through the axis, the pressure acting in both directions perpendicular to this plane will tend to disrupt or pull apart the cylinder at the section, and will therefore produce a tensile stress in a tangential direction on the metal throughout the section. The pressure acting against the ends of the cylinder will tend to pull it apart longitudinally.

The metal of any elementary cube in the cylinder is therefore subjected to three principal stresses: a radial stress of compression, a tangential stress of extension, and a longitudinal stress of extension.

If the cylinder be subjected to a uniform exterior pressure stresses will be similarly developed in the three directions.

In the following discussion we will always understand by the term stress, the stress per unit of area, and by the term strain, the strain per unit of length, unless these terms are qualified by the word total or other qualifying word.

Assume a closed cylinder affected by uniform interior and exterior pressures. At any point of the cylinder

Let t be the tangential stress,
p the radial stress,
q the longitudinal stress.

Substituting these letters in equations (3) for X, Y, and Z, respectively, and changing the sign of Y, since the interior and exterior pressures act toward each other radially, so that the stress, p, acts in a direction opposite to that assumed for Y in deducing equations (3), we obtain the following equations.

$$\left.\begin{aligned} l_t &= \frac{1}{E}\left(t+\frac{p}{3}-\frac{q}{3}\right) \\ l_p &= -\frac{1}{E}\left(p+\frac{t}{3}+\frac{q}{3}\right) \\ l_q &= \frac{1}{E}\left(q-\frac{t}{3}+\frac{p}{3}\right) \end{aligned}\right\} \qquad (4)$$

which express the values of the strains in the directions of the three stresses. These values may be positive or negative, depending upon the resultant of the stresses. A positive value of a strain represents elongation and a negative value contraction, as a positive value of a stress represents a tensile stress and a negative value a compressive stress.

103. Relations between the Stresses t, p, and q. Lamé's Laws.—The stresses and strains in equations (4) form six unknown quantities which cannot be determined from the three equations.

Lamé, a distinguished investigator in the subject of elasticity of solid bodies, has established relations between the stresses, by means of which the equations may be solved and the values of the stresses and strains determined. He assumes that the longitudinal stress q and the longitudinal strain l_q are constant throughout the cross section. The last of equations (4) may then be written

$$t-p=3(q-l_qE)=\text{constant} \qquad (5)$$

which equation is true whether q has a value or is zero. As t and p apply to any point in the walls of the cylinder, we have Lamé's first law.

In a cylinder under uniform pressure the difference between the tangential tension and the radial pressure is the same at all points in the section of the cylinder.

Now let us consider a right section of the cylinder, *of unit length*, Fig. 38.

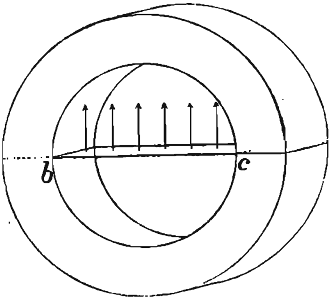

FIG. 38.

Let P_0 be the pressure per unit of area acting on the interior of the cylinder,
P_1 the pressure per unit of area on the exterior,
R_0 the interior radius of the cylinder,
R_1 the exterior radius,
r the radius of any point in the cylinder.

The total interior pressure acting normally on either side of the diametral plane bc is $2P_0R_0$. The total pressure acting on the outer circumference on either side of the plane and normal to it is $2P_1R_1$. The difference of these pressures is the resultant pressure acting on the metal in the sectional plane bc. The total tangential stress on the metal at the section will therefore be

$$2(P_0R_0-P_1R_1)$$

But since t represents this stress per unit of area, the total stress is equal to $2\int_{R_0}^{R_1} t\,dr$. Therefore

$$\int_{R_0}^{R_1} t\,dr = P_0R_0-P_1R_1$$

Assuming that t is a function of r, it must be such a function that $t\,dr$ when integrated between the limits R_1 and R_0 will be equal to $P_0R_0-P_1R_1$. $t\,dr$ must then be equal to $-d(pr)$ because the integral of this expression taken between the given limits is $P_0R_0-P_1R_1$. The substitution of the pressures P_0 and P_1 for the radial stress p, in integrating the expression $-d(pr)$, may be made because, as will be found later, p varies proportionately with P_0 and P_1.

We therefore have

$$t\,dr = -d(pr) = -p\,dr - r\,dp$$

From which by combination with equation (5) and integration, *see foot note*, we obtain

$$t+p=C/r^2 \tag{6}$$

in which C is a constant.

Equation (6) expresses Lamé's second law:

In a cylinder under uniform pressure the sum of the tangential tension and the radial pressure varies inversely as the square of the radius.

Both laws are based on the assumption that the longitudinal stress is constant or zero.

104. Stresses in the Cylinder.—By means of Lamé's laws we may now determine the values for the stresses at all points in the cylinder. We may write for t, p, and r in equations (5) and (6) the coordinate values referring to any point in the cylinder and thus form the equations

$$t-p=T_0-P_0=T_1-P_1$$

$$(t+p)r^2=(T_0+P_0)R_0^2=(T_1+P_1)R_1^2$$

Eliminating T_0 and T_1 from these equations we may obtain

$$t = \frac{P_0R_0^2-P_1R_1^2}{R_1^2-R_0^2} + \frac{R_0^2R_1^2(P_0-P_1)}{R_1^2-R_0^2}\frac{1}{r^2} \tag{7}$$

$$p = -\frac{P_0R_0^2-P_1R_1^2}{R_1^2-R_0^2} + \frac{R_0^2R_1^2(P_0-P_1)}{R_1^2-R_0^2}\frac{1}{r^2} \tag{8}$$

$$t\,dr=-pdr-rdp$$

$$-(t+p)dr=rdp$$

From equation (5)

$$t+p=2p+k$$

Therefore

$$-\frac{dr}{r}=\frac{dp}{2p+k}$$

Integrating

$$\log_e\frac{1}{r}=\frac{1}{2}\log_e(2p+k)+\log_e A$$

Replacing $2p+k$ by its value $t+p$ we obtain

$$1/r^2=A^2(t+p)$$

or

$$t+p=1/A^2r^2$$

From these equations we may obtain the values of the tangential and radial stresses at any point in the section of the cylinder by substituting for r its value for the point.

Longitudinal Stress.—The longitudinal stress has been assumed as constant over the cross section of the cylinder. Under this assumption when applied to a gun the total longitudinal stress due to the pressure on the face of the breech block is distributed uniformly over the cross section of the gun, producing a stress per unit of area that is small compared with the tangential and radial stresses. In the present discussion of the stresses acting on the cylinder the longitudinal stress will therefore be neglected, and q in equations (4) will be considered as zero. Later the value of the longitudinal stress will be deduced.

105. Resultant Stresses in the Cylinder.—Making $q=0$ in equations (4) and substituting for t and p their values from (7) and (8) we obtain

$$El_t = S_t = \frac{2}{3}\,\frac{P_0R_0^2 - P_1R_1^2}{R_1^2 - R_0^2} + \frac{4}{3}\,\frac{R_0^2R_1^2(P_0 - P_1)}{R_1^2 - R_0^2}\,\frac{1}{r^2} \qquad (9)$$

$$El_p = S_p = \frac{2}{3}\,\frac{P_0R_0^2 - P_1R_1^2}{R_1^2 - R_0^2} - \frac{4}{3}\,\frac{R_0^2R_1^2(P_0 - P_1)}{R_1^2 - R_0^2}\,\frac{1}{r^2} \qquad (10)$$

$$El_q = S_q = -\frac{2}{3}\,\frac{P_0R_0^2 - P_1R_1^2}{R_1^2 - R_0^2} \qquad (11)$$

In the above equations the first members are the respective strains multiplied by the modulus of elasticity. Referring to equation (2) we see that each product is equal to the stress which acting alone would produce the strain. The equations then give the values of the simple stresses that would produce the same strains as are caused by the stresses p and t acting together. Their values at any point in the cylinder are obtained from the above equations by giving to r the value for the point.

Basic Principle of Gun Construction.—The following principle is the foundation of the modern theory of gun construction.

No fiber of any cylinder in the gun must be strained beyond the elastic limit of the metal of the cylinder.

This principle is strictly adhered to in the construction of guns built up wholly of steel forgings. In the construction of wire-wound guns the tube is, in some constructions, purposely compressed beyond its elastic limit by the pressure exerted upon it by the wire.

The principle fixes a limit to the stresses to which any cylinder that forms part of a gun may be subjected. If we represent by

θ the tensile elastic limit of the metal,
ρ the compressive elastic limit of the metal,

the stresses represented by the first members of equations (9) to (11) may never exceed either θ or ρ, depending on whether the stress is one of extension or of compression; and the interior and exterior pressures, represented by P_0 and P_1 in those equations, must never have such values as to cause the stresses to exceed these limits.

106. Simplification of the Formulas of Gun Construction.—The formulas of gun construction are deduced from equations (9), (10), and (11). Heretofore, in the deduction, these equations have been used in the form in which they appear above, and the formulas resulting from them have been similarly extended and equally formidable in appearance, and much labor has been expended in writing them out.

We will introduce here, for the first time in any text, a simplification of equations (9), (10), and (11), which will result in a marked simplification of all the formulas of gun construction, making the formulas easier to handle, and greatly reducing the labor required in their use.

We will express in equations (9), (10), and (11) R_1^2 in terms of R_0^2, and in the future deductions we will always express R_2^2, R_3^2, R_n^2 in terms of R_0^2.

$$\left.\begin{array}{llll} \text{Make } & R_1^2=aR_0^2 & \text{or} & a=R_1^2/R_0^2 \\ & R_2^2=bR_0^2 & & b=R_2^2/R_0^2 \\ & R_3^2=cR_0^2 & & c=R_3^2/R_0^2 \\ & R_n^2=nR_0^2 & & n=R_n^2/R_0^2 \end{array}\right\} \qquad (12)$$

For convenience in future discussion we will call a, b, c, n the *radius ratios*.

Now if we divide numerator and denominator of each term of equations (9), (10), and (11) by R_0^2 and substitute for R_1^2/R_0^2 its value a from equations (12) we obtain

$$El_t = S_t = \frac{2}{3}\frac{(P_0 - aP_1)}{(a-1)} + \frac{4}{3}\frac{a(P_0 - P_1)}{(a-1)}\frac{R_0^2}{r^2} \qquad (13)$$

$$El_p = S_p = \frac{2}{3}\frac{(P_0 - aP_1)}{(a-1)} - \frac{4}{3}\frac{a(P_0 - P_1)}{(a-1)}\frac{R_0^2}{r^2} \qquad (14)$$

$$El_q = S_q = -\frac{2}{3}\frac{(P_0 - aP_1)}{(a-1)} \qquad (15)$$

Rules for Transformation.—We will notice here, with reference to the transformation, two facts on which we will base rules for future transformations. In what follows we will understand by the words *term factor* a factor that affects a whole term, in contradistinction to a factor that affects a part of a term only.

Comparing the first term of the second member of equation (13) with the corresponding term of equation (9) we can write the first rule.

Rule 1. The non-appearance of R_0^2 in any term involving the radius ratios indicates that the term from which it was formed had in the numerator the same number of *term factors involving the squares of the limiting radii* as in the denominator.

In the first term of the second member of equation (9) the numerator contains a single *term factor* involving the squares of the radii. The denominator similarly contains but one such term factor.

Comparing the last terms of equations (13) and (9) we see,

Rule 2. When R_0^2 appears in the numerator of a term involving the radius ratios, it indicates that the original term contained in the numerator one more *term factor involving the squares of the limiting radii* than in the denominator.

Though the last term of equation (13) contains in numerator and denominator the same number of term factors that involve the radius ratios, the presence of R_0^2 in the numerator indicates that the term from which it was formed had one more such term factor. That factor was R_0^2, and since $R_0^2/R_0^2 = 1$ the factor has disappeared from equation (13).

107. Stresses in a Simple Cylinder.—In a cylinder forming a part of a gun we have three cases to consider. There may be a pressure on the interior of the cylinder and none on the exterior, the atmospheric pressure being considered zero. There may be a pressure on the exterior of the cylinder and none on the interior. Or both exterior and interior pressures may be acting at once, the interior pressure being usually the greater. We will consider the simple cylinder under these circumstances.

Differentiating equation (13) we obtain

$$\frac{dS_t}{dr} = -\frac{8}{3}\,\frac{a(P_0 - P_1)}{a-1}\,\frac{R_0^2}{r^3} \tag{16}$$

and differentiating again,

$$\frac{d^2S_t}{dr^2} = \frac{8a(P_0 - P_1)}{a-1}\,\frac{R_0^2}{r^4} \tag{17}$$

Similarly from equation (14) we obtain

$$\frac{dS_p}{dr} = \frac{8}{3}\,\frac{a(P_0 - P_1)}{a-1}\,\frac{R_0^2}{r^3} \tag{18}$$

$$\frac{d^2S_p}{dr^2} = -\frac{8a(P_0 - P_1)}{a-1}\,\frac{R_0^2}{r^4} \tag{19}$$

First Case. Interior Pressure Only.—Making $P_1 = 0$ in equation (13) and remembering that r may vary between the limits R_0 and R_1 we see that the smaller the value of r the greater will be the value of the resultant tangential stress. This is more readily seen in equation (16) in which the first differential coefficient of the stress as a function of the radius is negative when $P_1 = 0$, showing that S_t decreases as r increases. R_0 being the least value of r the tangential stress is greatest at the interior of the cylinder. Since, when $P_1 = 0$, S_t in equation (13) is positive for all values of r, the stress is one of extension throughout the cross section of the cylinder. When $P_1 = 0$ in equation (17) the second member is positive, showing that the curve of stress is concave upwards, the axis of r being taken as horizontal. The curve of tangential stress due to an interior pressure only may then be represented in general by the curve t_1 in Fig. 39, the ordinates being the values of the stress, the abscissas the values of the radius.

The numbers at the extremities of the curve are the actual stresses due to an interior pressure $P_0=36{,}000$ pounds per square inch in a cylinder one caliber thick. They are calculated from equation (13) by making $P_1=0$ and $R_1=3R_0$. When $R_1=3R_0$ we have $a=R_1^2/R_0^2=9$. The equation becomes with these substitutions

$$S_t=\frac{P_0}{12}\left(1+18\frac{R_0^2}{r^2}\right) \tag{20}$$

Making $P_0=36{,}000$ and $r=R_0$ we obtain $S_t=57{,}000$; and for $r=3R_0$, $S_t=9000$.

Similarly from equations (14), (18), and (19) we determine for the radial stress produced by an interior pressure the general curve p_1, Fig. 39, which shows radial compression throughout the cross section with the greatest stress at the interior. Equations (14) and (15) become for the cylinder one caliber thick

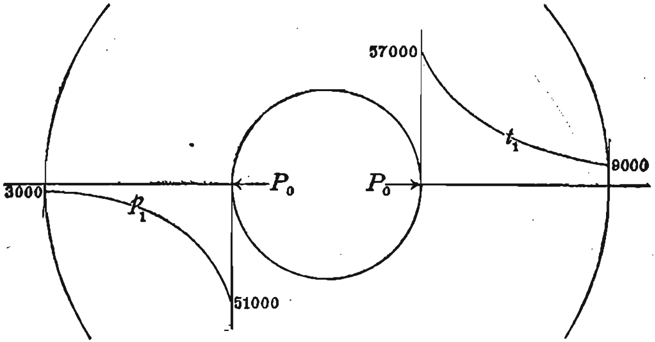

FIG. 39.

$$S_p=\frac{P_0}{12}\left(1-18\frac{R_0^2}{r^2}\right) \tag{21}$$

$$S_q=-\frac{P_0}{12} \tag{22}$$

and comparing these with equation (20) we see that for equal values of r the radial stress from an interior pressure is always less than the tangential stress. The longitudinal stress is less than either.

The radial stresses produced by a pressure $P_0 = 36{,}000$ are noted on the curve p_1.

We may observe from equations (20), (21), and (22) that the thickness of the cylinder being expressed in calibers, or, what is the same thing, in terms of the interior radius, the stresses developed by an interior pressure are entirely independent of the caliber, and are the same for all cylinders the same number of calibers thick.

108. Second Case. Exterior Pressure Only.—Making $P_0 = 0$ in equations (13) to (19) we may determine the curves of stress for an exterior pressure acting alone. In this case the value of S_t, equation (13), is always negative. The stress is therefore compressive throughout the cylinder. dS_t/dr, equation (16), is positive. S_t therefore increases algebraically with r. d^2S_t/dr^2, equation (17), is negative. The curve is therefore concave downwards. The general curve t_2, in Fig. 40, therefore results.

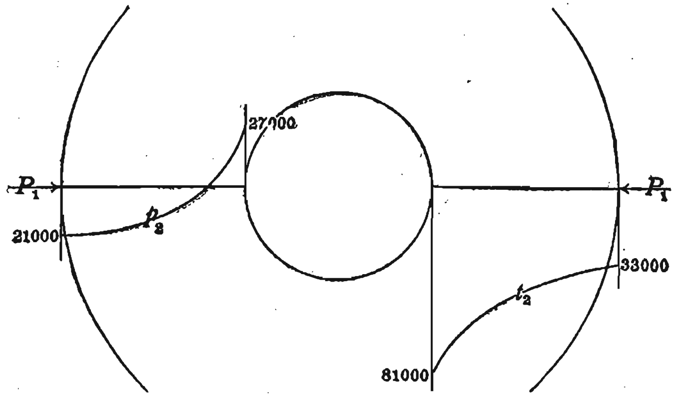

FIG. 40.

In the same way the general curve p_2 is obtained from equations (14), (18), and (19).

The numbers on the curves are the values for the stresses caused by an exterior pressure $P_1 = 36{,}000$ lbs. on a cylinder one caliber thick, for which $R_1 = 3R_0$ and $a = R_1^2/R_0^2 = 9$.

We see as before that the greatest stresses are at the interior of the cylinder, and that the tangential stress is greater than the radial. The tangential stress is one of compression throughout.

The radial stress is one of compression on the exterior and of extension on the interior.

109. Third Case. Interior and Exterior Pressures Acting.—The curves of stress due to interior and exterior pressures acting at once may be found from the equations, or by combination of the curves of stress due to the pressures acting separately. Thus in Fig. 41, in which the curves from Figs. 39 and 40 are repeated, the lines p_3 and t_3 represent the stresses due to the equal interior and exterior pressures, $P_0 = P_1 = 36{,}000$ lbs.

The position of the resultant curves of stress from interior and exterior pressures acting together will, of course, depend on the relative values of the two pressures. In Fig. 41 the pressures are equal. In Fig. 42 are shown the curves resulting when the interior pressure is twice the exterior pressure; $P_0 = 36{,}000$, $P_1 = 18{,}000$.

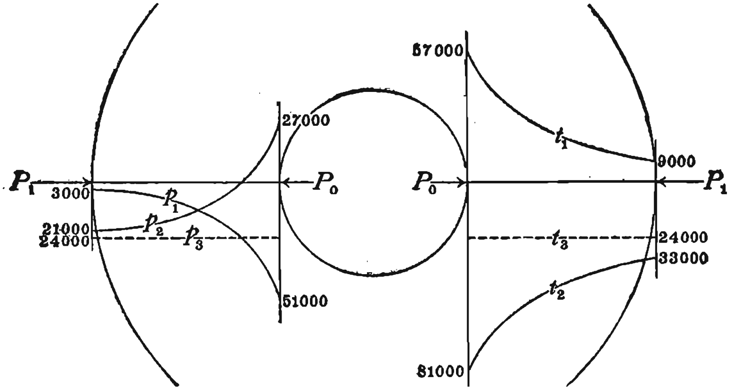

Fig. 41.

We may see at once from these figures that the tangential resistance of a cylinder to an interior pressure may be greatly increased by the application of an exterior pressure. Assuming that the maximum ordinates of the curves t_1 and t_2, in Fig. 41, are the elastic limits θ and ρ respectively, the interior pressure acting alone would produce the limit of tangential extension. But with the exterior pressure acting the interior pressure has first to overcome the existing compression, and as ρ is usually greater than θ the interior pressure required to produce the stress $\rho + \theta$ would be more than twice as great as the pressure required to produce the stress θ alone. That is to say, that by the application of an exterior pressure we may more than double the tangential resistance of a cylinder to an interior pressure.

Similarly it is seen that the tangential resistance of a cylinder to an exterior pressure is increased by the application of an interior pressure.

110. Limiting Interior Pressures.—In determining the maximum safe pressure that can be applied to the interior of a cylinder there are two cases to be considered; for, as we have just seen, a greater interior pressure may be applied when there is an exterior pressure acting than when the interior pressure acts alone.

INTERIOR AND EXTERIOR PRESSURES ACTING.—In Figs. 41 and 42 we see that when both interior and exterior pressures are acting on a given cylinder the maximum values of the resultant tangential and radial stresses depend upon the relative values of the pressures. In Fig. 41 the maximum values of the two resultant stresses are equal. In Fig. 42 the resultant radial stress of compression has a greater maximum value than the resultant tangential stress of extension. Therefore when both pressures are acting, in order to determine the maximum permissible interior pressure we must find the values of the interior pressures that will produce the limiting stresses both of extension and of compression, and then adopt the smaller value as the greatest permissible pressure. The maximum stress in either case occurs when $r=R_0$. Therefore make this substitution in equations (13) and (14). Write θ for S_t and $-\rho$ for S_p and solve the equations for P_0. The negative sign is given to ρ,

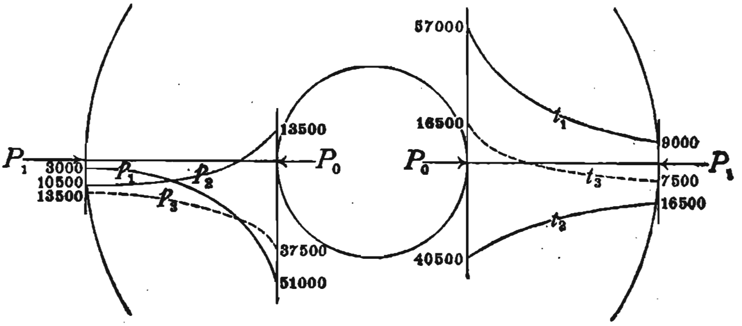

FIG. 42.

since ρ is an absolute value only, while S_p now represents a stress of compression, which is negative.

$$P_{0\theta}=\frac{3(a-1)\theta+6aP_1}{4a+2} \tag{23}$$

$$P_{0\rho}=\frac{3(a-1)\rho+2aP_1}{4a-2} \tag{24}$$

$P_{0\theta}$ is the interior pressure that acting with the exterior pressure P_1 will produce the limiting tangential stress of extension θ; and $P_{0\rho}$ is the interior pressure that acting with the exterior pressure P_1 will produce the limiting radial stress of compression ρ. The lesser of these two values should, according to our premises, always be used, but it will be seen later, that in practice it is usual to neglect consideration of $P_{0\rho}$ and to make use of $P_{0\theta}$ even when it is the greater. Assuming that $\theta=\rho$ we will find by equating the second members of the above equations that $P_{0\theta}$ will be less than, equal to, or greater than $P_{0\rho}$ as follows.

$$P_{0\theta} \overset{<}{\underset{>}{=}} P_{0\rho} \quad \text{as} \quad aP_1 \overset{<}{\underset{>}{=}} \tfrac{3}{4}\theta \tag{25}$$

III. INTERIOR PRESSURE ONLY.—We have seen in Fig. 39 that the greatest stress from an interior pressure acting alone is a tangential stress of extension at the interior of the cylinder. This must never exceed θ, the elastic limit for extension. Therefore to find the greatest permissible value of an interior pressure acting alone make $S_t=\theta$ in equation (13), $P_1=0$, $r=R_0$, and solve for P_0.

$$P_{0\theta}=\frac{3(a-1)}{4a+2}\theta \tag{26}$$

If the cylinder is one caliber thick $R_1=3R_0$, $a=9$, and

$$P_{0\theta}=0.63\theta$$

If the cylinder has infinite thickness $R_1=\infty$ and

$$P_{0\theta}=0.75\theta \tag{27}$$

From which we conclude that the greatest possible safe value for an interior pressure acting alone in a simple cylinder is 0.75θ;

and also that comparatively little benefit is derived by increasing the thickness of the cylinder to more than one caliber.

Now if we assume an exterior force applied to the cylinder and assume the effect of this force to be the stress ρ of compression, the tangential stress that must be produced by the interior pressure to reach the limit of safety becomes $\rho+\theta$, and this being substituted for θ in equation (26) it becomes

$$P_{0\theta}=\frac{3(a-1)}{4a+2}(\theta+\rho) \tag{28}$$

From equations (26) and (28) the advantage derived by the interior cooling of cast guns formed of a single cylinder becomes apparent. When the gun is cooled from the interior the layer of metal immediately surrounding the bore cools first and contracts. The cooling and contraction of the subsequent layers then produce a stress of compression on the layers of metal immediately surrounding the bore similar to the stress that would be produced by the application of an exterior pressure. The limiting interior pressure in this case would be obtained by substituting for ρ in equation (28) the value of the stress resulting from the initial compression.

112. Graphic Representation of Limiting Interior Pressures.—The system of graphics devised by Lieutenant Commander Louis M. Nulton, U. S. Navy, for the representation of the relation between the pressures and the shrinkages in cannon helps materially towards a ready understanding of the subject.

We will begin the study of the graphic system with the representation of the limiting interior pressures whose values are given by equations (23) and (24).

We will consider, as is customary in gun construction, that $\theta=\rho$.

Equations (23) and (24) may be put in the following forms, in which A, B, C, and D are constants for any given cylinder.

$$P_{0\theta}=A+BP_1 \tag{23a}$$

$$P_{0_\rho}=C+DP_1 \tag{24a}$$

These are the equations of right lines that do not pass through the origin of coordinates. The lines may be constructed, as shown in

Fig. 43, from the axes of P_0 and P_1, the line marked $P_1 P_{0\theta}$ from equation (23*a*) and the line $P_1 P_{0\rho}$ from (24*a*).

The abscissa of any point of the line $P_1 P_{0\theta}$ is the value of P_0, which, acting together with the pressure P_1, whose value is represented by the ordinate of the point, will produce the limiting interior tangential stress of extension θ. Similarly the abscissa and ordinate of any point of the line $P_1 P_{0\rho}$ represent the pressures P_0 and P_1 that acting together on the cylinder will produce the limiting interior radial stress of compression ρ.

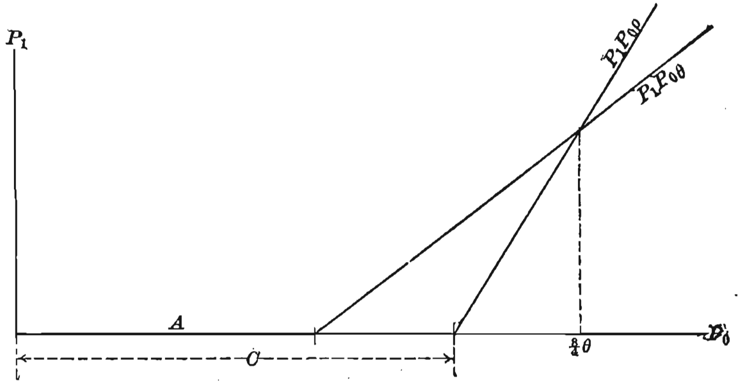

FIG. 43.

For any given value of either interior or exterior pressure, P_0 or P_1, we may at once determine from the figure the value of the corresponding exterior or interior pressure, P_1 or P_0, that will produce the limiting strain of compression or of extension.

For $P_0 = \frac{3}{4}\theta$ the pressure P_1, whose value is then $\frac{3}{4}\theta / a$, see equation (25), will produce in the interior of the cylinder the maximum permissible stresses both of extension and compression.

The figure also shows that the resistance of the cylinder to an interior pressure is increased by the application of an exterior pressure, since P_0 has its least value for $P_1 = 0$.

113. Limiting Exterior Pressure.—This is deduced only for the case of an exterior pressure acting alone, as we will have no occasion to use the limiting values of the exterior pressure when both interior and exterior pressures are acting.

From Fig. 40 we see that the greatest stress from an exterior pressure is a tangential stress of compression at the interior of the cylinder. This must not exceed ρ, the elastic limit for compression. Therefore make $S_t = -\rho$ in equation (13), $P_0=0$, $r=R_0$, and solve for P_1.

$$P_{1_\rho} = \frac{a-1}{2a}\rho \tag{29}$$

P_{1_ρ} being the exterior pressure that acting alone will produce the limiting tangential stress of compression ρ.

For the cylinder one caliber thick $R_1=3R_0$ in equation (29), $a=R_1^2/R_0^2=9$, and

$$P_{1_\rho}=0.44\rho$$

For the cylinder of infinite thickness $R_1=\infty$, and

$$P_{1_\rho}=0.50\rho$$

again showing how little is gained by increasing the thickness of the cylinder beyond one caliber.

114. Thickness of Cylinder.—The thickness H needed in a simple cylinder to withstand an interior pressure $P_{0\theta}$ is obtained by replacing a in equation (26) by its value R_1^2/R_0^2, solving the equation for R_1 and then subtracting R_0 from each member.

$$R_1 - R_0 = H = R_0\left(\sqrt{\frac{3\theta+2P_{0\theta}}{3\theta-4P_{0\theta}}}-1\right) \tag{30}$$

Similarly the necessary thickness to withstand an exterior pressure P_{1_ρ} is obtained from equation (29).

$$R_1 - R_0 = H = R_0\left(\sqrt{\frac{\rho}{\rho-2P_{1_\rho}}}-1\right)$$

Longitudinal Strength of a Simple Closed Cylinder.—The total pressure acting on each of the end walls is $\pi R_0^2 P_0$. This is assumed to be uniformly distributed over the cross section of the cylinder, $\pi(R_1^2-R_0^2)$. The longitudinal stress per unit of area is therefore

$$q=\frac{P_0}{a-1}$$

Substituting this value of q in the third equation (4), and for t and p their values from (7) and (8), we obtain for the longitudinal stress in the cylinder

$$El_q = \frac{P_0 - 2aP_1}{3(a-1)}$$

Giving El_q its maximum value, θ or ρ, and solving for P_0, using θ we obtain

$$P_{0\theta} = 3(a-1)\theta + 2aP_1$$

for the interior pressure that will produce the maximum permissible longitudinal stress.

If $P_1 = 0$

$$P_{0\theta} = 3(a-1)\theta$$

a value considerably greater than that expressed in equation (26).

Problems.—1. What is the maximum permissible interior pressure on a steel gun hoop the interior diameter of which is 20 inches and the exterior diameter 28 inches, the elastic limit of the metal being 60,000 pounds per square inch?

Ans. 17,561 lbs. per sq. in.

2. The steel tubes of a water tube boiler are 2 inches in interior diameter and 2.4 inches in exterior diameter. The elastic limit of the metal is 30,000 lbs. per sq. in. What is the limiting interior water pressure? *Ans.* 5103.2 lbs. per sq. in.

3. Using a factor of safety of $1\frac{1}{2}$, what is the limiting interior pressure in an air compressor tank with interior and exterior diameters of 15 and 17 inches respectively? The elastic limit of the metal is 30,000 lbs. per sq. in. *Ans.* 2391 lbs. per sq. in.

4. An iron tube 3 inches in interior diameter is subjected to exterior pressure, 1326.5 lbs. per sq. in. The elastic limit of the metal is 20,000 lbs. per sq. in. What must be the exterior diameter of the tube in order that it may safely withstand the pressure? *Ans.* 3.25 inches.

5. The 6-inch wire-wound gun has the following dimensions at the powder chamber: $R_0 = 4.5$ inches, $R_1 = 12$ inches. If the gun were constructed of a single forging with an elastic limit of 60,000 lbs. per sq. in. what would be the maximum permissible powder pressure? *Ans.* 36,132 lbs. per sq. in.

6. A boiler 6 feet in interior diameter is required to withstand

a steam pressure of 350 lbs. per sq. in. The elastic limit of the metal is 20,000 lbs. per sq. in. What is the maximum thickness required in the shell? *Ans.* 0.64 inches.

7. The cylinder of a hydraulic jack has an interior diameter of 10 inches and a maximum working pressure of 10,000 lbs. per sq. in. The elastic limit of the metal is 40,000 lbs. per sq. in. What thickness of wall is required in order that the factor of safety may be $1\frac{1}{2}$? *Ans.* 2.9 inches.

115. Compound Cylinder, Built-up Guns.—It has been shown that the resistance of a cylinder to an interior pressure may be greatly increased by the application of pressure on the exterior of the cylinder. This is accomplished in practice by shrinking a second cylinder over the first. The shrinkage causes a uniform pressure over the exterior of the inner cylinder and an equal uniform pressure on the interior of the outer cylinder.

The exterior pressure strengthens the inner cylinder against an interior pressure, and at the same time weakens the outer cylinder.

That the full strength of the compound cylinder may be utilized it is important that the shrinkage, and therefore the pressure at the surfaces in contact, be so regulated that under the action of an interior pressure the interior of the weakened outer cylinder will not be stretched to its elastic limit before the inner cylinder has reached that limit. Otherwise we cannot employ the full strength of the inner cylinder. And if the inner cylinder is strained to the elastic limit before the outer cylinder, we cannot employ the full strength of the outer cylinder.

We have seen in Fig. 39 that the tangential stress produced in a single cylinder by an interior pressure diminishes in value as the thickness of the cylinder increases. It is therefore apparent that the stress transmitted to the outer cylinder may, by giving proper thickness to the inner cylinder, be so reduced that when added to the initial stress existing in the outer cylinder this cylinder will not be strained beyond its elastic limit. And by adjusting the thicknesses of the two cylinders and the pressure produced by the shrinkage, the system may be so constructed that the cylinders composing it will both be strained to the elastic limit at the same time.

There is evidently then a relation between the thicknesses of the cylinders and the shrinkage that must be applied in order that the inner and outer cylinders shall be stretched to their elastic limits by the same interior pressure. This relation must be established if we desire to utilize the full elastic strength of the cylinders. And if a third and a fourth cylinder are added the proper relation between the thickness and the shrinkage must be established for these as well.

A modern gun is built up of a number of cylinders assembled by shrinkage, the number of the cylinders, from two to four, depending upon the size and power of the gun. The shrinkage of each cylinder is so adjusted that under the action of the powder pressure, if the pressure becomes sufficiently great, all the cylinders will be strained to the elastic limit at once.

When the powder pressure is acting in a compound cylinder the system is said to be *in action*. When the powder pressure is not acting the system is *at rest*. In action each elementary cylinder except the outer one is subjected to both interior and exterior pressures. At rest the inner cylinder is subjected to exterior pressure only, the outer cylinder to interior pressure only, and the intermediate cylinders to both pressures.

116. System Composed of Two Cylinders.—Assume a system so assembled that under the action of an interior pressure both cylinders will be strained to their elastic limits.

Let R_0, R_1, R_2, Fig. 44, be the radii of the successive surfaces from the interior outwards,

P_0, P_1, P_2, the normal pressures on the successive surfaces when the system is in action,

p_0, p_1, p_2, variations in P_0, P_1, P_2, as the system passes from a state of action to a state of rest,

θ_0, θ_1, the tensile elastic limits of the inner and outer cylinders respectively,

ρ_0, ρ_1, the compressive elastic limits.

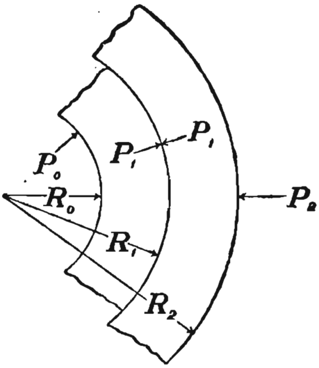

FIG. 44.

E, the modulus of elasticity, assumed the same for both cylinders,

P_{1s}, the normal pressure at the surface of contact *wher the system is at rest.*

Application of Formulas to Outer Cylinders.—It will be well, before proceeding further, to show how the formulas deduced for a single cylinder are made applicable to outer cylinders in compound systems.

Thus equation (23)

$$P_{0\theta}=\frac{3(a-1)\theta_0+6aP_1}{4a+2} \tag{31}$$

gives the value of the limiting pressure in a single cylinder when the pressure P_1 acts on the exterior.

Let us make this apply to the second cylinder of a compound system.

Substituting for a its value R_1^2/R_0^2 and clearing of fractions in numerator and denominator,

$$P_{0\theta}=\frac{3(R_1^2-R_0^2)\theta_0+6R_1^2P_1}{4R_1^2+2R_0^2} \tag{32}$$

Now to apply this equation to the second cylinder change all zero subscripts to 1, and subscripts 1 to 2. Making these changes, dividing numerator and denominator by R_0^2, we obtain, since $R_1^2/R_0^2=a$ and $R_2^2/R_0^2=b$,

$$P_{1\theta}=\frac{3(b-a)\theta_1+6bP_2}{4b+2a} \tag{33}$$

Comparing this equation with (31), from which it has been deduced, we see that the transformation may be immediately made by substituting b for a, and by writing a after the numerical quantities that are affected when we substitute R_1^2/R_0^2 for a and clear of fractions.

We have made this transformation under transformation rule 1, page 197. In equation (31) the numerator forms but one *term factor* and the denominator another. As R_0^2 does not appear in

(31) we know that the equation from which it is derived, equation (32), is of the same form.

The following equation, which refers to pressures in the inner cylinder of a compound system,

$$p_1=\frac{(b-a)}{a(b-1)}p_0 \quad \text{becomes} \quad p_2=\frac{a(c-b)}{b(c-a)}p_1 \qquad (34)$$

for the second cylinder, since the absence of R_0^2 in the first equation indicates that its original equation had two term factors involving the squares of the radii in the numerator as well as in the denominator. Therefore consider $1=R_0^2/R_0^2$ as present as a term factor in the numerator of the first equation, change to R_1^2/R_0^2, and write a for this quantity in the second equation.

The equation

$$P_{0\theta}=3(a-1)\theta_0+2aP_1$$

becomes, if made applicable to the second cylinder,

$$P_{1\theta}=\frac{3(b-a)\theta_1+2bP_2}{a} \qquad (35)$$

since the absence of R_0^2 indicates that the original equation had one term factor in the denominator as well as in the numerator.

Equation (13) is, for the first cylinder,

$$El_t=S_t=\frac{2(P_0-aP_1)}{3(a-1)}+\frac{4a(P_0-P_1)}{3(a-1)}\frac{R_0^2}{r^2}$$

and becomes for the second cylinder

$$El_t=S_t=\frac{2(aP_1-bP_2)}{3(b-a)}+\frac{4ab(P_1-P_2)}{3(b-a)}\frac{R_0^2}{r^2} \qquad (36)$$

Under transformation rule 2 the presence of R_0^2 in the numerator of the last term indicates that the original term had two term factors involving the squares of the limiting radii in the numerator and one in the denominator. Therefore supply the missing factor $1=R_0^2/R_0^2$, change to R_1^2/R_0^2, write a in (36) for this quantity and change the a in (13) to b. R_0^2 is itself not affected in the transformation, as in reality it disappears during the transformation and reappears later by reinsertion.

Whenever in doubt as to a transformation replace the radius ratios by their values, clear the resulting fractions, make the transformation, and rewrite the ratios.

117. **System in Action.**—When the system is in action the outer cylinder is strained to its elastic limit by an interior pressure. The limiting pressure is given by equation (26), changing the subscripts to conform to the nomenclature above.

$$P_{1\theta}=\frac{3(b-a)}{4b+2a}\theta_1 \qquad (37)$$

The pressure $P_{1\theta}$ will extend the inner layer of the outer cylinder to its elastic limit. It is therefore the greatest safe pressure that can be applied to the interior of this cylinder.

The pressure $P_{1\theta}$ just found also acts upon the exterior of the inner cylinder, and the pressure P_0 upon the interior. For the limiting values of the interior pressure we have, under these circumstances, from equations (23) and (24),

$$P_{0\theta}=\frac{3(a-1)\theta_0+6aP_{1\theta}}{4a+2} \qquad (38)$$

$$P_{0\rho}=\frac{3(a-1)\rho_0+2aP_{1\theta}}{4a-2} \qquad (39)$$

The smaller of these values as determined by the test, equation (25), must be used as the limiting interior pressure. Acting with the pressure $P_{1\theta}$ it brings the inner layer of the inner cylinder to its elastic limit of tension or compression according as $P_{0\theta}$ or $P_{0\rho}$ is the less. At the same time the pressure $P_{1\theta}$ stretches the inner layer of the outer cylinder to its elastic limit.

Equation (37), containing in the second member known quantities only, is solved first, and the value of $P_{1\theta}$ obtained is substituted in equation (38) or (39) as determined by the test. The maximum permissible value of P_0 results.

118. System at Rest.—We have seen in Figs. 40 and 41 that an exterior pressure acting alone on a cylinder may produce a greater stress than when an interior pressure is also acting.

It may be, therefore, that the pressure $P_{1\theta}$ deduced as a safe pressure for the system in action may produce a higher pressure than the inner cylinder can safely withstand when the system is at rest, that is, when the interior pressure P_0 is zero. This must be determined before we can assume, as safe values for the pressures, the values obtained from the consideration of the system in action.

As the system passes from a state of action to a state of rest variations occur in the pressures acting, and consequent variations in the stresses at the various surfaces. p_0 and p_1 represent the variations in the pressures P_0 and P_1 respectively. Since the interior pressure changes from P_0 to 0 we have

$$p_0 = -P_0 \tag{40}$$

because $P_0 - P_0 = 0$; that is, the algebraic sum of the pressure in action and the variation in the pressure is the pressure at rest.

The variations in the tangential stresses due to the variations in the pressures may be determined from equation (13). For the exterior of the inner cylinder, the pressures $-P_0$ and p_1 acting, write $-P_0$ for P_0, p_1 for P_1 and make $r = R_1$.

It will be noticed that when $r = R_1$ in equations (13) and (14) the last factor becomes R_0^2/R_1^2 or $1/a$, which cancels the a in the numerator of the last term.

$$S_t = \frac{-6P_0 - (2a+4)p_1}{3(a-1)} \tag{41}$$

For the outer cylinder equation (13) takes the form of equation (36). For the interior of the outer cylinder, the pressure p_1 acting alone, write p_1 for P_1, make $P_2 = 0$, and $r = R_1$.

$$S_t = \frac{(2a+4b)p_1}{3(b-a)} \tag{42}$$

As the surfaces of contact of the two cylinders form virtually one surface the two values for the variation in the stress at this surface must be equal. Equating the second members of equations (41) and (42) and solving for p_1, we obtain

$$p_1 = -\frac{(b-a)P_0}{a(b-1)} \tag{43}$$

which expresses the relation between the variations in pressure at the interior and exterior of the inner cylinder.

We have designated the pressure at the surface of contact of the two cylinders, system at rest, by P_{1s}. The variation in pressure from the state of action to the state of rest must therefore be

$$p_1 = -(P_{1\theta} - P_{1s}) = P_{1s} - P_{1\theta} \tag{44}$$

because $P_{1\theta} - (P_{1\theta} - P_{1s}) = P_{1s}$. Solving (44) for P_{1s}

$$P_{1s} = P_{1\theta} + p_1$$

and substituting the value of p_1 from equation (43) we obtain

$$P_{1s} = P_{1\theta} - \frac{(b-a)P_0}{a(b-1)} \tag{45}$$

for the value of the pressure on the exterior of the inner cylinder, system at rest.

119. This value of P_{1s} must not exceed the maximum permissible value of an exterior pressure acting alone on the inner cylinder, as given by equation (29).

$$P_{1\rho} = \frac{a-1}{2a}\rho_0 \tag{46}$$

If it does the inner cylinder at rest will be crushed by the pressure applied to strengthen it in action.

The condition that P_{1s} shall not exceed $P_{1\rho}$ may be expressed

$$P_{1s} = P_{1\theta} - \frac{(b-a)P_0}{a(b-1)} \leqq \frac{a-1}{2a}\rho_0 = P_{1\rho}. \tag{47}$$

If the values of $P_{1\theta}$ from equation (37) and of P_0 from (38) or (39) do not fulfill the above conditions these values for the pressures cannot be used for the system in action.

To find the safe values for the pressures in this case we must reduce the value of the first member of (47), $P_{1\theta}$, until it is equal to the second member, P_{1_ρ}. $P_{1\theta}$ becomes then P_1 and we have

$$P_1 = P_{1_\rho} + \frac{(b-a)P_0}{a(b-1)} \qquad (48)$$

This is the relation that must exist between P_1 and P_0 in order that these pressures may be safe for the system at rest.

Equations (38) and (39) express the relations between the safe pressures for the system in action.

If therefore we substitute the lesser value, P_1 from (48), for $P_{1\theta}$ in equations (38) and (39) and solve for P_0 we will obtain the values of P_0 that will be safe both in action and at rest.

$$P_{0\theta} = \frac{3(a-1)\theta_0 + 6aP_{1_\rho}}{(4a+2) - 6\dfrac{b-a}{b-1}} \qquad (49)$$

$$P_{0_\rho} = \frac{3(a-1)\rho_0 + 2aP_{1_\rho}}{(4a-2) - 2\dfrac{b-a}{b-1}} \qquad (50)$$

The lesser of these two values will be the limiting safe interior pressure that can be applied to the system.

Assuming θ and ρ equal, we will find by equating the second members of equations (49) and (50) that $P_{0\theta}$ will be less than, equal to, or greater than P_{0_ρ} according as

$$a(b-1)P_{1_\rho} \overset{<}{\underset{>}{=}} \frac{3}{4}(a-1)\theta_0 \qquad (51)$$

120. Graphic Representation. System at Rest and in Action.—Equation (43) expresses the value of the variation p_1 in the exterior pressure for a variation P_0 in the interior pressure. Dropping the negative sign for convenience this equation may be written, for a given cylinder,

$$p_1 = KP_0$$

and may be represented by the line p_1P_0 in Fig. 45. The variation in exterior pressure increases directly with the interior pressure at a rate represented by the inclination of the line p_1P_0.

The lines $P_1P_{0\theta}$ and $P_1P_{0\rho}$ represent, as in Fig. 43, the coordinate limiting pressures for the inner cylinder. $P_{1\theta}$ is the limiting pressure at the surface of contact in action obtained from equation (37). Considering only the tangential stresses, the abscissa of the point c, $P_0 = 42{,}955$, is the limiting value of the interior pressure in action. As the system passes from action to rest the exterior pressure falls at the rate represented by the inclination of the line p_1P_0. Therefore drawing through c a line parallel to p_1P_0, the point where it cuts the axis P_1 will be the value of P_{1s}, the pressure at rest, P_0 being zero at this point. If the value of P_{1s} is less than $P_{1\rho}$, the limiting value of the pressure at rest calculated from equation (46), the value $P_{1\theta}$ is a safe value. If P_{1s} is greater than $P_{1\rho}$ we cannot use $P_{1\theta}$ in action. In this case we would find the permissible value of P_1 in action by drawing a line from $P_{1\rho}$ parallel to p_1P_0. Its intersection with $P_1P_{0\theta}$ would give the values of the coordinate limiting exterior and interior pressures in action.

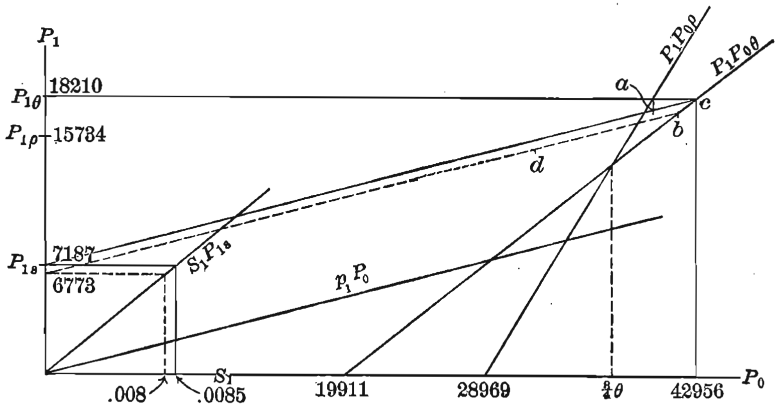

FIG. 45.

121. Maximum Value of the Safe Interior Pressure in a Compound Cylinder.—The stresses and strains produced by any pressure applied to a compound cylinder are exactly the same as would be produced by the same pressure applied to a single cylinder of the same dimensions.

The resultant stresses in the compound cylinder are the algebraic sums of the stresses already existing in the cylinder and those induced by the application of the pressure, and similarly for the strains.

As the resultant stresses may never exceed the elastic limits of extension and compression, the maximum permissible pressure in any cylinder is given by equation (28).

Changing a into b to make of the compound cylinder a single cylinder whose outer radius is R_2, we have

$$P_{0\theta}=\frac{3(b-1)}{4b+2}(\theta+\rho)$$

Making $R_2=\infty$, and therefore $b=R_2^2/R_0^2=\infty$, we obtain

$$P_{0\theta}=\tfrac{3}{4}(\theta_0+\rho_0)$$

which is the greatest possible value of the safe interior pressure in a compound cylinder.

The same result is obtained by substituting $\theta_0+\rho_0$ for θ in equation (27).

122. Shrinkage.—*The absolute shrinkage* is the difference between the exterior diameter of the inner cylinder and the interior diameter of the outer cylinder before heated for assembling, $2ab$, Fig. 46.

The relative shrinkage is the absolute shrinkage divided by the diameter, or the shrinkage per unit of length, ab/R_1. The shrinkages are so small that it is unnecessary to distinguish between the lengths of the radii as affected by the shrinkage.

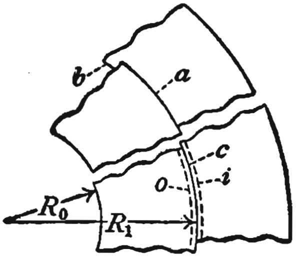

FIG. 46.

The shrinkage diminishes the exterior radius of the inner cylinder, when cold, and increases the interior radius of the outer cylinder, so that the radius R_1 of the surfaces in contact is of a length intermediate between the lengths of the original radii.

The relative shrinkage is, Fig. 46,

$$\phi=ab/R_1=\frac{ci+co}{R_1} \qquad (52)$$

The relative compression ci/R_1 is the strain per unit of length produced by the pressure P_{1s} acting on the exterior of the inner cylinder. As the circumference is proportional to the radius the diminution of the circumference per unit of length will be the same as the unit shortening of the radius, and the value of the tangential strain produced by the pressure P_{1s} may be obtained from equation (13), by making $P_0=0$ and $r=R_1$.

$$ci/R_1=l_t=\frac{(2a+4)P_{1s}}{3E(a-1)}$$

The negative sign is omitted, as it simply indicates compression.

The tangential strain co/R_1 at the interior of the outer cylinder is similarly obtained from equation (13), which for the second cylinder takes the form of equation (36). Making $P_1=P_{1s}$, $P_2=0$, and $r=R_1$,

$$co/R_1=l_t=\frac{(2a+4b)P_{1s}}{3E(b-a)}$$

Therefore from equation (52) we have for the relative shrinkage

$$\phi=\frac{2a(b-1)P_{1s}}{E(a-1)(b-a)} \tag{53}$$

The absolute shrinkage is

$$S_1=2R_1\phi=\frac{4R_1a(b-1)P_{1s}}{E(a-1)(b-a)} \tag{54}$$

The exterior diameter of the inner cylinder before shrinkage should be

$$2R_1'=2R_1+S_1 \tag{55}$$

R_1 representing here the interior radius of the outer cylinder before assembling.

The relative tangential compression of the bore due to the shrinkage pressure P_{1s} is found from equation (13) by making $P_0=0$, $P_1=P_{1s}$, and $r=R_0$.

$$l_t=\frac{2aP_{1s}}{E(a-1)}$$